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**SHOCKS AND FRICTIONS  
IN US BUSINESS CYCLES**

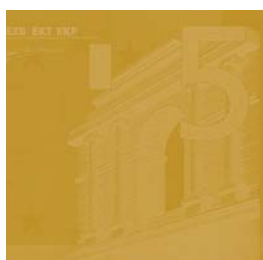
**A BAYESIAN  
DSGE APPROACH**

by Frank Smets  
and Rafael Wouters



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### A BAYESIAN DSGE APPROACH <sup>1</sup>

by Frank Smets <sup>2</sup>  
and Rafael Wouters <sup>3</sup>



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## **Abstract**

Using a Bayesian likelihood approach, we estimate a dynamic stochastic general equilibrium model for the US economy using seven macro-economic time series. The model incorporates many types of real and nominal frictions and seven types of structural shocks. We show that this model is able to compete with Bayesian Vector Autoregression models in out-of-sample prediction. We investigate the relative empirical importance of the various frictions. Finally, using the estimated model we address a number of key issues in business cycle analysis: What are the sources of business cycle fluctuations? Can the model explain the cross-correlation between output and inflation? What are the effects of productivity on hours worked? What are the sources of the “Great Moderation”?

**Key words:** DSGE models; monetary policy

**JEL-classification:** E4-E5

## Non-technical summary

Over the past decade, a new generation of small-scale monetary micro-founded business cycle models with sticky prices and wages (the New Keynesian or New Neoclassical Synthesis (NNS) models) has become popular in monetary policy analysis. This paper estimates an extended version of these models on US data covering the period 1966:1-2004:4 and using a Bayesian estimation methodology. The estimated model contains many shocks and frictions. It features sticky nominal price and wage setting that allow for backward inflation indexation, habit formation in consumption and investment adjustment costs that create hump-shaped responses of aggregate demand, and variable capital utilisation and fixed costs in production. The stochastic dynamics is driven by seven orthogonal structural shocks: total factor productivity shocks, risk premium shocks, investment-specific technology shocks, wage mark-up shocks, price mark-up shocks, exogenous spending shocks and monetary policy shocks.

The objectives of the paper are threefold. First, as the NNS models have become the standard workhorse for monetary policy analysis, it is important to verify whether they can explain the main features of the US macro data: real GDP, hours worked, consumption, investment, real wages, prices and the short-term nominal interest rate. We show that the NNS model has a fit comparable to that of Bayesian VAR models. These results are confirmed by a simple out-of-sample forecasting exercise. The restrictions implied by the NNS model lead to an improvement of the forecasting performance compared to standard VARs, in particular, at medium-term horizons. Bayesian NNS models therefore combine a sound, micro-founded structure suitable for policy analysis with a good probabilistic description of the observed data and good forecasting performance.

Second, the introduction of a large number of frictions raises the question whether each of those frictions are really necessary to describe the seven data series. The Bayesian estimation methodology provides a natural framework for testing which frictions are empirically important by comparing the marginal

likelihood of the various models. We find that price and wage stickiness are found to be equally important. Indexation, on the other hand, is relatively unimportant in both goods and labour markets. While all the real frictions help in reducing the prediction errors of the NNS model, empirically the most important are the investment adjustment costs. In the presence of wage stickiness, the introduction of variable capacity utilisation is less important.

Finally, we use the estimated NNS model to address a number of key business cycle issues. First, what are the main driving forces of output developments in the US? We find that “demand” shocks such as the risk premium, exogenous spending and investment-specific technology shocks explain a significant fraction of the short-run forecast variance in output. However, wage mark-up (or labour supply) and to a lesser extent productivity shocks explain most of its variation in the medium to long run. Second, do positive productivity shocks increase or reduce employment. We find that they have a significant short-run negative impact on hours worked. This is the case even in the economy with flexible prices and wages because of the slow adjustment of the two demand components following a positive productivity shock. Third, inflation developments are mostly driven by the price mark-up shocks in the short run and the wage mark-up shocks in the long run. Nevertheless, the model is able to capture the cross correlation between output and inflation at business cycle frequencies. Finally, in order to investigate the stability of the results, we estimate the NNS model for two subsamples: the “Great Inflation” period from 1966:2 to 1979:2 and the “Great Moderation” period from 1984:1-2004:4. We find that most of the structural parameters are stable over those two periods. The biggest difference concerns the variances of the structural shocks. In particular, the standard deviations of the productivity, monetary policy and price mark-up shocks seem to have fallen in the second sub sample, explaining the fall in the volatility of output growth and inflation in this period. We also detect a fall in the monetary policy response to output developments in the second sub-period.

## 1. Introduction

A new generation of small-scale monetary business cycle models with sticky prices and wages (the New Keynesian or New Neoclassical Synthesis (NNS) models) has become popular in monetary policy analysis.<sup>1</sup> Following Smets and Wouters (2003), this paper estimates an extended version of these models, largely based on Christiano, Eichenbaum and Evans (CEE, 2005), on US data covering the period 1966:1-2004:4 and using a Bayesian estimation methodology. The estimated model contains many shocks and frictions. It features sticky nominal price and wage setting that allow for backward inflation indexation, habit formation in consumption and investment adjustment costs that create hump-shaped responses of aggregate demand, and variable capital utilisation and fixed costs in production. The stochastic dynamics is driven by seven orthogonal structural shocks. In addition to total factor productivity shocks, the model includes two shocks that affect the intertemporal margin (risk premium shocks and investment-specific technology shocks), two shocks that affect the intratemporal margin (wage and price mark-up shocks), and two policy shocks (exogenous spending and monetary policy shocks). Compared to the model used in Smets and Wouters (2003), there are three main differences. First, the number of structural shocks is reduced to the number of seven observables used in estimation. For example, there is no time-varying inflation target, nor a separate labour supply shock. Second, the model features a deterministic growth rate driven by labour-augmenting technological progress, so that the data do not need to be detrended before estimation. Third, the Dixit-Stiglitz aggregator in the intermediate goods and labour market is replaced by the more general aggregator developed in Kimball (1995). This aggregator implies that the demand elasticity of differentiated goods and labour depends on their relative price. As shown in Eichenbaum and Fischer (forthcoming), the introduction of this real rigidity allows us to estimate a more reasonable degree of price and wage stickiness.

The objectives of the paper are threefold. First, as the NNS models have become the standard workhorse for monetary policy analysis, it is important to verify whether they can explain the main features of the US macro data: real GDP, hours worked, consumption, investment, real wages, prices and the short-term nominal interest rate. CEE (2005) show that a version of the model estimated in this paper can replicate

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<sup>1</sup> See Goodfriend and King (1997), Rotemberg and Woodford (1995), Clarida, Gali and Gertler (1999) and Woodford (2003).



the impulse responses following a monetary policy shock identified in an unrestricted Vector Autoregression (VAR). As in Smets and Wouters (2003), the introduction of a larger number of shocks allows us to estimate the full model using the seven data series mentioned above. The marginal likelihood criterion, which captures the out-of sample prediction performance, is used to test the NNS model against standard and Bayesian VAR models. We find that the NNS model has a fit comparable to that of Bayesian VAR models. These results are confirmed by a simple out-of-sample forecasting exercise. The restrictions implied by the NNS model lead to an improvement of the forecasting performance compared to standard VARs, in particular, at medium-term horizons. Bayesian NNS models therefore combine a sound, micro-founded structure suitable for policy analysis with a good probabilistic description of the observed data and good forecasting performance.

Second, the introduction of a large number of frictions raises the question whether each of those frictions are really necessary to describe the seven data series. For example, CEE (2005) show that once one allows for nominal wage rigidity, there is no need for additional price rigidity in order to capture the impulse responses following a monetary policy shock. The Bayesian estimation methodology provides a natural framework for testing which frictions are empirically important by comparing the marginal likelihood of the various models. In contrast to CEE (2005), price and wage stickiness are found to be equally important. Indexation, on the other hand, is relatively unimportant in both goods and labour markets, confirming the single-equation results of Galí and Gertler (1999). While all the real frictions help in reducing the prediction errors of the NNS model, empirically the most important are the investment adjustment costs. In the presence of wage stickiness, the introduction of variable capacity utilisation is less important.

Finally, we use the estimated NNS model to address a number of key issues. First, what are the main driving forces of output developments in the US? Broadly speaking we confirm the analysis of Shapiro and Watson (1988), who use a structural VAR methodology to examine the sources of business cycle fluctuations. While “demand” shocks such as the risk premium, exogenous spending and investment-specific technology shocks explain a significant fraction of the short-run forecast variance in output, both wage mark-up (or labour supply) and to a lesser extent productivity shocks explain most of its variation in the medium to long run. Second, in line with Galí (1999) and Francis and Ramey (2004), productivity shocks have a significant short-run negative impact on hours worked. This is the case even in the flexible

price economy because of the slow adjustment of the two demand components following a positive productivity shock. Third, inflation developments are mostly driven by the price mark-up shocks in the short run and the wage mark-up shocks in the long run. Nevertheless, the model is able to capture the cross correlation between output and inflation at business cycle frequencies. Finally, in order to investigate the stability of the results, we estimate the NNS model for two subsamples: the “Great Inflation” period from 1966:2 to 1979:2 and the “Great Moderation” period from 1984:1-2004:4. We find that most of the structural parameters are stable over those two periods. The biggest difference concerns the variances of the structural shocks. In particular, the standard deviations of the productivity, monetary policy and price mark-up shocks seem to have fallen in the second sub sample, explaining the fall in the volatility of output growth and inflation in this period. We also detect a fall in the monetary policy response to output developments in the second sub-period.

In the next section, we discuss the linearized DSGE model that is subsequently estimated. In section three, the prior and posterior distribution of the structural parameters and the shock processes are discussed. The model statistics and forecast performance are compared to those of unconstrained VAR (and BVAR) models, in section four. In section five, the empirical importance of the different frictions are discussed. Finally, in section six, we use the estimated model to discuss a number of key issues in business cycle analysis. Section seven contains the concluding remarks.

## **2. The linearized DSGE model**

The DSGE model contains many frictions that affect both nominal and real decisions of households and firms. The model is based on CEE (2005) and Smets and Wouters (2003). As in Smets and Wouters (2005), we extend the model so that it is consistent with a balanced steady state growth path driven by deterministic labour-augmenting technological progress. Households maximise a non-separable utility function with two arguments (goods and labour effort) over an infinite life horizon. Consumption appears in the utility function relative to a time-varying external habit variable. Labour is differentiated by a union, so that there is some monopoly power over wages, which results in an explicit wage equation and allows for the introduction of sticky nominal wages à la Calvo (1983). Households rent capital services to firms and decide how much capital to accumulate given the capital adjustment costs they face. As the

rental price of capital changes, the utilisation of the capital stock can be adjusted at increasing cost. Firms produce differentiated goods, decide on labour and capital inputs, and set prices, again according to the Calvo model. The Calvo model in both wage and price setting is augmented by the assumption that prices that are not re-optimised are partially indexed to past inflation rates. Prices are therefore set in function of current and expected marginal costs, but are also determined by the past inflation rate. The marginal costs depend on wages and the rental rate of capital. Similarly, wages depend on past and expected future wages and inflation.

There are a few differences with respect to the model developed in Smets and Wouters (2005). First, the number of structural shocks is reduced to seven in order to match the number of observables that are used in estimation. Second, in both goods and labour markets we replace the Dixit-Stiglitz aggregator with an aggregator which allows for a time-varying demand elasticity which depends on the relative price as in Kimball (1995). As shown by Eichenbaum and Fischer (forthcoming), the introduction of this real rigidity allows us to estimate a more reasonable degree of price and wage stickiness.

In the rest of this section, we describe the log-linearized version of the DSGE model that we subsequently estimate using US data. All variables are log-linearized around their steady-state balanced growth path. Starred variables denote steady state values.<sup>2</sup> We first describe the aggregate demand side of the model and then turn to the aggregate supply.

The aggregate resource constraint is given by:

$$(1) \quad y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g,$$

Output ( $y_t$ ) is absorbed by consumption ( $c_t$ ), investment ( $i_t$ ), capital-utilisation costs that are a function of the capital utilisation rate ( $z_t$ ) and exogenous spending ( $\varepsilon_t^g$ ).  $c_y$  is the steady-state share of consumption in output and equals  $1 - g_y - i_y$ , where  $g_y$  and  $i_y$  are respectively the steady-state exogenous spending-output ratio and investment-output ratio. The steady-state investment-output ratio in turn equals  $(\gamma - 1 + \delta)k_y$  where  $\gamma$  is the steady-state growth rate,  $\delta$  stands for the depreciation rate of

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<sup>2</sup> Some details of the decision problems faced by the agents in the economy are given in the model appendix. An appendix with the full derivation of the steady state and the linearized model equations is available upon request.

capital and  $k_y$  is the steady-state capital-output ratio. Finally,  $z_y = R_*^k k_y$  where  $R_*^k$  is the steady-state rental rate of capital. We assume that exogenous spending follows a first-order autoregressive process with an IID-Normal error term and is also affected by the productivity shock as follows:  $\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g + \rho_{ga} \eta_t^a$ . The latter is empirically motivated by the fact that in estimation exogenous spending also includes net exports, which may be affected by domestic productivity developments.

The dynamics of consumption follows from the consumption Euler equation and is given by:

$$(2) \quad c_t = c_1 c_{t-1} + (1 - c_1) E_t c_{t+1} + c_2 (l_t - E_t l_{t+1}) - c_3 (r_t - E_t \pi_{t+1} + \varepsilon_t^b)$$

where  $c_1 = \frac{\lambda / \gamma}{1 + \lambda / \gamma}$ ,  $c_2 = \frac{(\sigma_c - 1)(W_*^h L_* / C_*)}{\sigma_c (1 + \lambda / \gamma)}$  and  $c_3 = \frac{1 - \lambda / \gamma}{(1 + \lambda / \gamma) \sigma_c}$ . Current consumption ( $c_t$ )

depends on a weighted average of past and expected future consumption, and on expected growth in hours worked ( $l_t - E_t l_{t+1}$ ), the ex-ante real interest rate ( $r_t - E_t \pi_{t+1}$ ) and a disturbance term  $\varepsilon_t^b$ . Under the assumption of no external habit formation ( $\lambda = 0$ ) and log utility in consumption ( $\sigma_c = 1$ ),  $c_1 = c_2 = 0$  and the traditional purely forward-looking consumption equation is obtained. With steady-state growth, the growth rate  $\gamma$  marginally affects the reduced-form parameters in the linearized consumption equation. When the elasticity of intertemporal substitution (for constant labour) is smaller than one ( $\sigma_c > 1$ ), consumption and hours worked are complements in utility and consumption depends positively on current hours worked and negatively on expected growth in hours worked (see Basu and Kimball, 2002). Finally, the disturbance term  $\varepsilon_t^b$  represents a wedge between the interest rate controlled by the central bank and the return on assets held by the households. A positive shock to this wedge increases the required return on assets and reduces current consumption. At the same time, it also increases the cost of capital and reduces the value of capital and investment, as shown below.<sup>3</sup> This shock has similar effects as so-called net-worth shocks in Bernanke, Gertler and Gilchrist (1999) and Christiano, Motto and Rostagno (2003), which explicitly model the external finance premium. The disturbance is

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<sup>3</sup> This latter effect makes this shock different from a discount factor shock (as in Smets and Wouters, 2003), which only affects the intertemporal consumption Euler equation. In contrast to a discount factor shock, the risk premium shock helps explaining the comovement of consumption and investment.



assumed to follow a first-order autoregressive process with an IID-Normal error term:

$$\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b.$$

The dynamics of investment comes from the investment Euler equation and is given by:

$$(3) \quad i_t = i_1 i_{t-1} + (1 - i_1) E_t i_{t+1} + i_2 q_t + \varepsilon_t^i$$

where  $i_1 = \frac{1}{1 + \beta \gamma^{(1-\sigma_c)}}$ ,  $i_2 = \frac{1}{(1 + \beta \gamma^{(1-\sigma_c)}) \gamma^2 \varphi}$ ,  $\varphi$  is the steady-state elasticity of the capital adjustment

cost function and  $\beta$  is the discount factor applied by households. As in CEE (2005), a higher elasticity of the cost of adjusting capital reduces the sensitivity of investment ( $i_t$ ) to the real value of the existing capital stock ( $q_t$ ). Modelling capital adjustment costs as a function of the change in investment rather than its level introduces additional dynamics in the investment equation, which is useful in capturing the hump-shaped response of investment to various shocks. Finally,  $\varepsilon_t^i$  represents a disturbance to the investment-specific technology process and is assumed to follow a first-order autoregressive process with an IID-Normal error term:  $\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i$ .

The corresponding arbitrage equation for the value of capital is given by:

$$(4) \quad q_t = q_1 E_t q_{t+1} + (1 - q_1) E_t r_{t+1}^k - (r_t - \pi_{t+1} + \varepsilon_t^b)$$

where  $q_1 = \beta \gamma^{-\sigma_c} (1 - \delta) = \frac{1 - \delta}{R_*^k + (1 - \delta)}$ . The current value of the capital stock ( $q_t$ ) depends positively

on its expected future value and the expected real rental rate on capital ( $E_t r_{t+1}^k$ ) and negatively on the ex-ante real interest rate and the risk premium disturbance.

Turning to the supply side, the aggregate production function is given by:

$$(5) \quad y_t = \phi_p (\alpha k_t^s + (1 - \alpha) l_t + \varepsilon_t^a)$$

Output is produced using capital ( $k_t^s$ ) and labour services (hours worked,  $l_t$ ). Total factor productivity ( $\varepsilon_t^a$ ) is assumed to follow a first-order autoregressive process:  $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$ . The parameter  $\alpha$

captures the share of capital in production and the parameter  $\phi_p$  is one plus the share of fixed costs in production, reflecting the presence of fixed costs in production.

As newly installed capital becomes only effective with a one-quarter lag, current capital services used in production ( $k_t^s$ ) are a function of capital installed in the previous period ( $k_{t-1}$ ) and the degree of capital utilisation ( $z_t$ ):

$$(6) \quad k_t^s = k_{t-1} + z_t$$

Cost minimisation by the households that provide capital services implies that the degree of capital utilisation is a positive function of the rental rate of capital:

$$(7) \quad z_t = z_1 r_t^k$$

where  $z_1 = \frac{1-\psi}{\psi}$  and  $\psi$  is a positive function of the elasticity of the capital utilisation adjustment cost function and normalized to be between zero and one. When  $\psi = 1$ , it is extremely costly to change the utilisation of capital and as a result the utilisation of capital remains constant. In contrast, when  $\psi = 0$ , the marginal cost of changing the utilisation of capital is constant and as a result in equilibrium the rental rate on capital is constant as is clear from equation (7).

The accumulation of installed capital ( $k_t$ ) is not only a function of the flow of investment but also of the relative efficiency of these investment expenditures as captured by the investment-specific technology disturbance:

$$(8) \quad k_t = k_1 k_{t-1} + (1 - k_1) i_t + k_2 \varepsilon_t^i$$

with  $k_1 = (1 - \delta) / \gamma$  and  $k_2 = (1 - (1 - \delta) / \gamma) (1 + \beta \gamma^{(1-\sigma_c)}) \gamma^2 \varphi$ .

Turning to the monopolistic competitive goods market, cost minimisation by firms implies that the price mark-up ( $\mu_t^p$ ), defined as the difference between the average price and the nominal marginal cost or the negative of the real marginal cost, is equal to the difference between the marginal product of labour ( $mpl_t$ ) and the real wage ( $w_t$ ):

$$(9) \quad \mu_t^p = mpl_t - w_t = \alpha(k_t^s - l_t) + \varepsilon_t^a - w_t$$

As implied by the second equality in (9), the marginal product of labour is itself a positive function of the capital-labour ratio and total factor productivity.

Due to price stickiness as in Calvo (1983) and partial indexation to lagged inflation of those prices that can not be re-optimised as in Smets and Wouters (2003), prices adjust only sluggishly to their desired mark-up. Profit maximisation by price-setting firms gives rise to the following New-Keynesian Phillips curve:

$$(10) \quad \pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \mu_t^p + \varepsilon_t^p$$

where  $\pi_1 = \frac{\iota_p}{1 + \beta\gamma^{1-\sigma_c}\iota_p}$ ,  $\pi_2 = \frac{\beta\gamma^{1-\sigma_c}}{1 + \beta\gamma^{1-\sigma_c}\iota_p}$  and  $\pi_3 = \frac{1}{1 + \beta\gamma^{1-\sigma_c}\iota_p} \frac{(1 - \beta\gamma^{1-\sigma_c}\xi_p)(1 - \xi_p)}{\xi_p((\phi_p - 1)\varepsilon_p + 1)}$ . Inflation

( $\pi_t$ ) depends positively on past and expected future inflation, negatively on the current price mark-up and positively on a price mark-up disturbance ( $\varepsilon_t^p$ ). The price mark-up disturbance is assumed to follow an ARMA(1,1) process:  $\varepsilon_t^p = \rho_p \varepsilon_t^p + \eta_t^p - \mu_p \eta_{t-1}^p$ , where  $\eta_t^p$  is an IID-Normal price mark-up shock. The inclusion of the MA term is designed to capture the high-frequency fluctuations in inflation.

When the degree of indexation to past inflation is zero ( $\iota_p = 0$ ), equation (10) reverts to a standard purely forward-looking Phillips curve ( $\pi_1 = 0$ ). The assumption that all prices are indexed to either lagged inflation or the steady state inflation rate ensures that the Phillips curve is vertical in the long run. The speed of adjustment to the desired mark-up depends among others on the degree of price stickiness ( $\xi_p$ ), the curvature of the Kimball goods market aggregator ( $\varepsilon_p$ ) and the steady-state mark-up, which in equilibrium is itself related to the share of fixed costs in production ( $\phi_p - 1$ ) through a zero-profit condition. A higher  $\varepsilon_p$  slows down the speed of adjustment because it increases the strategic complementarity with other price setters. When all prices are flexible ( $\xi_p = 0$ ) and the price-mark-up shock is zero, equation (10) reduces to the familiar condition that the price mark-up is constant or equivalently that there are no fluctuations in the wedge between the marginal product of labour and the real wage.

Cost minimisation by firms will also imply that the rental rate of capital is negatively related to the capital-labour ratio and positively to the real wage (both with unitary elasticity):

$$(11) \quad r_t^k = -(k_t - l_t) + w_t$$

In analogy with the goods market, in the monopolistically competitive labour market the wage mark-up will be equal to the difference between the real wage and the marginal rate of substitution between working and consuming ( $mrs_t$ ):

$$(12) \quad \mu_t^w = w_t - mrs_t = w_t - (\sigma_l l_t + \frac{1}{1-\lambda}(c_t - \lambda c_{t-1}))$$

where  $\sigma_l$  is the elasticity of labour supply with respect to the real wage and  $\lambda$  is the habit parameter in consumption.

Similarly, due to nominal wage stickiness and partial indexation of wages to inflation, real wages only adjust gradually to the desired wage mark-up:

$$(13) \quad w_t = w_1 w_{t-1} + (1 - w_1)(E_t w_{t+1} + E_t \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + \varepsilon_t^w$$

$$\text{with } w_1 = \frac{1}{1 + \beta \gamma^{1-\sigma_c}}, w_2 = \frac{1 + \beta \gamma^{1-\sigma_c} \iota_w}{1 + \beta \gamma^{1-\sigma_c}}, w_3 = \frac{\iota_w}{1 + \beta \gamma^{1-\sigma_c}} \text{ and } w_4 = \frac{1}{1 + \beta \gamma^{1-\sigma_c}} \frac{(1 - \beta \gamma^{1-\sigma_c} \xi_w)(1 - \xi_w)}{\xi_w((\phi_w - 1)\varepsilon_w + 1)}.$$

The real wage  $w_t$  is a function of expected and past real wages, expected, current and past inflation, the wage mark-up and a wage-markup disturbance ( $\varepsilon_t^w$ ). If wages are perfectly flexible ( $\xi_w = 0$ ), the real wage is a constant mark-up over the marginal rate of substitution between consumption and leisure. In general, the speed of adjustment to the desired wage mark-up depends on the degree of wage stickiness ( $\xi_w$ ) and the demand elasticity for labour, which itself is a function of the steady-state labour market mark-up ( $\phi_w - 1$ ) and the curvature of the Kimball labour market aggregator ( $\varepsilon_w$ ). When wage indexation is zero ( $\iota_w = 0$ ), real wages do not depend on lagged inflation ( $w_3 = 0$ ). The wage-markup disturbance ( $\varepsilon_t^w$ ) is assumed to follow an ARMA(1,1) process with an IID-Normal error term:  $\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w$ . As in the case of the price mark-up shock, the inclusion of an MA term allows us to pick up some of the high frequency fluctuations in wages.<sup>4</sup>

Finally, the model is closed by adding the following empirical monetary policy reaction function:

$$(14) \quad r_t = \rho r_{t-1} + (1 - \rho)\{r_\pi \pi_t + r_y (y_t - y_t^p)\} + r_{\Delta y} [(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + \varepsilon_t^r$$

<sup>4</sup> Alternatively, we could interpret this disturbance as a labour supply disturbance coming from changes in preferences for leisure.



The monetary authorities follow a generalised Taylor rule by gradually adjusting the policy-controlled interest rate ( $r_t$ ) in response to inflation and the output gap, defined as the difference between actual and potential output (Taylor, 1993). Consistently with the DSGE model, potential output is defined as the level of output that would prevail under flexible prices and wages in the absence of the two “mark-up” shocks.<sup>5</sup> The parameter  $\rho$  captures the degree of interest rate smoothing. In addition, there is also a short-run feedback from the change in the output gap. Finally, we assume that the monetary policy shocks ( $\varepsilon_t^r$ ) follows a first-order autoregressive process with an IID-Normal error term:

$$\varepsilon_t^r = \rho_R \varepsilon_{t-1}^r + \eta_t^r .$$

Equations (1) to (14) determine fourteen endogenous variables:  $y_t, c_t, i_t, q_t, k_t^s, k_t, z_t, r_t^k, \mu_t^p, \pi_t, \mu_t^w, w_t, l_t$  and  $r_t$ . The stochastic behaviour of the system of linear rational expectations equations is driven by seven exogenous disturbances: total factor productivity ( $\varepsilon_t^a$ ), investment-specific technology ( $\varepsilon_t^i$ ), risk premium ( $\varepsilon_t^b$ ), exogenous spending ( $\varepsilon_t^s$ ), price mark-up ( $\varepsilon_t^p$ ), wage mark-up ( $\varepsilon_t^w$ ) and monetary policy ( $\varepsilon_t^r$ ) shocks. Next we turn to the estimation of the model.

### 3. Parameter estimates

The model presented in the previous section is estimated with Bayesian estimation techniques using seven key macro-economic quarterly US time series as observable variables: the log difference of real GDP, real consumption, real investment and the real wage, log hours worked, the log difference of the GDP deflator and the federal funds rate. A full description of the data used is given in the appendix. The corresponding measurement equation is:

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<sup>5</sup> In practical terms, we expand the model consisting of equations (1) to (14) with a flexible-price-and-wage version in order to calculate the model-consistent output gap. Note that the assumption of treating the wage equation disturbance as a wage mark-up disturbance rather than a labour supply disturbance coming from changed preferences has implications for our calculation of potential output.

$$(15) \quad Y_t = \begin{bmatrix} dlGDP_t \\ dlCONS_t \\ dlINV_t \\ dlWAG_t \\ lHOURS_t \\ dlP_t \\ FEDFUNDS_t \end{bmatrix} = \begin{bmatrix} \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{l} \\ \bar{\pi} \\ \bar{r} \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ l_t \\ \pi_t \\ r_t \end{bmatrix}$$

where  $l$  and  $dl$  stand for log and log difference respectively,  $\bar{\gamma} = 100(\gamma - 1)$  is the common quarterly trend growth rate to real GDP, consumption, investment and wages,  $\bar{\pi} = 100(\Pi_* - 1)$  is the quarterly steady-state inflation rate and  $\bar{r} = 100(\beta^{-1}\gamma^{\sigma_c}\Pi_* - 1)$  is the steady-state nominal interest rate. Given the estimates of the trend growth rate and the steady-state inflation rate, the latter will be determined by the estimated discount rate. Finally,  $\bar{l}$  is steady-state hours-worked, which is normalized to be equal to zero.

First, we estimate the mode of the posterior distribution by maximising the log posterior function, which combines the prior information on the parameters with the likelihood of the data. In a second step, the Metropolis-Hastings algorithm is used to get a complete picture of the posterior distribution and to evaluate the marginal likelihood of the model.<sup>6</sup> The model is estimated over the full sample period from 1966:1 till 2004:4. In Section 5.4 we estimate the model over two subperiods (1966:1-1979:2 and 1984:1-2004:4) in order to investigate the stability of the estimated parameters.<sup>7</sup>

<sup>6</sup> See Smets and Wouters (2003) for a more elaborate description of the methodology. A sample of 250.000 draws was created (neglecting the first 10.000 draws). The Hessian resulting from the optimisation procedure was used for defining the transition probability function that generates the new proposed draw. A step size of 0.3 resulted in a rejection rate of 0.65. The resulting sample properties are not sensitive to the step size. Two methods were used to test the stability of the sample. The difference between the means of the two sub-samples (100.000 first and 100.000 last drawings) should be Normal distributed with a standard error proportional to the square of the summed sub-sample variances divided by the sample size, if the draws are i.i.d. distributed. However the drawings of the MH algorithm are highly correlated. By selecting only each  $n$  drawing from the overall sample, the i.i.d. distribution is approximated. For bigger step sizes, the hypothesis of equality in mean of the two sub-samples can no longer be rejected for most of the parameters. The second method to evaluate the stability is a graphical test based on the cumulative mean minus the overall mean (see Bauwens et al, 2000). Starting from a minimal cumulative mean of 25.000 drawings, this ratio remains within a 10% band for all the parameters. In sum, an exact statistical test for the stability of the sample is complicated by the highly autocorrelated nature of the MH-sampler. However from an economic point of view the differences between subsamples and independent samples of size 100.000 or more are negligible.

<sup>7</sup> The data set used generally starts in 1947. However, in previous versions of this paper we found that the first ten years are not representative of the rest of the sample, so that we decided to shorten the sample to 1957:1 – 2004:4. In addition, below in Section 4 we use the first 10 years as a training sample for calculating the marginal likelihood of unconstrained VARs, so that the effective sample starts in 1966:1.

### 3.1 Prior distribution of the parameters

The priors on the stochastic processes are harmonised as much as possible. The standard errors of the innovations are assumed to follow an inverse-gamma distribution with a mean of 0.10 and two degrees of freedom, which corresponds to a rather loose prior. The persistence of the AR(1) processes is beta distributed with mean 0.5 and standard deviation 0.2. A similar distribution is assumed for the MA parameter in the process for the price and wage mark-up. The quarterly trend growth rate is assumed to be Normal distributed with mean 0.4 (quarterly growth rate) and standard deviation 0.1. The steady-state inflation rate and the discount rate are assumed to follow a gamma distribution with a mean of 2.5% and 1% on an annual basis.

Five parameters are fixed in the estimation procedure. The depreciation rate  $\delta$  is fixed at 0.025 (on a quarterly basis) and the exogenous spending-GDP ratio  $g_y$  is set at 18%. Both of these parameters would be difficult to estimate unless the investment and exogenous spending ratios would be directly used in the measurement equation. Three other parameters are clearly not identified: the steady-state mark-up in the labour market ( $\lambda_w$ ), which is set at 1.5, and the curvature parameters of the Kimball aggregators in the goods and labour market ( $\varepsilon_p$  and  $\varepsilon_w$ ), which are both set at 10.

The parameters describing the monetary policy rule are based on a standard Taylor rule: the long run reaction on inflation and the output gap are described by a Normal distribution with mean 1.5 and 0.125 (0.5 divided by 4) and standard errors 0.125 and 0.05 respectively. The persistence of the policy rule is determined by the coefficient on the lagged interest rate rate which is assumed to be Normal around a mean of 0.75 with a standard error of 0.1. The prior on the short run reaction coefficient to the change in the output-gap is 0.125.

The parameters of the utility function are assumed to be distributed as follows. The intertemporal elasticity of substitution is set at 1.5 with a standard error of 0.375; the habit parameter is assumed to fluctuate around 0.7 with a standard error of 0.1 and the elasticity of labour supply is assumed to be around 2 with a standard error of 0.75. These are all quite standard calibrations. The prior on the adjustment cost parameter for investment is set around 4 with a standard error of 1.5 (based on CEE, 2005) and the capacity utilisation elasticity is set at 0.5 with a standard error of 0.15. The share of fixed

costs in the production function is assumed to have a prior mean of 0.25. Finally, there are the parameters describing the price and wage setting. The Calvo probabilities are assumed to be around 0.5 for both prices and wages, suggesting an average length of price and wage contracts of half a year. This is compatible with the findings of Bils and Klenow (2004) for prices. The prior mean of the degree of indexation to past inflation is also set at 0.5 in both goods and labour markets.<sup>8</sup>

### 3.2 Posterior estimates of the parameters

Table 1 gives the mode, the mean and the 5 and 95 percentiles of the posterior distribution of the parameters obtained by the Metropolis-Hastings algorithm.

The trend growth rate is estimated to be around 0.43, which is somewhat smaller than the average growth rate of output per capita over the sample. The posterior mean of the steady state inflation rate over the full sample is about 3% on an annual basis. The mean of the discount rate is estimated to be quite small (0.65% on an annual basis). The implied mean steady state nominal and real interest rates are respectively about 6 % and 3% on an annual basis.

{Insert Table 1a-b}

A number of observations are worth making regarding the estimated processes for the exogenous shock variables (Table 1b). Overall, the data appears to be very informative on the stochastic processes for the exogenous disturbances. The productivity, the government spending and the wage mark-up processes are estimated to be the most persistent with an AR(1) coefficient of 0.95, 0.97 and 0.96 respectively. The mean of the standard error of the shock to the productivity process is 0.45. The high persistence of the productivity and wage mark-up processes implies that at long horizons most of the forecast error variance of the real variables will be explained by those two shocks. In contrast, both the persistence and the standard deviation of the risk premium and monetary policy shock are relatively low (0.18 and 0.12 respectively).

Turning to the estimates of the main behavioural parameters, it turns out that the mean of the posterior distribution is typically relatively close to the mean of the prior assumptions. There are a few notable

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<sup>8</sup> We have analysed the sensitivity of the estimation results to the prior assumptions by increasing the standard errors of the prior distributions of the behavioural parameters by 50 %. Overall, the estimation results are very similar.

exceptions. Both the degree of price and wage stickiness are estimated to be quite a bit higher than 0.5. The average duration of wage contracts is somewhat less than a year; whereas the average duration of price contracts is about 3 quarters. The mean of the degree of price indexation (0.24) is on the other hand estimated to be much less than 0.5.<sup>9</sup> Also the elasticity of the cost of changing investment is estimated to be higher than assumed a priori, suggesting an even slower response of investment to changes in the value of capital. Finally, the posterior mean of the fixed cost parameter is estimated to be much higher than assumed in the prior distribution (1.6) and the share of capital in production is estimated to be much lower (0.19). Overall, it appears that the data is quite informative on the behavioural parameters as indicated by the lower variance of the posterior distribution relative to the prior distribution. Two exceptions are the elasticity of labour supply and the elasticity of the cost of changing the utilisation of capital, where the posterior and prior distributions are quite similar.<sup>10</sup>

Finally, turning to the monetary policy reaction function parameters, the mean of the long-run reaction coefficient to inflation is estimated to be relatively high (2.0). There is a considerable degree of interest rate smoothing as the mean of the coefficient on the lagged interest rate is estimated to be 0.81. Policy does not appear to react very strongly to the output gap level (0.09), but does respond strongly to changes in the output-gap (0.22) in the short run.

#### **4. Forecast performance: comparison with VAR models**

In this Section we compare the out-of-sample forecast performance of the estimated DSGE model with that of various VARs estimated on the same data set. The marginal likelihood, which can be interpreted as a summary statistic for the model's out-of-sample prediction performance, forms a natural benchmark for comparing the DSGE model with alternative specifications and other statistical models.<sup>11</sup> However, as

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<sup>9</sup> When relaxing the prior distributions, it turns out that the degree of wage stickiness rise even more, whereas the degree of price indexation falls by more.

<sup>10</sup> Figures with the prior and posterior distributions of all the parameters are available upon request.

<sup>11</sup> As discussed in Geweke (1998), the Metropolis-Hastings-based sample of the posterior distribution can be used to evaluate the marginal likelihood of the model. Following Geweke (1998), we calculate the modified harmonic mean to evaluate the integral over the posterior sample. An alternative approximation is the Laplace approximation around the posterior mode, which is based on a normal distribution. In our experience the results of both approximations are very close in the case of our estimated DSGE model. This is not too surprising given the generally close correspondence between the histograms of the posterior sample and the normal distribution around the estimated mode for the individual parameters. Given the large advantage of the Laplace approximation in terms of computational costs, we will use this approximation for comparing alternative model specifications in the next section.

Sims (2003) has pointed out it is important to use a training sample in order to standardize the prior distribution across widely different models. In order to check for robustness, we also consider a more traditional out-of-sample RMSE forecast exercise in this section.

{Insert Table 2}

Table 2 compares the marginal likelihood of the DSGE model and various unconstrained VAR models, all estimated over the full sample period (1966:1 – 2004:4) and using the period 1956:1 – 1965:4 as a training sample. Several results are worth emphasizing. First, the tightly parameterized DSGE model performs much better than an unconstrained VAR in the same vector of observable variables,  $Y_t$  (first column of Table 2). The bad empirical performance of unconstrained VARs may not be too surprising, as it is known that over-parameterized models typically perform poorly in out-of-sample forecast exercises. One indication of this is that the marginal likelihood of the unconstrained VAR model deteriorates quickly as the lag order increases. For that reason, in the second column of Table 2, we consider the Bayesian VAR model proposed by Sims and Zha (1998). This BVAR combines a Minnesota-type prior (see Litterman, 1984) with priors that take into account the degree of persistence and cointegration in the variables. In order to allow the data to decide on the degree of persistence and cointegration, in this BVAR we enter real GDP, consumption, investment and the real wage in log levels. When setting the tightness of the prior, we choose a set of parameters recommended by Sims (2003) for quarterly data.<sup>12</sup> The second column of Table 2 shows that the marginal likelihood of the Sims-Zha BVAR increases significantly compared to the unconstrained VAR. Moreover, the best BVAR model (BVAR(4)) does as well as the DSGE model.<sup>13</sup>

Overall, the comparison of marginal likelihoods shows that the estimated DSGE model can compete with standard BVAR models in terms of empirical one-step-ahead prediction performance. These results are confirmed by a more traditional out-of sample forecasting exercise reported in Table 3. Table 3 reports out-of-sample RMSEs for different forecast horizons over the period 1990:1 to 2004:4. For this exercise,

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<sup>12</sup> In order to determine the tightness of the priors, we use standard values as suggested in Sims (2003) (See also Sims and Zha, 1998). In particular, the decay parameter is set at 1.0, the overall tightness is set at 10, the parameter determining the weight on the "sum of coefficients" or "own-persistence" is set at 2.0 and the parameter determining the weight on the "co-persistence" at 5. Moreover, the vector of prior standard deviations of the equation shocks is based on the VAR(1) residuals estimated over the training period.

the VAR(1), BVAR(4) and DSGE model were initially estimated over the sample 1966:1 - 1989:4. The models were then used to forecast the seven data series contained in  $Y_t$  from 1990:1 to 2004:4, whereby the VAR(1) and BVAR(4) models were re-estimated every quarter, whereas the DSGE model was re-estimated every year. The measure of overall performance reported in the last column of Table 3 is the log determinant of the uncentered forecast error covariance matrix.

{Insert Table 3}

The out-of-sample forecast statistics confirm the good forecast performance of the DSGE model relative to the VAR and BVAR models. At the one-quarter ahead horizon, the BVAR(4) and the DSGE model improve with about the same magnitude over the VAR(1) model, confirming the results from Table 2. However, over longer horizons up to three years, the DSGE model does considerably better than both the VAR(1) and BVAR(4) model. Somewhat surprisingly, the BVAR(4) model performs worse than the simple VAR(1) model at longer horizons. Moreover, the improvement appears to be quite uniform across the seven macro variables.

## 5. Model sensitivity: which frictions are empirically important?

The introduction of a large number of frictions raises the question which of those are really necessary to capture the dynamics of the data. In this Section we examine the contribution of each of the frictions to the marginal likelihood of the DSGE model.

Table 4 presents the estimates of the mode of the parameters and the marginal likelihood when each of frictions (price and wage stickiness, price and wage indexation, investment adjustment costs and habit formation, capital utilisation and fixed costs in production) are drastically reduced one at a time. This table also gives an idea of the robustness of the parameters and the model performance with respect to the various frictions included in the model. For comparison, the first column reproduces the baseline estimates (mode of the posterior) and the marginal likelihood based on the Laplace approximation.

{Insert Table 4}

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<sup>13</sup> The marginal likelihood can be further increased by optimising the tightness of the “own persistence” prior. For example, setting this parameter equal to 10 increases the marginal likelihood of the BVAR(4) model to -896.

We first focus on the nominal frictions. Reducing the degree of nominal price and wage stickiness to a Calvo probability of 0.10 is about equally costly in terms of a deterioration of the marginal likelihood. In both cases the marginal likelihood falls very significantly by about 50. A lower degree of price stickiness leads to a strong increase in the estimated degree of price indexation from 0.22 to 0.84. Also the variance and the persistence of the price mark-up shocks increases as a result. The other parameters are less affected. The main impact of reducing the degree of wage stickiness on the other parameters concerns the elasticity of wages with respect to employment: the labour supply elasticity becomes much smaller and falls from a value of 1.92 to 0.25. In terms of short run dynamics, these changes more or less cancel out, leaving the impact of labour effort on wage dynamics unaffected. In this case, the variance and the persistence of the wage mark-up shock increases.

While both Calvo frictions are empirically quite important, neither price nor wage indexation do play a very important role in the model dynamics. On the contrary, restricting the price indexation parameter to a very low value of 0.01 leads to an improvement of the marginal likelihood, suggesting that empirically it would be better to leave this friction out. Moreover, leaving out either friction does not have any noticeable impact on the other parameters.

Turning to the real frictions, the most important in terms of the marginal likelihood are the investment adjustment costs. Reducing the elasticity of adjustment costs to a very low level leads to a deterioration of the marginal likelihood by 160. Also reducing habit formation in consumption is quite costly, although much less so than reducing investment adjustment costs. The reduced hump-shaped endogenous dynamics of the model due to these restrictions is compensated mainly by higher and more persistent exogenous shocks to productivity, investment, consumption and government spending. The other real frictions fall, while the nominal rigidities increase. The presence of variable capital utilisation does not seem to matter for the model's performance. Shutting this off comes at no cost. What is costly is to reduce the share of fixed costs in production to 10%. Contrary to the discussion in King and Rebelo (2000), the absence of variable capital utilisation does not increase the standard error of the productivity shock in our model. In contrast, reducing the fixed costs in production does mechanically increase the standard deviation of the productivity shock.



Overall, the results from this sensitivity exercise illustrate that the estimated parameters appear relatively robust to changes in the frictions one by one. Price and wage indexation and variable capital utilisation are of minor importance in terms of the overall empirical performance of the model. On the real side, investment adjustment costs are the most important friction. On the nominal side, both wage and price stickiness are very important.

## 6. Applications

After having shown that the estimated model fits the US macro-economic data quite well, we use it to investigate a number of key macro-economic issues. In this section, we address the following questions. First, what are the main driving forces of output? Second, can the model replicate the cross correlation between output and inflation? Third, what is the effect of a productivity shock on hours worked? Fourth, why have output and inflation become less volatile? We study these issues in each subsection in turn.

### 6.1 What are the main driving forces of output?

Figure 1 gives the forecast error variance decomposition of output, inflation and the federal funds rate at various horizons based on the mode of the model's posterior distribution reported in Section 3. In the short run (within a year) movements in real GDP are primarily driven by the exogenous spending shock and the two shocks that affect the intertemporal Euler equations, i.e. the risk premium shock which affects both the consumption and investment Euler equation and the investment-specific technology shock which affects the investment Euler equation. Together they account for more than 50 percent of the forecast error variance of output up to one year. Each of those shocks can be categorised as “demand” shocks in the sense that they have a positive effect on output, hours worked, inflation and the nominal interest rate under the estimated policy rule. This is illustrated in Figure 2 which shows the estimated mean impulse response functions to each of those three shocks. Not surprisingly, the risk premium shock explains a big part of the short-run variations in consumption, while the investment shock explains the largest part of investment in the short run (not shown).<sup>14</sup>

{Insert Figure 1}

{Insert Figure 2}

However, in line with the results of Shapiro and Watson (1989), it is mostly two “supply” shocks, the productivity and the wage mark-up shock, that account for most of the output variations in the medium to long run. Indeed, even at the two year horizon, together the two shocks account for more than 50% of the variations in output. In the longer run, the wage mark-up shock dominates the productivity shock. Those shocks also become dominant forces in the long-run developments of consumption and to a lesser extent investment. Not surprisingly, the wage-markup shock is also the dominant factor behind long-run movements in hours worked. As shown in Figure 3, a typical positive wage mark-up shock gradually reduces output and hours worked by 0.8 and 0.6 percent respectively. Confirming the large identified VAR literature on the role of monetary policy shocks (e.g. Christiano, Eichenbaum and Evans, 2000), monetary policy shocks contribute only a small fraction of the forecast variance of output at all horizons.

{Insert Figure 3}

Figure 4 shows the historical contribution of each of four types of shocks (productivity, demand, monetary policy and mark-up shocks) to annual output growth over the sample period. It is interesting to compare the main sources of the various recessions over this period. While the recessions of the early 1990s and the beginning of the new millennium are mainly driven by demand shocks, the recession of 1974 is mainly due to positive mark-up shocks (associated with the oil crisis). Monetary policy shocks only play a dominant role in the recession of the early 1980s when the Federal Reserve under the chairmanship of Paul Volker started the disinflation process.

{Insert Figure 4}

## 6.2 Determinants of inflation and the output-inflation cross correlation?

Figure 1 also contains the variance decomposition of inflation. It is quite clear that at all horizons, price and wage mark-ups are the most important drivers of inflation. In the short run, price mark-ups dominate, whereas in the medium to long run wage mark-ups become relatively more important. Even at the medium to long-run horizons, the other shocks only explain a minor fraction of the total variation in inflation. Similarly, monetary policy shocks account for only a small fraction of inflation volatility. This

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<sup>14</sup> The full set of impulse response functions, as well as the associated confidence sets, are available in the appendix on request.

is also clear from Figure 4, which depicts the historical contribution of the different types of shocks to inflation over the sample period. The dominant source of secular shifts in inflation is driven by price and wage mark-up shocks. However, also monetary policy did play a role in the rise of inflation in the 1970s and the subsequent disinflation during the Volker period. Moreover, negative demand shocks did contribute to low inflation in the early 1990s and the start of the new millennium.

There are at least two reasons why the various demand and productivity shocks have only limited effects on inflation. First, the estimated slope of the New Keynesian Phillips curve is very small, so that only large and persistent changes in the marginal cost will have an impact on inflation. Second, and more importantly, under the estimated monetary policy reaction function the Fed responds quite aggressively to emerging output gaps and their impact on inflation. This is reflected in the fact that at the short and medium-term horizon more than 60 percent of variations in the nominal interest rate are due to the various demand and productivity shocks, in particular the risk premium shock (third panel of Figure 1). Only in the long run, does the wage mark-up shock become a dominant source of movements in nominal interest rates.

{Insert Figure 5 }

In the light of these results it is interesting to see to what extent our model can replicate the empirical correlation function between output and inflation as, for example, highlighted in Gali and Gertler (1999). Figure 5 plots the empirical correlation function of output (detrended using the Hodrick-Prescott filter) and inflation (estimated over the period 1966:1-2004:4), as well as the median and the 5 and 95% equivalent generated by the model's posterior distribution. In order to generate this distribution, 1000 draws from the posterior distribution of the model parameters are used to generate artificial samples of output and inflation of the same sample size as the actual data set. For each of those 1000 artificial samples, the autocorrelation function is calculated and the median and 5 and 95 percentiles are derived. Figure 5 clearly shows that the DSGE model is able to replicate both the negative correlation between inflation one to two years in the past and current output and the positive correlation between current output and inflation one year ahead. Moreover, the correlations generated by the DSGE model are significantly different from zero. Decomposing the cross-covariance function in contributions by the different types of shocks, we find that the negative correlation between current inflation and future output

is mostly driven by the price and wage mark-up shocks. In contrast, the positive correlation between the current output gap and future inflation is the result of both demand shocks and mark-up shocks. Monetary policy shocks do not play a role for two reasons. First, they account for only a small fraction of inflation and output developments. Second, as shown in Figure 6, according to the estimated DSGE model the peak effect of a policy shock on inflation occurs before its peak effect on output.

{Insert Figure 6}

### **6.3 The effect of a productivity shock on hours worked**

Following Galí (1999), there has been a lively debate about the effects of productivity shocks on hours worked and about the implications of this finding for the role of those shocks in US business cycles. Galí (1999), Francis and Ramey (2005) and Galí and Rabanal (2004) have argued that due to the presence of nominal price rigidities, habit formation and adjustment costs to investment, positive productivity shocks lead to an immediate fall in hours worked. Given the strongly positive correlation between output and hours worked over the business cycle this implies that productivity shocks can not play an important role in the business cycle. In contrast, using alternative VAR specifications and identification strategies, Christiano et al (2004), Dedola and Neri (2004) and Peersman and Straub (2005) have argued that the empirical evidence on the effect of a productivity shock on hours worked is not very robust and could be consistent with a positive impact on hours worked.

{Insert Figure 7}

In Section 6.1 we have already discussed that productivity shocks play an important, but not dominant role in driving output developments beyond the one year horizon in our estimated model. At business cycle frequencies, they account for about 25 to 30% of the forecast error variance. Figure 7 presents the response of the actual and the flexible-price level of output, hours worked and nominal interest rate to a productivity shock in the estimated model. Overall, the estimates confirm the analysis of Gali (1999) and Francis and Ramey (2004). A positive productivity shock leads to an expansion of aggregate demand, output and real wages, but an immediate and significant reduction in hours worked. Hours worked turn

only significantly positive after two years.<sup>15</sup> Under the estimated monetary policy reaction function, nominal and real interest rates fall, but not enough to prevent the opening up of an output gap and a fall in inflation. Moreover, our estimation results show that it is mainly the estimated degree of habit persistence and the importance of capital adjustment costs that explain the negative impact of productivity on hours worked, thereby confirming the analysis of Francis and Ramey (2004). Indeed, also under flexible prices, hours worked would fall significantly as indicated in the upper right-hand panel of Figure 7. Given these estimates, it is unlikely that a more accommodative monetary policy would lead to positive employment effects. The relatively low medium-run positive effects on hours worked are due to two factors. First, although persistent, the productivity shock is temporary. As a result, output already starts returning to baseline when the effects on hours worked start materialising. A different stochastic process for the productivity shock which implies a gradual introduction of higher total factor productivity could increase the effect on hours worked.<sup>16</sup> Second, a positive productivity shock reduces the fixed cost per unit of production and therefore less labour is required for a given output.

#### **6.4 The “Great Inflation” and the “Great Moderation”: sub-sample estimates**

In this Section we first compare the estimates for two sub-samples in order to investigate the stability of the full-sample estimates and then examine using those estimates why output and inflation volatility has fallen in the most recent period. The first sub-sample, corresponding to the period 1966:2-1979:2, captures the period of the “Great Inflation” and ends with the appointment of Paul Volcker as chairman of the Federal Reserve Board. The second sub-sample, 1984:1-2004:4 captures the more recent period of the “Great Moderation”, in which not only inflation was relatively low and stable, but also output and inflation volatility fell considerably (e.g. McConnell and Perez-Quiros, 2000). Table 5 compares the mode of the posterior distribution of the DSGE model parameters over both periods.

{Insert Table 5}

The most significant differences between the two sub-periods concern the variances of the stochastic processes. In particular the standard errors of the productivity, monetary policy and price mark-up shocks

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<sup>15</sup> This picture does not change very much when we do not allow for a positive effect of productivity on exogenous spending

<sup>16</sup> See for instance Rotemberg (2003) for arguments favoring a slow appearance of major productivity advances in output growth.

(and to a lesser extent the investment shock) seem to have fallen. The persistence of those processes has changed much less. One exception is the risk premium shock which has become even less persistent in the second sub-period.

Somewhat surprisingly, the steady state inflation rate is only marginally lower in the second subperiod (2.6) versus the first period (2.9). What is different is the central bank's reaction coefficient to the output gap, which is halved and is no longer significant in the second period. In contrast, the response to inflation is only marginally higher in the second period and the response to the change in the output gap is the same. These results are consistent with the findings of Orphanides (2003), who shows using real-time data estimates that what has changed in US monetary policy behaviour since the early 1980s is the relative response to output. They are, however, at odds with the results of Boivin and Giannoni (2006), which finds that a stronger central bank response to inflation in the second subperiod can account for a smaller output response to monetary policy shocks estimated in identified VARs. In our case, the lower response to the output gap actually increases the output response of a monetary policy shock in the second period.

Interestingly, it turns out that the degree of price and wage stickiness has increased in the second period, while the degree of indexation has fallen. The latter is consistent with single-equation sub-sample estimates of a hybrid New Keynesian Phillips curve by Gali and Gertler (1999). This finding is also consistent with the story that low and stable inflation may reduce the cost of not adjusting prices and therefore lengthen the average price duration leading to a flatter Phillips curve. At the same time, it may also reduce rule-of-thumb behaviour and indexation leading to a lower coefficient on lagged inflation in the Phillips curve. The effects are most visible in the goods market, less in the labour market. Finally, there is also some limited evidence of increased real rigidities in the second sub-sample. For example, the elasticity of adjusting capital increases from 3.6 to 6.4 in the second sub-sample.

{Insert Table 6}

In order to assess, the sources behind the great moderation of the last two decades, Table 6 provides the results of a counterfactual exercise in which we examine what the standard deviation of output growth and inflation would have been in the most recent period if the US economy had faced the same shocks as in the 1970s, if the monetary policy reaction function as estimated in the pre-1979 period would have

been the same, or if the structure of the economy would have remained unchanged. Table 6 first of all confirms that both output growth and inflation were significantly less volatile in the second sub sample. The estimated DSGE model captures this reduction in volatility, although it overestimates the standard deviation somewhat in both periods. Turning to the counterfactual exercise, it turns out that the most important drivers behind the reduction in volatility are the shocks, which appear to have been more benign in the last period. A reversal to the monetary policy reaction function of the 1970s would have contributed to somewhat higher inflation volatility and lower output growth volatility, but these effects are very small compared to the overall reduction in volatility. Finally, also the changes in the structural parameters do not appear to have contributed to a major change in the volatility of the economy. Overall, these results appear to confirm recent findings of Stock and Watson (2003) and Sims and Zha (2006) that most of the structural change can be assigned to changes in the volatility of the shocks. It remains an interesting research question whether policy has contributed to the reduction of those shocks.

## 7. Concluding remarks

In this paper, we have shown that modern micro-founded NNS models are able to fit the main US macro data very well, if one allows for a sufficiently rich stochastic structure and set of frictions. Our results support the earlier approaches by Rotemberg and Woodford (1997) and Christiano, Eichenbaum and Evans (2005). Although the estimated structural model is highly restricted, it is able to compete with standard VAR and BVAR models in out-of-sample forecasting, indicating that the theory embedded in the structural model is helpful in improving the forecasts of the main US macro variables, in particular at business cycle frequencies.

Of course, the estimated model remains stylised and should be further developed. In particular, a deeper understanding of the various nominal and real frictions that have been introduced would increase the confidence in using this type of models for welfare analysis. Our analysis also raises questions about the deeper determinants of the various “structural” shocks such as productivity and wage mark-up shocks that are identified as being important driving factors of output and inflation developments? However, we hope to have shown that the Bayesian approach followed in this paper offers an effective tool for comparing and selecting between such alternative micro-founded model specifications.

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**Table 1a: Prior and posterior distribution of structural parameters**

	Prior distribution			Posterior distribution			
	Distr.	Mean	St.Dev.	Mode	Mean	5%	95%
$\varphi$	Normal	4.00	1.50	5.48	5.74	3.97	7.42
$\sigma_c$	Normal	1.50	0.37	1.39	1.38	1.16	1.59
$h$	Beta	0.70	0.10	0.71	0.71	0.64	0.78
$\xi_w$	Beta	0.50	0.10	0.73	0.70	0.60	0.81
$\sigma_L$	Normal	2.00	0.75	1.92	1.83	0.91	2.78
$\xi_p$	Beta	0.50	0.10	0.65	0.66	0.56	0.74
$l_w$	Beta	0.50	0.15	0.59	0.58	0.38	0.78
$l_p$	Beta	0.50	0.15	0.22	0.24	0.10	0.38
$\psi$	Beta	0.50	0.15	0.54	0.54	0.36	0.72
$\Phi$	Normal	1.25	0.12	1.61	1.60	1.48	1.73
$r_\pi$	Normal	1.50	0.25	2.03	2.04	1.74	2.33
$\rho$	Beta	0.75	0.10	0.81	0.81	0.77	0.85
$r_y$	Normal	0.12	0.05	0.08	0.08	0.05	0.12
$r_{\Delta y}$	Normal	0.12	0.05	0.22	0.22	0.18	0.27
$\bar{\pi}$	Gamma	0.62	0.10	0.81	0.78	0.61	0.96
$100(\beta^{-1} - 1)$	Gamma	0.25	0.10	0.16	0.16	0.07	0.26
$\bar{L}$	Normal	0.00	2.00	-0.1	0.53	-1.3	2.32
$\bar{\gamma}$	Normal	0.40	0.10	0.43	0.43	0.40	0.45
$\alpha$	Normal	0.30	0.05	0.19	0.19	0.16	0.21

Note: The posterior distributions is obtained using the Metropolis-Hastings algorithm

**Table 1b: Prior and posterior distribution of shock processes**

	Prior distribution			Posterior distribution			
	Distr.	Mean	St.Dev.	Mode	Mean	5%	95%
$\sigma_a$	Invgamma	0.10	2.00	0.45	0.45	0.41	0.50
$\sigma_b$	Invgamma	0.10	2.00	0.24	0.23	0.19	0.27
$\sigma_g$	Invgamma	0.10	2.00	0.52	0.53	0.48	0.58
$\sigma_l$	Invgamma	0.10	2.00	0.45	0.45	0.37	0.53
$\sigma_r$	Invgamma	0.10	2.00	0.24	0.24	0.22	0.27
$\sigma_p$	Invgamma	0.10	2.00	0.14	0.14	0.11	0.16
$\sigma_w$	Invgamma	0.10	2.00	0.24	0.24	0.20	0.28
$\rho_a$	Beta	0.50	0.20	0.95	0.95	0.94	0.97
$\rho_b$	Beta	0.50	0.20	0.18	0.22	0.07	0.36
$\rho_g$	Beta	0.50	0.20	0.97	0.97	0.96	0.99
$\rho_l$	Beta	0.50	0.20	0.71	0.71	0.61	0.80
$\rho_r$	Beta	0.50	0.20	0.12	0.15	0.04	0.24
$\rho_p$	Beta	0.50	0.20	0.90	0.89	0.80	0.96
$\rho_w$	Beta	0.50	0.20	0.97	0.96	0.94	0.99
$\mu_p$	Beta	0.50	0.20	0.74	0.69	0.54	0.85
$\mu_w$	Beta	0.50	0.20	0.88	0.84	0.75	0.93
$\rho_{ga}$	Beta	0.50	0.20	0.52	0.52	0.37	0.66

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

**Table 2: Comparison of the marginal likelihood of alternative VAR models and the DSGE model**

Order of the VAR	No other prior	Sims and Zha (1998) prior
VAR(1)	-928.0	-940.9
VAR(2)	-966.6	-915.8
VAR(3)	-1018.1	-908.7
VAR(4)	-1131.2	-906.6
VAR(5)	-	-907.7
Memo: DSGE model	-905.8	-905.8

Note: In order to increase the comparability of the marginal likelihood of the various models, all models are estimated using the period 1956:1-1965:4 as a training sample (Sims, 2002).

**Table 3: Out-of-sample prediction performance**

	GDP	dP	Fedfunds	Hours	Wage	CONS	INV	Overall
VAR(1)	RMSE-statistic for different forecast horizons							
1q	0.60	0.25	0.10	0.46	0.64	0.60	1.62	-12.87
2q	0.94	0.27	0.18	0.78	1.02	0.95	2.96	-8.19
4q	1.64	0.34	0.36	1.45	1.67	1.54	5.67	-3.25
8q	2.40	0.53	0.64	2.13	2.88	2.27	8.91	1.47
12q	2.78	0.63	0.79	2.41	4.09	2.74	10.97	2.36
BVAR(4)	Percentage gains (+) or losses (-) relative to VAR(1) model							
1q	2.05	14.14	-1.37	-3.43	2.69	12.12	2.54	3.25
2q	-2.12	15.15	-16.38	-7.32	-0.29	10.07	2.42	0.17
4q	-7.21	31.42	-12.61	-8.58	-3.82	1.42	0.43	0.51
8q	-15.82	33.36	-13.26	-13.94	-8.98	-8.19	-11.58	-4.10
12q	-15.55	37.59	-13.56	-4.66	-15.87	-3.10	-23.49	-9.84
DSGE	Percentage gains (+) or losses (-) relative to VAR(1) model							
1q	5.68	2.05	-8.24	0.68	5.99	20.16	9.22	3.06
2q	14.93	10.62	-17.22	10.34	6.20	25.85	16.79	2.82
4q	20.17	46.21	1.59	19.52	9.21	26.18	21.42	6.82
8q	22.55	68.15	28.33	22.34	15.72	21.82	25.95	11.50
12q	32.17	74.15	40.32	27.05	21.88	23.28	41.61	13.51

Notes: All models are estimated starting in 1966:1. The forecast period is 1990:1-2004:4. VAR(1) and BVAR(4) models are re-estimated each quarter; the DSGE model each year. The overall measure of forecast performance is the log determinant of the uncentered forecast error covariance matrix. Gains and losses in the overall measure are expressed as the difference in the overall measure divided by the number of variables and by two to convert the variance to standard errors (times 100).

**Table 4: Testing the empirical importance of the nominal and real frictions in the DSGE model**

	Base	$\xi_p = 0.1$	$\xi_w = 0.1$	$\iota_p = 0.0$	$\iota_w = 0.0$	$\varphi = 0.1$	$h = 0.1$	$\psi = 0.99$	$\Phi = 1.1$
Marginal likelihood									
	-923	-975	-973	-918	-927	-1084	-959	-924	-949
Mode of the structural parameters									
$\varphi$	5.48	4.41	2.78	5.45	5.62	0.10	1.26	5.33	5.19
$\sigma_c$	1.39	1.31	1.80	1.43	1.42	2.78	1.90	1.39	1.27
$h$	0.71	0.70	0.34	0.70	0.71	0.12	0.10	0.70	0.71
$\xi_w$	0.73	0.55	0.10	0.75	0.75	0.89	0.73	0.73	0.78
$\sigma_L$	1.92	1.48	0.25	1.91	1.91	5.24	1.21	1.79	2.33
$\xi_p$	0.65	0.10	0.48	0.66	0.69	0.86	0.62	0.59	0.80
$\iota_w$	0.59	0.71	0.68	0.61	0.01	0.39	0.61	0.63	0.58
$\iota_p$	0.22	0.84	0.24	0.01	0.24	0.08	0.21	0.21	0.19
$\psi$	0.54	0.82	0.66	0.54	0.50	0.02	0.69	0.99	0.45
$\Phi$	1.61	1.79	1.64	1.60	1.61	1.15	1.44	1.62	1.10
$r_\pi$	2.03	2.15	2.15	2.01	2.01	2.03	2.24	2.04	1.98
$\rho$	0.81	0.79	0.75	0.81	0.82	0.84	0.81	0.80	0.80
$r_y$	0.08	0.08	0.08	0.08	0.09	0.23	0.12	0.08	0.10
$r_{\Delta y}$	0.22	0.21	0.25	0.22	0.22	0.30	0.29	0.23	0.25
$\alpha$	0.19	0.21	0.20	0.19	0.19	0.20	0.19	0.18	0.13
Mode of the autoregressive parameters of the exogenous shock processes									
$\rho_a$	0.95	0.96	0.97	0.96	0.95	0.99	0.97	0.96	0.96
$\rho_b$	0.18	0.19	0.67	0.18	0.18	0.89	0.79	0.18	0.28
$\rho_g$	0.97	0.96	0.97	0.97	0.97	0.99	0.97	0.97	0.96
$\rho_l$	0.71	0.71	0.78	0.70	0.69	0.99	0.90	0.73	0.74
$\rho_r$	0.12	0.14	0.13	0.12	0.11	0.02	0.03	0.13	0.11
$\rho_p$	0.90	0.97	0.94	0.88	0.88	0.60	0.93	0.92	0.85
$\rho_w$	0.97	0.98	0.98	0.97	0.97	0.92	0.98	0.97	0.95
$\mu_p$	0.74	0.20	0.71	0.59	0.77	0.34	0.76	0.71	0.67
$\mu_w$	0.88	0.75	0.14	0.91	0.88	0.96	0.95	0.90	0.87

**Table 5: Sub-sample estimates**

	Structural parameters				Shock processes				
	1966:1-1979:2		1984:1-2004:4		1966:1-1979:2		1984:1-2004:4		
	Mode	SD	Mode	SD	Mode	SD	Mode	SD	
$\varphi$	3.61	1.03	6.23	1.12	$\sigma_a$	0.58	0.05	0.35	0.02
$\sigma_c$	1.39	0.22	1.47	0.13	$\sigma_b$	0.22	0.04	0.18	0.02
$h$	0.63	0.07	0.68	0.04	$\sigma_g$	0.54	0.05	0.41	0.03
$\xi_w$	0.65	0.07	0.74	0.13	$\sigma_l$	0.52	0.09	0.39	0.05
$\sigma_L$	1.52	0.65	2.30	0.67	$\sigma_r$	0.20	0.02	0.12	0.01
$\xi_p$	0.55	0.08	0.73	0.04	$\sigma_p$	0.22	0.03	0.11	0.01
$l_w$	0.58	0.13	0.46	0.16	$\sigma_w$	0.20	0.02	0.21	0.03
$l_p$	0.45	0.18	0.21	0.09	$\rho_a$	0.97	0.01	0.94	0.02
$\psi$	0.34	0.13	0.69	0.11	$\rho_b$	0.39	0.17	0.14	0.08
$\Phi$	1.43	0.09	1.54	0.09	$\rho_g$	0.91	0.03	0.96	0.01
$r_\pi$	1.65	0.19	1.77	0.29	$\rho_l$	0.60	0.10	0.64	0.07
$\rho$	0.81	0.03	0.84	0.02	$\rho_r$	0.22	0.10	0.29	0.10
$r_y$	0.17	0.03	0.08	0.05	$\rho_p$	0.51	0.24	0.74	0.13
$r_{\Delta y}$	0.20	0.03	0.16	0.02	$\rho_w$	0.96	0.02	0.82	0.15
$\bar{\pi}$	0.72	0.11	0.67	0.10	$\mu_p$	0.46	0.20	0.59	0.18
$\beta^{-1} - 1$	0.14	0.06	0.12	0.05	$\mu_w$	0.84	0.07	0.62	0.17
$\bar{L}$	0.03	0.62	-0.55	1.21	$\rho_{ga}$	0.58	0.11	0.39	0.11
$\bar{\gamma}$	0.33	0.04	0.44	0.02					
$\alpha$	0.19	0.02	0.21	0.02					

Note: SD stands for standard deviation.

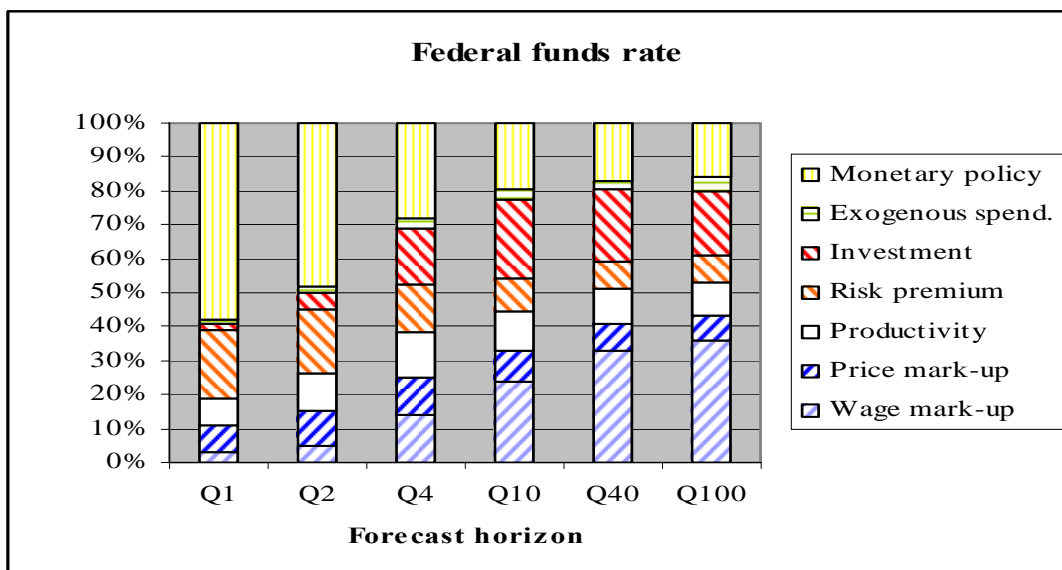
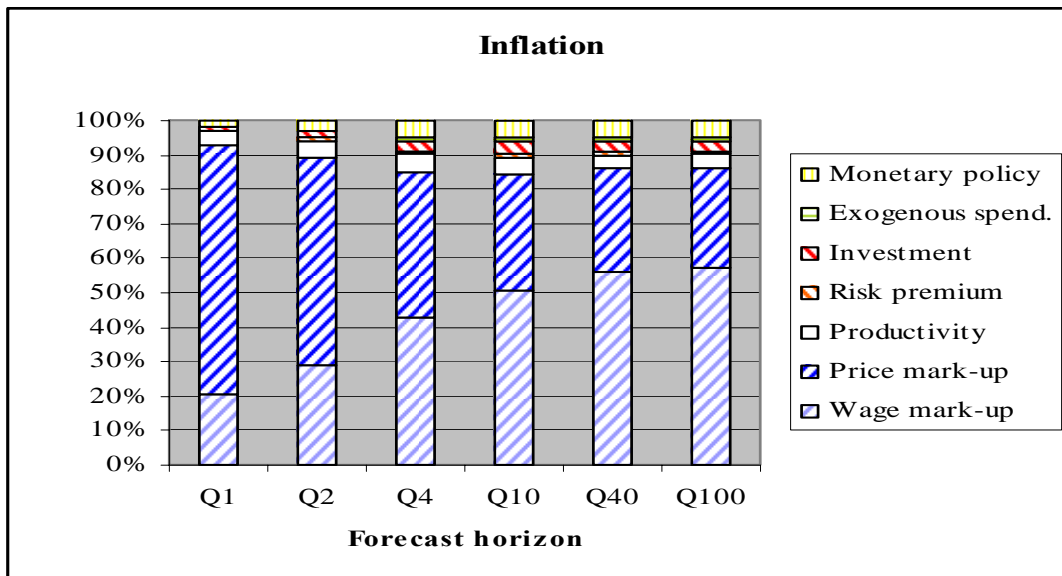
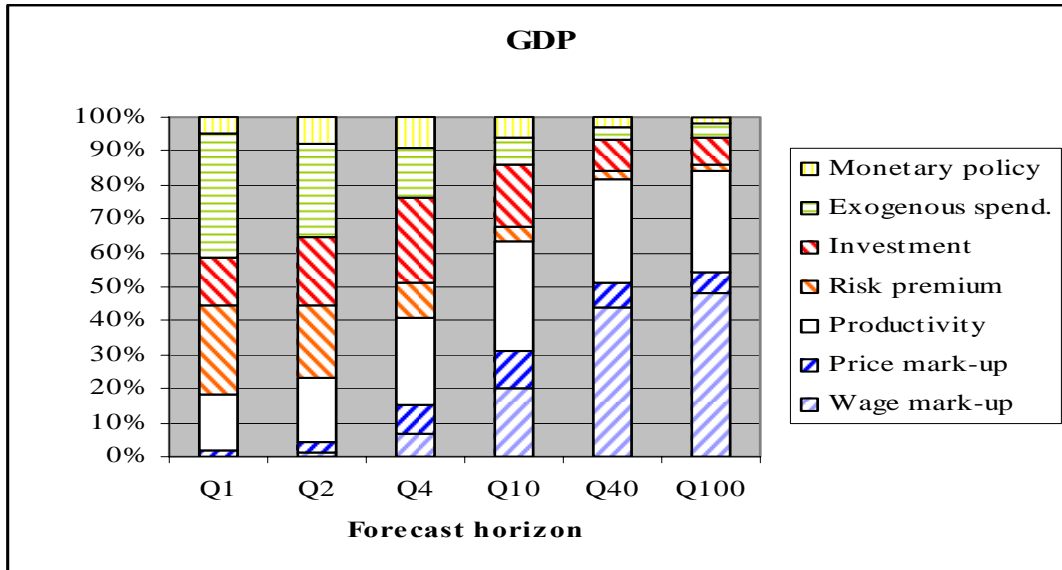


**Table 6: Actual, model-based and counterfactual standard deviations of GDP growth and inflation**

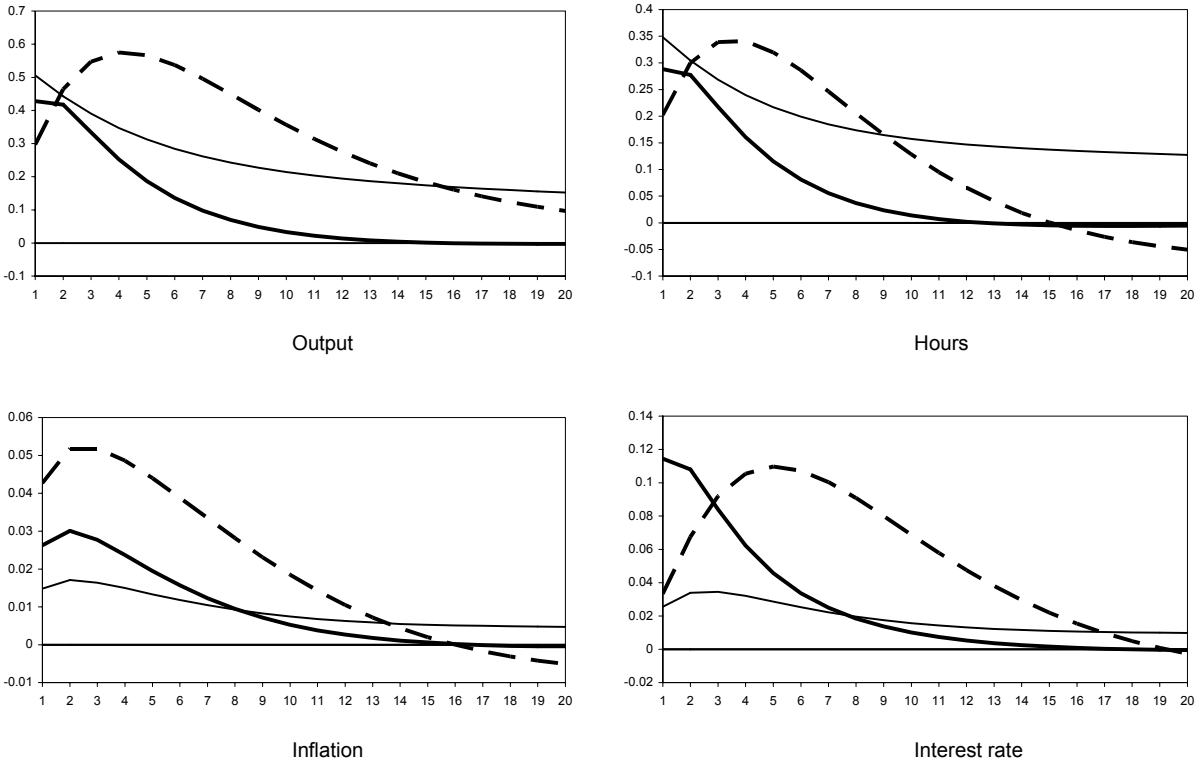
	1966:1-2004:4		1966:1-1979:2		1984:1-2004:4		Counterfactual 1984:1-2004:4		
	Actual	Model	Actual	Model	Actual	Model	Shocks	Policy	Structure
Growth	0.86	0.94	1.01	1.13	0.59	0.73	1.21	0.70	0.75
Inflation	0.62	0.57	0.55	0.81	0.25	0.34	1.30	0.39	0.32

Note: “Actual” refers to the data-based standard deviations over the indicated sample; “Model” refers to the standard deviations generated by the DSGE model estimated over the indicated sample. The counterfactual standard deviations for the period 1984:1-2004:4 refer to the standard deviations that would have occurred in this period if the shock processes (“Shocks”), the monetary policy rule (“Policy”) or the structural parameters (“Structure”) would have been the same as the ones estimated in 1966:1-1979:2 sample.

**Figure 1: Forecast error variance decomposition (at the mode of the posterior distribution)**

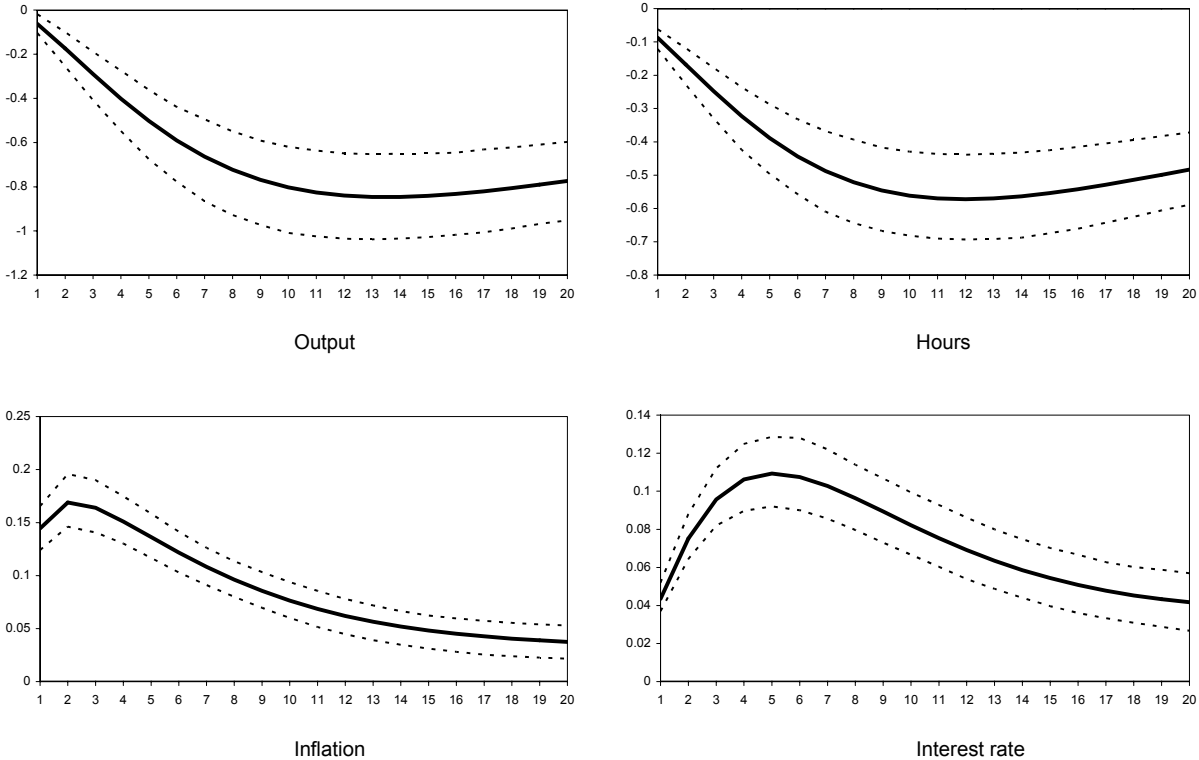


**Figure 2: The estimated mean impulse responses to “demand” shocks**



Notes: Bold solid line: risk premium shock; thin solid line: exogenous spending shock; dashed line: investment shock.

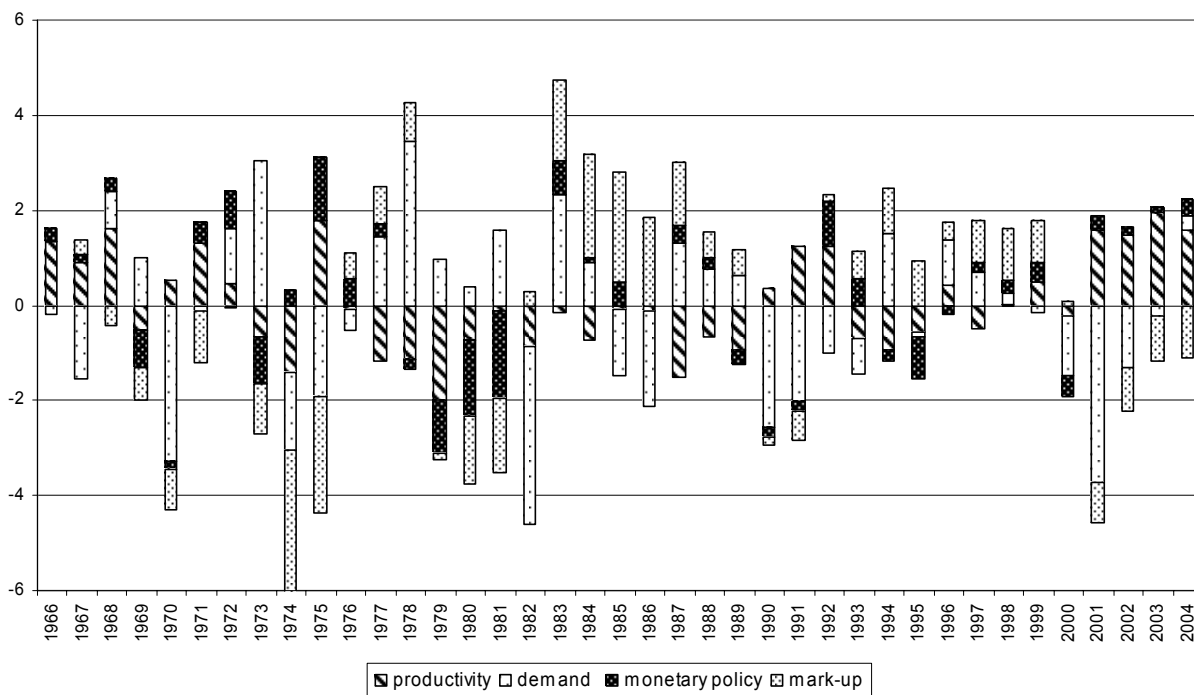
**Figure 3: The estimated impulse response to a wage mark-up shock**



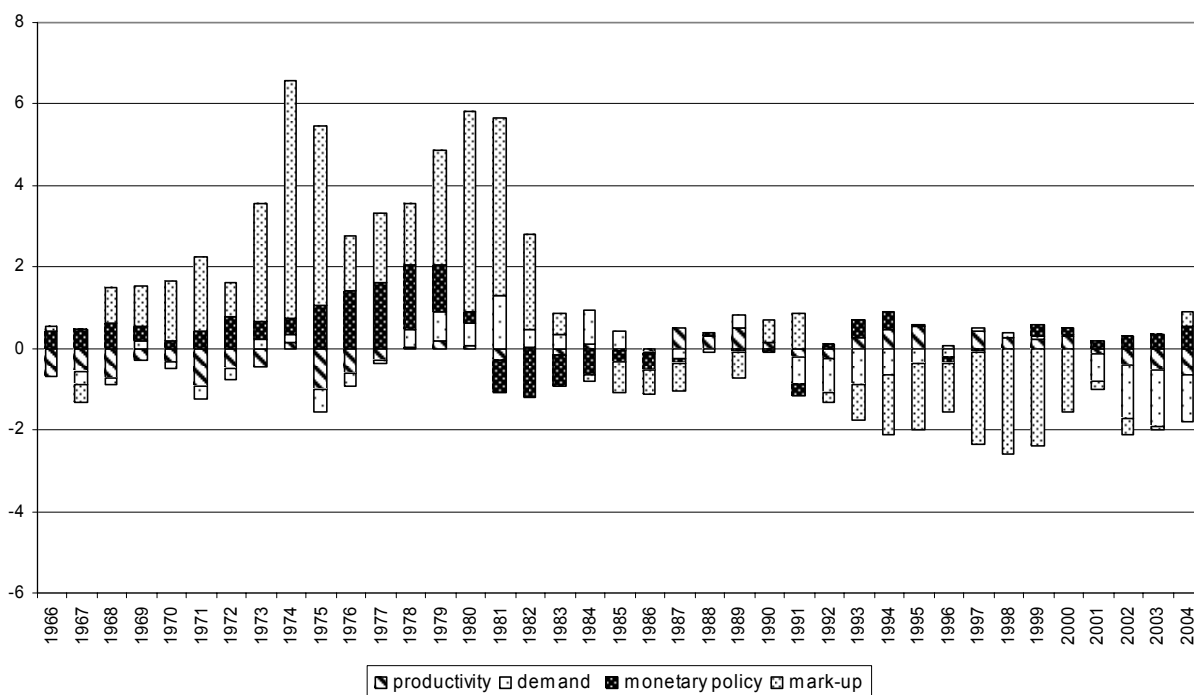
Notes: The solid line is the mean impulse response; the dotted lines are the 10 and 90% posterior intervals

**Figure 4: Historical decomposition of GDP growth and inflation**

Annual per capita GDP growth (deviation from trend growth)

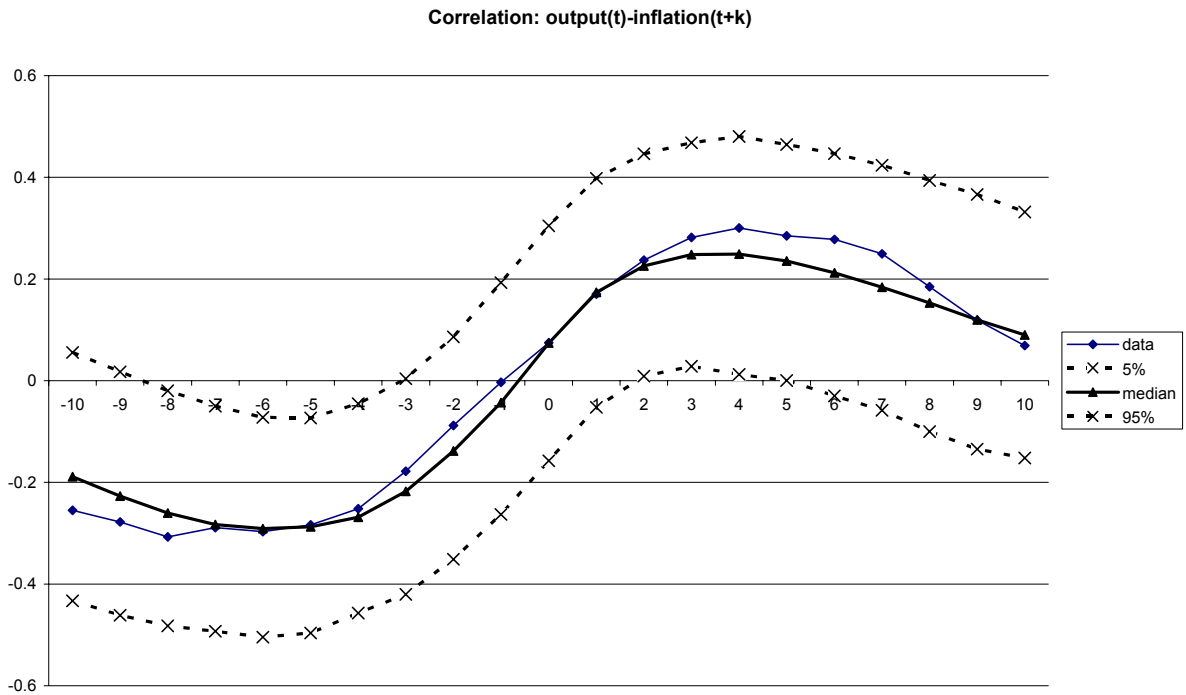


Annual inflation (GDP deflator - deviation from mean)



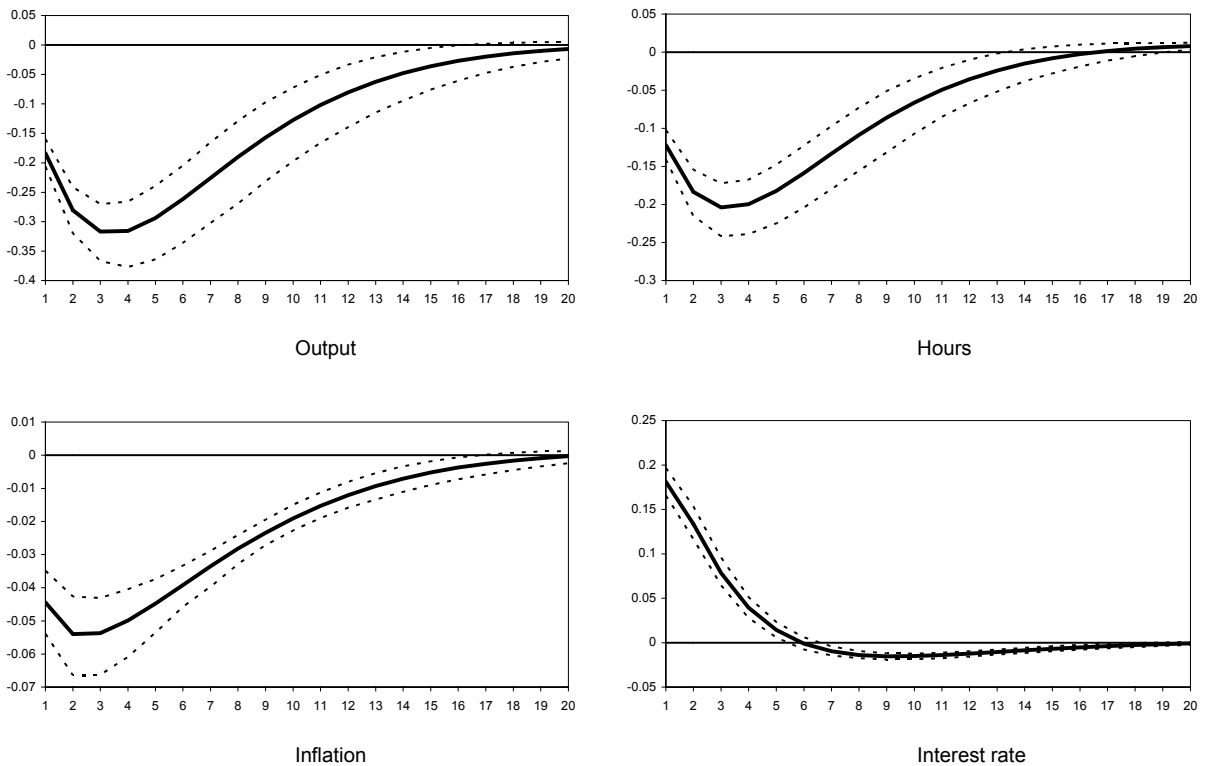
Notes: The demand shocks include the risk premium, investment-specific technology and exogenous spending shocks; the mark-up shocks include the price and wage mark-up shocks. Trend per-capita growth is estimated at 1.73%, whereas mean inflation is estimated at 3.17%.

**Figure 5: The actual and model-based cross-correlation function between output and inflation**



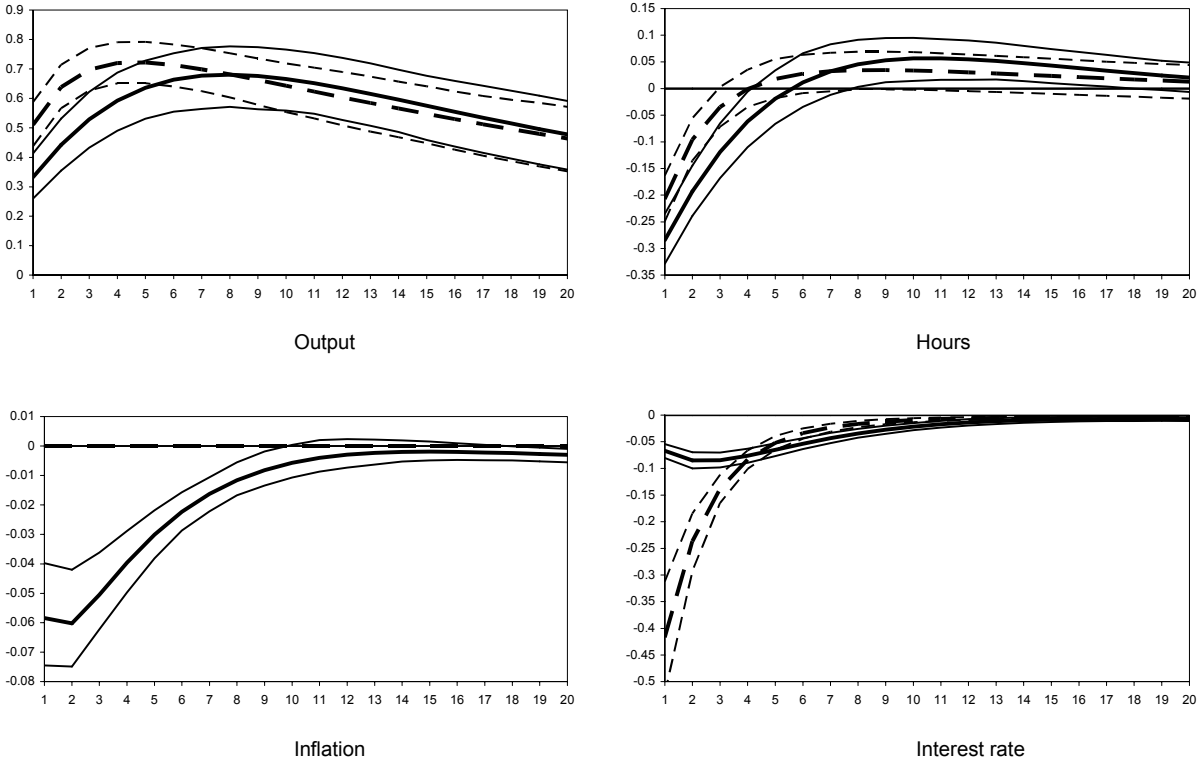
Note: Output is Hodrick-Prescott filtered real GDP.

**Figure 6: The impulse responses to a monetary policy shock**



Notes: The solid line is the mean impulse response; the dotted lines are the 10 and 90% posterior intervals

**Figure 7: The estimated impulse responses to a productivity shock**



Notes: The solid lines represent the estimated actual mean responses and the 10 and 90% posterior interval; the dashed lines represent the counterfactual flexible-wage-and-price responses.

## Data appendix

Equations (1) to (14) are estimated using seven key macro-economic time series: real GDP, consumption, investment, hours worked, real wages, prices and a short-term interest rate. The Bayesian estimation methodology is extensively discussed in Smets and Wouters (2003). GDP, consumption and investment are taken from the US Department of Commerce - Bureau of Economic Analysis databank. Real Gross Domestic Product is expressed in Billions of Chained 1996 Dollars. Nominal Personal Consumption Expenditures and Fixed Private Domestic Investment are deflated with the GDP-deflator.<sup>17</sup> Inflation is the first difference of the log of the Implicit Price Deflator of GDP. Hours and wages come from the BLS (hours and hourly compensation for the NFB sector for all persons). Hourly compensation is divided by the GDP price deflator in order to get the real wage variable. Hours are adjusted to take into account the limited coverage of the NFB sector compared to GDP (the index of average hours for the NFB sector is multiplied with the Civilian Employment (16 years and over)<sup>18</sup>). The aggregate real variables are expressed per capita by dividing with the population over 16. All series are seasonally adjusted. The interest rate is the Federal Funds Rate. Consumption, investment, GDP, wages and hours are expressed in 100 times log. The interest rate and inflation rate are expressed on a quarterly basis corresponding with their appearance in the model (in the figures the series are translated on an annual basis).

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<sup>17</sup> We follow Altig, Christiano, Eichenbaum and Linde (2004) here. This approach avoids the positive trend in the investment share of output that results from the decline in the relative investment expenditures deflator. A fully specified model would start from a two-sector model allowing for a separate trend in technological progress in the investment good sector. In such a two sector model, the relative price of investment and consumption goods can be used to identify the investment specific technological progress. In our one sector model, it was difficult to identify a separate deterministic trend in investment specific technology. However, there are significant short run shocks around this trend that influence investment in the short and medium run.

<sup>18</sup> This correction is also used in Chang, Gomes and Schorfheide (2002).



# Model appendix

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## 1 Decision problems of firms and households and equilibrium conditions

### 1.1 Final goods producers

The final good  $Y_t$  is a composite made of a continuum of intermediate goods  $Y_t(i)$  as in Kimball (1995). The final good producers buy intermediate goods, package them into  $Y_t$ , and sell the final good to consumers, investors and the government in a perfectly competitive market. They maximize profits:

$$\begin{aligned} \max_{Y_t, Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \\ \text{s.t. } \left[ \int_0^1 G \left( \frac{Y_t(i)}{Y_t}; \epsilon_t^p \right) di \right] = 1 \quad (\mu_{f,t}) \end{aligned}$$

where  $P_t$  and  $P_t(i)$  are the price of the final and intermediate goods respectively, and  $G$  is a strictly concave and increasing function characterised by  $G(1) = 1$ .  $\epsilon_t^p$  is an exogenous process that reflects shocks to the aggregator function that result in changes in the elasticity of demand and therefore in the markup. We will constrain  $\epsilon_t^p \in (0, \infty)$ .  $\epsilon_t^p$  follows the exogenous ARMA process:

$$\ln \epsilon_t^p = (1 - \rho_p) \ln \epsilon^p + \rho_p \ln \epsilon_{t-1}^p - \theta_p \eta_{t-1}^p + \eta_t^p, \quad \eta_t^p \sim N(0, \sigma_p)$$

Combining the first-order conditions with respect to  $Y_t(i)$  and  $Y_t$  results in:

$$Y_t(i) = Y_t G'^{-1} \left[ \frac{P_t(i)}{P_t} \int_0^1 G' \left( \frac{Y_t(i)}{Y_t} \right) \frac{Y_t(i)}{Y_t} di \right]$$

As in Kimball (1995), the assumptions on  $G$  imply that the demand for input  $Y_t(i)$  is decreasing in its relative price, while the elasticity of demand is a positive function of the relative price (or a negative function of the relative output).

### 1.2 Intermediate goods producers

Intermediate good producer  $i$  uses the following technology:

$$Y_t(i) = \varepsilon_t^a K_t^s(i)^\alpha [\gamma^t L_t(i)]^{1-\alpha} - \gamma^t \Phi \tag{1}$$

where  $K_t^s(i)$  is capital services used in production,  $L_t(i)$  is a composite labour input and  $\Phi$  is a fixed cost.  $\gamma^t$  represents the labour-augmenting deterministic growth rate in the economy and  $\varepsilon_t^a$  is total factor productivity and follows the process:

$$\ln \varepsilon_t^a = (1 - \rho_z) \ln \varepsilon^a + \rho_z \ln \varepsilon_{t-1}^a + \eta_t^a, \quad \eta_t^a \sim N(0, \sigma_a) \quad (2)$$

The firm's profit is given by:

$$P_t(i)Y_t(i) - W_t L_t(i) - R_t^k K_t^s(i).$$

where  $W_t$  is the aggregate nominal wage rate and  $R_t^k$  is the rental rate on capital.

Cost minimization yields the following first-order conditions:

$$(\partial L_t(i)) : \Theta_t(i) \gamma^{(1-\alpha)t} (1 - \alpha) \varepsilon_t^a K_t^s(i)^\alpha L_t(i)^{-\alpha} = W_t \quad (3)$$

$$(\partial K_t^s(i)) : \Theta_t(i) \gamma^{(1-\alpha)t} \alpha \varepsilon_t^a K_t^s(i)^{\alpha-1} L_t(i)^{1-\alpha} = R_t^k \quad (4)$$

where  $\Theta_t(i)$  is the Lagrange multiplier associated with the production function and equals marginal cost  $MC_t$ .

Combining these FOCs and noting that the capital-labour ratio is equal across firms implies:

$$K_t^s = \frac{\alpha}{1 - \alpha} \frac{W_t}{R_t^k} L_t \quad (5)$$

The marginal cost  $MC_t$  is the same for all firms and equal to:

$$MC_t = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} W_t^{1-\alpha} R_t^k \alpha \gamma^{-(1-\alpha)t} (\varepsilon_t^a)^{-1} \quad (6)$$

Under Calvo pricing with partial indexation, the optimal price set by the firm that is allowed to re-optimize results from the following optimisation problem:

$$\begin{aligned} \max_{\tilde{P}_t(i)} E_t \sum_{s=0}^{\infty} \xi_p^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \left[ \tilde{P}_t(i) (\prod_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p}) - MC_{t+s} \right] Y_{t+s}(i) \\ \text{s.t. } Y_{t+s}(i) = Y_{t+s} G'^{-1} \left( \frac{P_t(i) X_{t,s}}{P_{t+s}} \tau_{t+s} \right) \end{aligned}$$

where  $\tilde{P}_t(i)$  is the newly set price,  $\xi_p$  is the Calvo probability of being allowed to optimize

one's price,  $\pi_t$  is inflation defined as  $\pi_t = P_t/P_{t-1}$ ,  $[\frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}}]$  is the nominal discount factor for firms (which equals the discount factor for the households that are the final owners of the firms),  $\tau_t = \int_0^1 G' \left( \frac{Y_t(i)}{Y_t} \right) \frac{Y_t(i)}{Y_t} di$  and

$$X_{t,s} = \begin{cases} 1 & \text{for } s = 0 \\ (\prod_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p}) & \text{for } s = 1, \dots, \infty \end{cases}$$

The first-order condition is given by:

$$E_t \sum_{s=0}^{\infty} \xi_p^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} Y_{t+s}(i) \left[ X_{t,s} \tilde{P}_t(i) + \left( \tilde{P}_t(i) X_{t,s} - MC_{t+s} \right) \frac{1}{G'^{-1}(z_{t+s})} \frac{G'(x_{t+s})}{G''(x_{t+s})} \right] = 0 \quad (7)$$

where  $x_t = G'^{-1}(z_t)$  and  $z_t = \frac{P_t(i)}{P_t} \tau_t$ .

The aggregate price index is in this case given by:

$$P_t = (1 - \xi_p) P_t(i) G'^{-1} \left[ \frac{P_t(i) \tau_t}{P_t} \right] + \xi_p \pi_{t-1}^{\iota_p} \pi_*^{1-\iota_p} P_{t-1} G'^{-1} \left[ \frac{\pi_{t-1}^{\iota_p} \pi_*^{1-\iota_p} P_{t-1} \tau_t}{P_t} \right] \quad (8)$$

### 1.3 Households

Household  $j$  chooses consumption  $C_t(j)$ , hours worked  $L_t(j)$ , bonds  $B_t(j)$ , investment  $I_t(j)$  and capital utilisation  $Z_t(j)$ , so as to maximise the following objective function:

$$E_t \sum_{s=0}^{\infty} \beta^s \left[ \frac{1}{1 - \sigma_c} (C_{t+s}(j) - \lambda C_{t+s-1})^{1-\sigma_c} \right] \exp \left( \frac{\sigma_c - 1}{1 + \sigma_l} L_{t+s}(j)^{1+\sigma_l} \right)$$

subject to the budget constraint:

$$\begin{aligned} & C_{t+s}(j) + I_{t+s}(j) + \frac{B_{t+s}(j)}{\varepsilon_t^b R_{t+s} P_{t+s}} - T_{t+s} \\ & \leq \frac{B_{t+s-1}(j)}{P_{t+s}} + \frac{W_{t+s}^h(j) L_{t+s}(j)}{P_{t+s}} + \frac{R_{t+s}^k Z_{t+s}(j) K_{t+s-1}(j)}{P_{t+s}} - a(Z_{t+s}(j)) K_{t+s-1}(j) + \frac{Div_{t+s}}{P_{t+s}} \end{aligned} \quad (9)$$

and the capital accumulation equation:

$$K_t(j) = (1 - \delta)_{t-1}(j) + \varepsilon_t^i \left[ 1 - S \left( \frac{I_t(j)}{I_{t-1}(j)} \right) \right] I_t(j) \quad (10)$$

There is external habit formation captured by the parameter  $\lambda$ . The one-period bond is expressed on a discount basis.  $\varepsilon_t^b$  is an exogenous premium in the return to bonds, which might reflect inefficiencies in the financial sector leading to some premium on the deposit rate versus the risk free rate set by the central bank, or a risk premium that households require to hold the one period bond.  $\varepsilon_t^b$  follows the stochastic process:

$$\ln \varepsilon_t^b = \rho_b \ln \varepsilon_{t-1}^b + \eta_t^b, \eta_t^b \sim N(0, \sigma_b) \quad (11)$$

$\delta$  is the depreciation rate,  $S(\cdot)$  is the adjustment cost function, with  $S(\gamma) = 0$ ,  $S'(\gamma) = 0$ ,  $S''(\cdot) > 0$ , and  $\varepsilon_t^i$  is a stochastic shock to the price of investment relative to consumption goods and follows an exogenous process:

$$\ln \varepsilon_t^i = \rho_i \ln \varepsilon_{t-1}^i + \eta_t^i, \eta_t^i \sim N(0, \sigma_i) \quad (12)$$

$T_{t+s}$  are lump sum taxes or subsidies and  $Div_t$  are the dividends distributed by the intermediate goods producers and the labour unions.

Finally, households choose the utilisation rate of capital. The amount of effective capital that households can rent to the firms is:

$$K_t^s(j) = Z_t(j) K_{t-1}(j) \quad (13)$$

The income from renting capital services is  $R_t^k Z_t(j) K_{t-1}(j)$ , while the cost of changing

capital utilisation is  $P_t a(Z_t(j)) K_{t-1}(j)$ .

In equilibrium households will make the same choices for consumption, hours worked, bonds, investment and capital utilization. The first-order conditions can be written as (dropping the  $j$  index):

$$(\partial C_t) \quad \Xi_t = \exp\left(\frac{\sigma_c - 1}{1 + \sigma_l} L_t(j)^{1 + \sigma_l}\right) (C_t - \lambda C_{t-1})^{-\sigma_c} \quad (14)$$

$$(\partial L_t) \quad \left[ \frac{1}{1 - \sigma_c} (C_t - h C_{t-1})^{1 - \sigma_c} \right] \exp\left(\frac{\sigma_c - 1}{1 + \sigma_l} L_t^{1 + \sigma_l}\right) (\sigma_c - 1) L_t^{\sigma_l} = -\Xi_t \frac{W_t^h}{P_t} \quad (15)$$

$$(\partial B_t) \quad \Xi_t = \beta \varepsilon_t^b R_t E_t \left[ \frac{\Xi_{t+1}}{\pi_{t+1}} \right] \quad (16)$$

$$(\partial I_t) \quad \Xi_t = \Xi_t^k \varepsilon_t^i \left( 1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}} \right) + \beta E_t \left[ \Xi_{t+1}^k \varepsilon_{t+1}^i S'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2 \right] \quad (17)$$

$$(\partial \bar{K}_t) \quad \Xi_t^k = \beta E_t \left[ \Xi_{t+1} \left( \frac{R_{t+1}^k}{P_{t+1}} Z_{t+1} - a(Z_{t+1}) \right) + \Xi_{t+1}^k (1 - \delta) \right] \quad (18)$$

$$(\partial u_t) \quad \frac{R_t^k}{P_t} = a'(Z_t) \quad (19)$$

where  $\Xi_t$  and  $\Xi_t^k$  are the Lagrange multipliers associated with the budget and capital accumulation constraint respectively. Tobin's  $Q_t = \Xi_t^k / \Xi_t$  and equals one in the absence of adjustment costs.

#### 1.4 Intermediate labour unions and labour packers

Households supply their homogenous labour to an intermediate labour union which differentiates the labour services, sets wages subject to a Calvo scheme and offers those labour services to intermediate labour packers. Labour used by the intermediate goods producers  $L_t$  is a composite made of those differentiated labour services  $L_t(i)$ . As with intermediate goods, the aggregator is the one proposed by Kimball (1995). The labour packers buy the differentiated labour services, package  $L_t$ , and offer it to the intermediate goods producers.

The labour packers maximize profits:

$$\begin{aligned} & \max_{L_t, L_t(i)} W_t L_t - \int_0^1 W_t(i) L_t(i) di \\ & s.t. \left[ \int_0^1 H\left(\frac{L_t(i)}{L_t}; \varepsilon_t^w\right) di \right] = 1 \quad (\mu_{l,t}) \end{aligned}$$

where  $W_t$  and  $W_t(i)$  are the price of the composite and intermediate labour services respectively, and  $H$  is a strictly concave and increasing function characterised by  $H(1) = 1$ .  $\varepsilon_t^w$  is an exogenous process that reflects shocks to the aggregator function that result in changes

in the elasticity of demand and therefore in the markup. We will constrain  $\epsilon_t^w \in (0, \infty)$ .  $\epsilon_t^w$  follows the exogenous ARMA process:

$$\ln \epsilon_t^w = (1 - \rho_w) \ln \epsilon_t^w + \rho_w \ln \epsilon_{t-1}^w - \theta_w \eta_{t-1}^w + \eta_t^w, \quad \eta_t^w \sim N(0, \sigma_w) \quad (20)$$

Combining FOCs results in:

$$L_t(i) = L_t H'^{-1} \left[ \frac{W_t(i)}{W_t} \int_0^1 H' \left( \frac{L_t(i)}{L_t} \right) \frac{L_t(i)}{L_t} di \right]$$

The labour unions are an intermediate between the households and the labor packers. Under Calvo pricing with partial indexation, the optimal wage set by the union that is allowed to re-optimize its wage results from the following optimisation problem:

$$\begin{aligned} \max_{\widetilde{W}_t(i)} E_t \sum_{s=0}^{\infty} \xi_w^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \left[ \widetilde{W}_t(i) (\Pi_{l=1}^s \gamma \pi_{t+l-1}^{\iota_w} \pi_*^{1-\iota_w} - W_{t+s}^h) \right] L_{t+s}(i) \\ \text{s.t. } L_{t+s}(i) = L_{t+s} H'^{-1} \left( \frac{W_t(i) X_{t,s}^w}{W_{t+s}} \tau_{t+s}^w \right) \end{aligned}$$

where  $\widetilde{W}_t(i)$  is the newly set wage,  $\xi_w$  is the Calvo probability of being allowed to optimise one's wage,  $\tau_t^w = \int_0^1 H' \left( \frac{L_t(i)}{L_t} \right) \frac{L_t(i)}{L_t} di$  and

$$X_{t,s}^w = \begin{cases} 1 & \text{for } s = 0 \\ (\Pi_{l=1}^s \gamma \pi_{t+l-1}^{\iota_w} \pi_*^{1-\iota_w}) & \text{for } s = 1, \dots, \infty \end{cases}$$

The first-order condition is given by:

$$E_t \sum_{s=0}^{\infty} \xi_w^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} L_{t+s}(i) \left[ X_{t,s}^w \widetilde{W}_t(i) + \left( \widetilde{W}_t(i) X_{t,s}^w - W_{t+s}^h \right) \frac{1}{H'^{-1}(z_{t+s}^w)} \frac{H'(x_{t+s}^w)}{H''(x_{t+s}^w)} \right] = 0 \quad (21)$$

where  $x_t^w = H'^{-1}(z_t^w)$  and  $z_t^w = \frac{W_t(i)}{W_t} \tau_t^w$ .

The aggregate wage index is in this case given by:

$$W_t = (1 - \xi_w) \widetilde{W}_t H'^{-1} \left[ \frac{\widetilde{W}_t \tau_t^w}{W_t} \right] + \xi_w \gamma \pi_{t-1}^{\iota_w} \pi_*^{1-\iota_w} W_{t-1} H'^{-1} \left[ \frac{\gamma \pi_{t-1}^{\iota_w} \pi_*^{1-\iota_w} W_{t-1} \tau_t^w}{W_t} \right] \quad (22)$$

The markup of the aggregate wage over the wage received by the households is distributed to the households in the form of dividends (see the budget constraint of households).

## 1.5 Government Policies

The central bank follows a nominal interest rate rule by adjusting its instrument in response to deviations of inflation and output from their respective target levels:

$$\frac{R_t}{R^*} = \left( \frac{R_{t-1}}{R^*} \right)^\rho \left[ \left( \frac{\pi_t}{\pi_*} \right)^{r_\pi} \left( \frac{Y_t}{Y_t^*} \right)^{r_y} \right]^{1-\rho} \left( \frac{Y_t/Y_{t-1}}{Y_t^*/Y_{t-1}^*} \right)^{r_{\Delta y}} \epsilon_t^r \quad (23)$$

where  $R^*$  is the steady state nominal rate (gross rate) and  $Y_t^*$  is natural output. The parameter  $\rho$  determines the degree of interest rate smoothing. The exogenous monetary policy shock  $\epsilon_t^r$  is determined as

$$\ln \epsilon_t^r = \rho_r \ln \epsilon_{t-1}^r + \eta_t^r \quad (24)$$

The government budget constraint is of the form

$$P_t G_t + B_{t-1} = T_t + \frac{B_t}{R_t} \quad (25)$$

where  $T_t$  are nominal lump-sum taxes (or subsidies) that also appear in household's budget constraint. Government spending expressed relative to the steady state output path  $\epsilon_t^g = G_t/(Y\gamma^t)$  follows the process:

$$\ln \epsilon_t^g = (1 - \rho_g) \ln \epsilon_t^g + \rho_g \ln \epsilon_{t-1}^g + \rho_{ga} \ln \epsilon_t^a - \rho_{ga} \ln \epsilon_{t-1}^a + \eta_t^g, \eta_t^g \sim N(0, \sigma_g) \quad (26)$$

where we allow for a reaction of government spending to respond to the productivity process.

## 1.6 The natural output level

The natural output level is defined as the output in the flexible price and wage economy without mark-up shock in prices and wages. Persistent markup shocks may therefore result in persistent conflicts between the stabilising inflation and the output gap and therefore in persistent deviations of inflation from the inflation target.

## 1.7 Resource constraint

Integrating the budget constraint across households and combining with the government budget constraint and the expressions for the dividends of intermediate goods producers and labour unions gives the overall resource constraint:

$$C_t + I_t + G_t + a(Z_t)K_{t-1} = Y_t \quad (27)$$

## 2 Detrending and linearization

The model can be detrended with the deterministic trend  $\gamma$  and nominal variables can be replaced by their real counterparts. The non-linear system is then linearised around the stationary steady state of the detrended variables.

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