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In 2004 all publications will carry a motif taken from the €100 banknote.

1 Comments by an anonymous referee are gratefully acknowledged. All possible remaining errors are our own.
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CONTENTS

Abstract

The rank of the spectral density matrix conveys relevant information in a variety of statistical modelling scenarios. This note shows how to estimate the rank of the spectral density matrix at any given frequency. The method presented is valid for any hermitian positive definite matrix estimate that has a normal asymptotic distribution with a covariance matrix whose rank is known.

Keywords: Tests of Rank, Spectral Density Matrix.
JEL classification: C12, C32 and C52.

NON-TECHNICAL SUMMARY

The rank of the spectral density matrix conveys relevant information in a variety of statistical modelling scenarios. First, Phillips (1986) showed that a necessary condition for cointegration of a multivariate time series is that the spectral density matrix of the innovation sequence at frequency zero is of reduced rank. Second, knowing the rank of the spectral density matrix allows to identify a simplifying structure of a vector times series under the approach suggested by Pena and Box (1987), i.e. the common driving forces behind the system. Third, knowledge of the rank of the spectral density matrix is also relevant in the context of the reduction of large multiple input multiple output (MIMO) systems. Fourth, it enables restricting the dimensionality of cyclical components at individual frequencies. If a vector series share common cycles over certain frequencies, then it must hold that the spectral density matrix is of reduced rank for those frequencies.

This paper discusses the estimation of the rank of the spectral density matrix using a similar approach to that in Cragg and Donald (1996). The Cragg and Donald (1996) approach is a very general method to test for the rank of a matrix as it only requires that an estimate of that matrix exists having a normal asymptotic distribution with a covariance matrix whose rank is known.

The presentation of the paper focuses in the particular case of the spectral density matrix. However, the test presented extends to any hermitian positive definite matrices. It is worth pointing that our method is also valid for testing the rank of a positive semidefinite Toeplitz matrix. The rank of this matrix conveys very relevant information in a number of signal processing applications, see, e.g., Pisarenko (1973) and Tryphou (2000).

1 Introduction

The rank of the spectral density matrix conveys relevant information in a variety of statistical modelling scenarios. Phillips (1986) showed that a necessary condition for cointegration of a multivariate time series is that the spectral density matrix of the innovation sequence at frequency zero is of reduced rank. Tests of the rank of the spectral density matrix are also relevant to identify a simplifying structure of a vector times series under the approach suggested by Pena and Box (1987), i.e. the common driving forces behind the system. Also, the knowledge of the rank of the spectral density matrix is relevant in the context of the reduction of large MIMO systems.

This paper discusses the estimation of the rank of the spectral density matrix using a similar approach to that in Cragg and Donald (1996). The Cragg and Donald (1996) approach is a very general method to test for the rank of a matrix as it only requires that an estimate of that matrix exists having a normal asymptotic distribution with a covariance matrix whose rank is known. The structure of the paper is as follows:

Section 2 presents some areas of work where a procedure that estimates the rank of the spectral density matrix may be of use. Section 3 presents the analytical framework and background material on the estimation of the spectral density matrix together with the asymptotic properties of the estimates. A method to estimate the rank of the spectral density matrix is described in section 4. Section 5 presents some Monte Carlo experiments that provide some intuition on the potential merits of this new method as a valid tool for the estimation of the cointegrating rank. Section 6 concludes.

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2 Motivation

The analysis of a number of statistical and econometric issues may be helped by a procedure that determines the rank of a spectral density matrix. Here we give some examples.

Common driving forces. Denote an m-vector zero mean stationary process by $\{\boldsymbol{x}_t\}_{t=1}^{\infty}$, and assume that there exists a representation:

$$\boldsymbol{x}_t = \boldsymbol{P}\boldsymbol{z}_t \tag{1}$$

where $\boldsymbol{P}$ is a $m \times r$ matrix of parameters, and $\boldsymbol{z}_t$ is a r-vector stationary process, with $r < m$, i.e. there is a reduction in dimensionality, which follows an ARMA(p,q) process, i.e.

$$\boldsymbol{\Phi}(L)\boldsymbol{z}_t = \boldsymbol{\Theta}(L)\boldsymbol{u}_t$$

where $\boldsymbol{\Phi}(L)$ and $\boldsymbol{\Theta}(L)$ are matrix lag polynomials with all their roots outside the unit circle, and $\boldsymbol{u}_t$ is an *iid* random process with zero mean and positive definite covariance matrix $\boldsymbol{\Gamma}_u$. A further identification restriction imposed in this model is that the r factors are independent, and that all $\boldsymbol{\Phi}_i$ and $\boldsymbol{\Theta}_i$ matrices are diagonal. $\boldsymbol{P}$ is usually referred to as the matrix of factor loadings. For identification purposes it is assumed that $\boldsymbol{P}'\boldsymbol{P} = \boldsymbol{I}$. Denote $\boldsymbol{\Gamma}_x(k) = E\{\boldsymbol{x}_t\boldsymbol{x}'_{t-k}\}$, and $\boldsymbol{\Gamma}_z(k) = E\{\boldsymbol{z}_t\boldsymbol{z}'_{t-k}\}$. Under the representation in equation (1), it follows that $\boldsymbol{\Gamma}_x(k) = \boldsymbol{P}\boldsymbol{\Gamma}_z(k)\boldsymbol{P}'$ for $k \geq 1$. The rank of $\boldsymbol{\Gamma}_x(k)$ for $k \geq 1$ is equal to r, the number of the common driving forces. Also, the spectral density matrix of $\boldsymbol{x}_t$ at frequency ω is denoted by and equal to $\boldsymbol{\Sigma}_{xx}(\omega) = \boldsymbol{P}\boldsymbol{\Sigma}_{zz}(\omega)\boldsymbol{P}$. The rank of this matrix is of reduced rank for all frequencies. The model in Pena and Box (1987) is equivalent to model (1) with added noise, i.e.

$$\boldsymbol{x}_t = \boldsymbol{P}\boldsymbol{z}_t + \boldsymbol{\varepsilon}_t \tag{2}$$

where, $\boldsymbol{\varepsilon}_t$ is an m-vector of *iid* zero mean processes with covariance matrix $\boldsymbol{\Gamma}_\varepsilon$. Identification of the number of common driving forces cannot be linked directly to the rank of the spectral density matrix, but to a transformation of this. It is easy to see that the number of driving forces is equivalent to the rank of $(\boldsymbol{\Sigma}_{\tilde{\boldsymbol{x}}\tilde{\boldsymbol{x}}}(\omega) - \boldsymbol{I})$, for all frequencies ω, where $\tilde{\boldsymbol{x}}_t = \boldsymbol{\Gamma}_x(0)^{-1/2}\boldsymbol{x}_t$.

Reduction of MIMO Systems. Given the multiple input multiple output (MIMO) system:

$$\begin{aligned} \boldsymbol{z}_t &= \sum_{k=0}^{\infty} \boldsymbol{A}_k \boldsymbol{u}_{t-k} \\ \boldsymbol{x}_t &= \boldsymbol{z}_t + \boldsymbol{\varepsilon}_t \end{aligned}$$

where $\boldsymbol{x}_t$ is the $p \times 1$ observed output, $\boldsymbol{z}_t$ is the $p \times 1$ true output vector, $\boldsymbol{u}_t$ is the $m \times 1$ input vector, and $\boldsymbol{\varepsilon}_t$ is a $p \times 1$ noise component. The transfer function of that system is given by $\boldsymbol{A}(e^{i\omega}) = \sum_{k=0}^{\infty} \boldsymbol{A}_k e^{ik\omega}$. For $\boldsymbol{u}_t$ and $\boldsymbol{\varepsilon}_t$ jointly stationary uncorrelated processes, it holds that $\boldsymbol{\Sigma}_{xu}(\omega) = \boldsymbol{A}(e^{i\omega})\boldsymbol{\Sigma}_{uu}(\omega)$, where $\boldsymbol{\Sigma}_{xu}(\omega)$ and $\boldsymbol{\Sigma}_{uu}(\omega)$ are the cross spectral density matrix between $\boldsymbol{x}_t$ and $\boldsymbol{u}_t$ and the spectral density matrix of $\boldsymbol{u}_t$ respectively. This suggests that an estimator of the transfer function could be given by:

$$\hat{\boldsymbol{A}}(e^{i\omega}) = \hat{\boldsymbol{\Sigma}}_{xu}(\omega)\hat{\boldsymbol{\Sigma}}_{uu}^{-1}(\omega)$$

See Priestley (1981) for further details. For systems with large numbers of input and output variables, this estimation strategy might contain redundant information, and under those circumstances it appears sensible to try to reduce the dimension of this system. Brillinger (1969) and Priestley, Rao, and Tong (1973) showed that a possible reduction strategy would be to apply principal component analysis to the Fourier components of the input and output. These so called Dynamic Principal Components are built from a spectral decomposition of the spectral density matrix. Knowledge of the rank of the spectral density matrix of the input vector and the output vector is useful to select the relevant number of dynamic principal components that provide an optimal representation of the input vector and the relevant number of dynamic principal components that provide an optimal representation of the output vector; and where optimal representation is defined as that which provides maximum linear predictive efficiency.

Cointegration. Let $\{\boldsymbol{y}_t\}_{t=0}^{\infty}$ be a $n \times 1$ vector stochastic process generated by:

$$\boldsymbol{y}_t = \boldsymbol{y}_{t-1} + \boldsymbol{u}_t$$

and where $\boldsymbol{y}_0$ is any random vector, and $\{\boldsymbol{u}_t\}$ is a zero mean, weakly stationary innovation process such that $E|\boldsymbol{u}_i|^{\beta}$ for $i = 1, \ldots, n$ and $\beta > 2$; and $\{\boldsymbol{u}_t\}_{t=0}^{\infty}$ is strong mixing with mixing numbers α_m that satisfy $\sum_{m=1}^{\infty} \alpha_m^{1-2/\beta} < \infty$. Under those conditions, Phillips (1986) showed that if the system is cointegrated, i.e. there exists a vector γ for which $\gamma' \boldsymbol{y}_t$ is stationary, then the spectral density matrix of the innovation sequence $\boldsymbol{u}_t$ at frequency zero, denoted by $\boldsymbol{\Sigma}_{uu}$, is of reduced rank equal to n minus the number of cointegrating vectors γ_i.

Clearly, for some of the cases considered above one requires a joint test that the rank of the spectral density matrix is of a given rank for all frequencies. Our method will consider given frequencies only. However, our methodology is viewed more as a diagnostic tool for further model development rather than a formal joint test of reduced rank. In that sense it can still provide vital information about the structure of the data considered by being used for a set of different frequencies. Further, identifying common components of multivariate data at particular frequencies is of interest in its own, e.g. macroeconomic analysis often focuses on business cycle frequencies of between 5 and 10 years. Our proposed procedure can be a useful tool in such circumstances.

3 Background Theory

Multivariate one sided tests. Let $\hat{\boldsymbol{\theta}}$ be a consistent estimator of the $q \times 1$ vector $\boldsymbol{\theta}$ such that $\sqrt{T} vec(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \xrightarrow{d} N(\mathbf{0}, \boldsymbol{\Omega})$, where $\boldsymbol{\Omega}$ is nonsingular and T denotes the size of the sample used to estimate $\boldsymbol{\theta}$. We would like to test the hypothesis $H_0 : \theta_i = 0$ $(i = 1, \ldots, q)$ against the alternative $H_1 : \theta_i \geq 0$ $(i = 1, \ldots, q)$ where the inequality is strict for at least one value of i. Kudo (1963) showed that a likelihood ratio statistic for the one sided hypothesis we consider and normally distributed random variables can be defined as:

$$\bar{\chi}^2 = T \hat{\boldsymbol{\theta}}' \boldsymbol{\Omega}^{-1} \hat{\boldsymbol{\theta}} - T \min_{\substack{\theta_i \geq 0 \\ i=1,\ldots,q}} (\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})' \boldsymbol{\Omega}^{-1} (\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \tag{3}$$

The minimum of the second summand can be computed by means of quadratic programming. Kudo (1963) further showed that the probability that the value of $\bar{\chi}^2$ exceeds $\bar{\chi}_0^2$ is given by:

$$Pr\left(\bar{\chi}^2 \geq \bar{\chi}_0^2\right) = \sum_{i=0}^{q} w_i Pr\left(\chi_i^2 \geq \bar{\chi}_0^2\right) \tag{4}$$

where χ_i^2 is the chi-squared random variable with i degrees of freedom, $\chi_0^2 = 0$, and w_i are nonnegative weights given by:

$$w_i = \sum_{Q_i} P\{(\boldsymbol{\Omega}_{Q_i'})^{-1}\} P\{\boldsymbol{\Omega}_{Q_i : Q_i'}\} \tag{5}$$

where the summation runs over all subsets Q_i of $K = \{1, \ldots, q\}$ of size i, and Q_i' is the complement of Q_i where $\mathbf{\Omega}_{Q_i}$ is the variance matrix of θ_j, $j \in Q_i$, and $\mathbf{\Omega}_{Q_i:Q_i'}$ is the same under the condition $\theta_j = 0$, $j \notin Q_i$, and $P\{\mathbf{\Omega}\}$ is the probability that the variables distributed in a multivariate normal distribution with mean zero and covariance matrix $\mathbf{\Omega}$ are all positive; finally, $P\{\mathbf{\Omega}_{\phi:K}\} = 1$ and $P\{(\mathbf{\Omega}_{K'})^{-1}\} = P\{(\mathbf{\Omega}_{\phi})^{-1}\} = 1$. The probabilities in 5 can be easily computed by means of the algorithm proposed in Sun (1988). Note that a simple expression for $\mathbf{\Omega}_{Q_i:Q_i'}$ is given by $\mathbf{\Omega}_{Q_i} - \mathbf{\Omega}_{Q_i,Q_i'}\mathbf{\Omega}_{Q_i'}^{-1}\mathbf{\Omega}_{Q_i,Q_i'}'$ where $\mathbf{\Omega}_{Q_i,Q_i'}$ is the covariance matrix of θ_j, $j \in Q_i$ and θ_j, $j \in Q_i'$ (see e.g. Anderson (2003, pp. 33-35)). Note that a similar analysis using the ideas of Kudo (1963), among others, has been carried out, by Gourieroux, Holly, and Monfort (1982), in the context of inequality constraints on coefficients in regression models.

Complex Multivariate Normal Distribution. A q-dimensional random variable $\boldsymbol{x}_t$ with complex valued components is complex multivariate normally distributed with mean $\boldsymbol{\mu}$ and covariance matrix $\mathbf{\Omega}$, and denoted as $N^C(\boldsymbol{\mu}, \mathbf{\Omega})$, if the $2q$-random variable with real components $(Re\ \boldsymbol{x}_t', Im\ \boldsymbol{x}_t')'$ is distributed as

$$N\left(\begin{bmatrix} Re\ \boldsymbol{\mu} \\ Im\ \boldsymbol{\mu} \end{bmatrix}, \frac{1}{2}\begin{bmatrix} Re\ \mathbf{\Omega} & -Im\ \mathbf{\Omega} \\ Im\ \mathbf{\Omega} & Re\ \mathbf{\Omega} \end{bmatrix}\right) \tag{6}$$

where Re and Im denote the real and imaginary part of a complex variate. Let us denote the covariance matrix in (6) by $\mathbf{\Omega}^r$. For a detailed exposition of the complex multivariate normal see Brillinger (1981, Sec. 4.2). If a set of vector random variables, $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n$ are i.i.d zero mean complex multivariate normal with covariance $\mathbf{\Omega}$, then $\sum_{i=1}^n \boldsymbol{x}_i \bar{\boldsymbol{x}}_i'$ (where $\bar{\boldsymbol{x}}_i$ is the complex conjugate of $\boldsymbol{x}_i$) is said to have a complex Wishart distribution with n degrees of freedom, and is denoted by $W^C(n, \mathbf{\Omega})$.

Spectral Density Matrix. Denote a zero mean, wide sense stationary m-vector process by $\{\boldsymbol{x}_t\}_{t=1}^{\infty}$. The spectral density matrix of $\boldsymbol{x}_t$ is defined as

$$\mathbf{\Sigma}(\omega) = (2\pi)^{-1} \sum_{k=-\infty}^{\infty} \mathbf{\Gamma}_k e^{-ik\omega}$$

for $\theta \in [-\pi, \pi]$ where $\mathbf{\Gamma}_k = E\{\boldsymbol{x}_t \boldsymbol{x}_{t-k}'\}$. Given a sample of T observations an estimate of the spectral density matrix is given by:

$$\overline{\mathbf{\Sigma}}(\omega) = (2\pi)^{-1} \sum_{k=-(T-1)}^{T-1} \hat{\mathbf{\Gamma}}_k e^{-ik\omega}$$

where $\hat{\mathbf{\Gamma}}_k = \frac{1}{T}\sum_{t=1}^{T-|k|} \boldsymbol{x}_t \boldsymbol{x}_{t-k}'$. $2\pi\overline{\mathbf{\Sigma}}(\omega)$ is the periodogram. The periodogram provides an inconsistent but asymptotically unbiased estimate of the spectral density matrix, and is asymptotically distributed as a complex Wishart variable with 1 degree of freedom. A standard approach for consistent estimation of the spectral density matrix[1] relies on 'smoothing' the periodogram itself over the frequencies, i.e. averaging adjacent frequency ordinates. These estimates take the form,

$$\hat{\mathbf{\Sigma}}(\omega) = \frac{1}{2M+1} \sum_{k=-M}^{M} \overline{\mathbf{\Sigma}}(\omega + k/T) \tag{7}$$

For M fixed as $T \to \infty$ this estimate is still inconsistent, asymptotically unbiased for the spectral density matrix and asymptotically distributed as $(2M+1)^{-1}W^C(2M+1, \mathbf{\Sigma}(\omega_j))$, (see Brillinger (1981, pp. 245)). This is the simplest form of a smoothed periodogram estimate for the spectral density matrix. Different weights can be assigned to the periodogram coordinates $\bar{\mathbf{\Sigma}}(\omega + k/T)$, see Brillinger (1981, Chapter 7). If we allow $M \to \infty$ as $T \to \infty$ but impose $M^4/T \to 0$ we get a consistent and asymptotically normal estimate (see e.g. Newey and West (1987)). In particular we get that $\sqrt{2M+1}(vec(\hat{\mathbf{\Sigma}}(\omega)) - vec(\mathbf{\Sigma}(\omega)))$ is asymptotically complex normal[2] with a covariance matrix whose element giving the asymptotic covariance between $\hat{\mathbf{\Sigma}}_{i,j}(\omega)$ and $\hat{\mathbf{\Sigma}}_{u,v}(\omega)$, is given by:

$$\begin{array}{ll} \mathbf{\Sigma}_{i,u}(\omega)\mathbf{\Sigma}_{j,v}(\omega) + \mathbf{\Sigma}_{i,v}(\omega)\mathbf{\Sigma}_{j,u}(\omega) & \text{if } \omega = 0, \pm\pi \\ \mathbf{\Sigma}_{i,u}(\omega)\mathbf{\Sigma}_{j,v}(\omega) & \text{if } \omega \neq 0, \pm\pi \end{array} \tag{8}$$

where $\mathbf{\Sigma}_{i,j}(\omega)$ is the (i,j)-th element of $\mathbf{\Sigma}(\omega)$. We will denote this covariance matrix by $\boldsymbol{V}$ and its estimate, obtained by using the estimated spectral density matrix, by $\hat{\boldsymbol{V}}$. More details may be found in e.g. Brillinger (1981, pp. 262) or Brockwell and Davis (1991, pp. 447). In what follows we will assume that the periodogram coordinate weights are such that the spectral density matrix estimate is nonnegative definite.

4 Testing the rank of $\mathbf{\Sigma}$

This paper deals with the issue of testing the rank of an $n \times n$ hermitian positive semi-definite matrix $\mathbf{\Sigma}$. In what follows we assume that in the following partition of $\mathbf{\Sigma}$ the $r \times r$ submatrix $\mathbf{\Sigma}_{11}$ is of full rank.

$$\begin{pmatrix} \mathbf{\Sigma}_{11} & \mathbf{\Sigma}_{12} \\ \mathbf{\Sigma}_{21} & \mathbf{\Sigma}_{22} \end{pmatrix}$$

If $\mathbf{\Sigma}_{11}$ is not initially of full rank r, a valid reordering of the columns and rows of $\mathbf{\Sigma}$ would guarantee this without affecting the overall rank of the matrix. Cragg and Donald

[1] As we are mainly interested in the rank of the spectral density matrix, in the rest of the discussion we drop the normalizing constant 2π.

[2] For more details on the choice of M and its effect on the asymptotic bias and variance of the estimator see also Brillinger (1981, Chapter 2).

(1996) propose the application of r steps of Gaussian elimination with complete pivoting on $\mathbf{\Sigma}$ to achieve the required result. This manipulation guarantees that $\mathbf{\Sigma}_{11}$ in the finally reordered matrix is of full rank r. In the case under study in this paper we need to preserve the symmetry of $\mathbf{\Sigma}$ and hence symmetric pivoting should be implemented. An algorithm to compute the factorization $\boldsymbol{P}\mathbf{\Sigma}\boldsymbol{P}' = \boldsymbol{G}\bar{\boldsymbol{G}}'$, where $\boldsymbol{P}$ is an $n \times n$ pivoting matrix and $\boldsymbol{G}$ is an $n \times r$ lower triangular matrix is available in the LINPACK, see Dongarra, Bunch, Moler, and Stewart (1979), and subroutine CCHDC for details. Without lack of generality we avoid the issue of pivoting in this section for ease of notation.

Given the linear dependance of the last $n-r$ columns on the first r columns it must hold that $\mathbf{\Lambda} = \mathbf{\Sigma}_{22} - \mathbf{\Sigma}_{21}\mathbf{\Sigma}_{11}^{-1}\mathbf{\Sigma}_{12} = \mathbf{0}$. This implies that a test of rank $H_0 : rk(\mathbf{\Sigma}) = r$ is equivalent to a test of the null hypothesis $H_0 : \mathbf{\Lambda} = \mathbf{0}$. This is the testing strategy adopted by Cragg and Donald (1996). We further have the following proposition which simplifies the problem considerably.

Proposition 1 $\mathbf{\Lambda} = \mathbf{0}$ *if and only if* $\Lambda_{i,i} = 0$, $i = 1, \ldots, n-r$ *where* $\Lambda_{i,i}$ *denotes the i-th diagonal element of* $\mathbf{\Lambda}$.

Proof: The 'if' part is obvious. The 'only if' part follows if we note the following. By the Schur Complement Theorem we know that if $\mathbf{\Sigma}$ is positive semidefinite then $\mathbf{\Lambda}$ will be positive semidefinite. Hence, all the eigenvalues of $\mathbf{\Lambda}$, denoted λ_i, $i = 1, \ldots, n-r$, will be nonnegative. For a positive semidefinite matrix it always holds that its trace is equal to the sum of its eigenvalues, implying $\sum_{i=1}^{n-r} \Lambda_{i,i} = \sum_{i=1}^{n-r} \lambda_i$. Then, by the fact that $\sum_{i=1}^{n-r} \Lambda_{i,i} = 0$ it must follow that $\lambda_i = 0$, $i = 1, \ldots, n-r$, i.e. the matrix has rank zero and is therefore a matrix of zeros. $\square$

We can therefore concentrate on testing the null hypothesis $H_0 : \boldsymbol{\theta} = 0$ where $\boldsymbol{\theta} = (\Lambda_{1,1}, \ldots, \Lambda_{n-r,n-r})'$. Note further that $\boldsymbol{\theta}$ is a real vector. Under the null hypothesis we show in the appendix that $\sqrt{2M+1}\, vec(\hat{\mathbf{\Lambda}}) \xrightarrow{d} N^C(\mathbf{0}, \boldsymbol{W})$ where $\xrightarrow{d}$ denotes convergence in distribution, and $\boldsymbol{W}$ is a matrix defined in the appendix. Hence

$$\sqrt{2M+1}\, \hat{\boldsymbol{\theta}} = \sqrt{2M+1}\, \boldsymbol{L} \left(Re\ vec(\hat{\mathbf{\Lambda}})', Im\ vec(\hat{\mathbf{\Lambda}})' \right)' \xrightarrow{d} N(\mathbf{0}, \boldsymbol{L}\boldsymbol{W}^r\boldsymbol{L}') \tag{9}$$

where $\boldsymbol{L}$ is a $n-r \times 2(n-r)^2$ selector matrix that picks the real part of the diagonal elements of $\hat{\mathbf{\Lambda}}$. Then, we have the following proposition

Proposition 2 *Under the null hypothesis,* $H_0 : r = r^*$, $(2M+1)\, \hat{\boldsymbol{\theta}}'\mathbf{\Psi}^{-1}\hat{\boldsymbol{\theta}}$ *is distributed as a weighted mixture of* χ_i^2, $i = 1, \ldots, n-r^*$, *where* $\mathbf{\Psi} = \boldsymbol{L}\boldsymbol{W}^r\boldsymbol{L}'$ *and the weights* w_i *are given by (5).*

Proof: Using the results of Kudo (1963) we can construct the test statistic for the null hypothesis $H_0 : \boldsymbol{\theta} = 0$ against the alternative $H_0 : \theta_i \geq 0, \; i = 1, \ldots n - r$ where at least one inequality is strict. Our estimate of the spectral density matrix guarantees that the diagonal elements are always nonnegative (Note that the spectral density matrix at frequency ω is simply the covariance matrix of a white noise process according to the spectral representation of a multivariate stationary process. See e.g. Brockwell and Davis (1991, Section 11.8 and (11.1.17))). This means that the second summand in the statistic $\bar{\chi}^2$ presented in (3), namely the quadratic programming problem, will always be zero. Therefore, the statistic of interest is simplified to $\bar{\chi}^2 = (2M + 1)\, \hat{\boldsymbol{\theta}}' \boldsymbol{\Psi}^{-1} \hat{\boldsymbol{\theta}}$. □

It is worth noting that the multivariate one sided test has been generalized by Kudo and Choi (1975) to cases where $\boldsymbol{\Psi}$ is singular. Further, we note that the following possibilities for simplifying the execution of the test, with respect to the calculation of the critical values, are possible. Firstly, Tang, Gnecco, and Geller (1989) provide an approximate likelihood ratio test which is distributed as a $\bar{\chi}^2$ statistic with weights that do not depend on $\boldsymbol{V}$ and are easily calculated. Secondly, since the weights in (5) add up to 1 (see, e.g., Bohrer and Chow (1978)) then a conservative test (i.e. a test whose true size is lower that the nominal significance level used) can usefully serve as a vehicle for deriving a consistent estimator for the rank. So we can set the weights, w_i such that the critical values of the assumed distribution are upper bounds of the critical values of the true distribution. This can be straightforwardly achieved by setting $w_i = 0$ for $i = 1, \ldots, n - r^* - 1$ and $w_i = 1$ for $i = n - r^*$. In other words the critical values of the $\chi^2_{n-r^*}$ distribution would be used.

A sequential application of this test of rank can provide a consistent estimate of the rank of $\boldsymbol{\Sigma}$ if the significance level used in the test converges to zero as the number of observations tends to infinity (See, e.g., Hosoya (1989)).

5 Monte Carlo Analysis

As stated above, one of the uses of estimating the rank of the spectral density matrix is identifying the cointegrating rank. The test developed by Johansen (1988) is the key reference in the econometric literature to search for the cointegrating rank. However, this method was developed under the assumption of normally distributed innovations. Non-normally distributed innovations lead to a loss in power of this method. It is thus of interest to see whether the method presented in this paper could have certain merits as a nonparametric test of cointegration. This section present a brief collection of Monte Carlo exercises that show that this is indeed the case. We note however that a more thourough study is beyond the scope of this paper and is left for future research.

The class of finite order linear vector error correction mechanism (VECM) models is not the most appropriate class to assess nonparametric procedures. Therefore, linear and nonlinear cointegrating systems will be considered. The data generation process for the vector simulated series $\boldsymbol{y}_t$ is defined as follows:

$$\Delta \boldsymbol{y}_t = F(\Delta \boldsymbol{y}_{t-1})\boldsymbol{\Pi} \boldsymbol{y}_{t-1} + \boldsymbol{\epsilon}_t \tag{10}$$

where we allow for three alternative specifications for $F(.)$:

$$F(\Delta \boldsymbol{y}_{t-1}) = \boldsymbol{I} \tag{11}$$

$$F(\Delta \boldsymbol{y}_{t-1}) = 1 - e^{-(\sum_{i=1}^{m} \Delta y_{i,t-1})^2} \tag{12}$$

$$F(\Delta \boldsymbol{y}_{t-1}) = 1\{|\sum_{i=1}^{m} \Delta y_{i,t-1}| > r\}\ ,\ \text{r=2} \tag{13}$$

These specifications lead to a linear model if (11), a pseudo-STAR model if (12), and a pseudo-SETAR model if (13). The last two lead to nonlinear VECM models where the speed of convergence to equilibrium depends on $\Delta \boldsymbol{y}_{t-1}$. As their name indicate the pseudo-STAR model is inspired by univariate smooth transition autoregressive (STAR) models, while the pseudo-SETAR by self-exciting threshold autoregressive (SETAR) models. Note that these nonlinear models still imply the existence of a Wold decomposition for the differenced data and therefore our suggested procedure is appropriate.

We concentrate on a multivariate model with 3 variables. We control the rank of the coefficient matrix $\boldsymbol{\Pi}$ in the error correction representation by specifying the vector of its eigenvalues. Two different vectors are considered: $(-0.6, 0, 0)$, i.e. one cointegrating vector, and $(-0.6, -0.6, 0)$, i.e. two cointegrating vectors. Note that all the eigenvalues are negative given the requirement that the eigenvalues of $\boldsymbol{I} + \boldsymbol{\Pi}$ are less than or equal to one. We then construct a standard normal random matrix of eigenvectors $\boldsymbol{E}$ which are almost surely linearly independent. These are transformed into an orthonormal basis, $\tilde{\boldsymbol{E}}$, using the Gram-Schmidt process. The coefficient matrix is then given by $\tilde{\boldsymbol{E}}\boldsymbol{\Lambda}\tilde{\boldsymbol{E}}'$ where $\boldsymbol{\Lambda}$ is a diagonal matrix containing the eigenvalues of the required coefficient matrix. Two alternative types of random disturbances are used for simulating $\boldsymbol{\epsilon}_t$. First, random normal disturbances with identity covariance matrix. Second, iid MA(1) processes with correlation coefficient 0.9. Using these random numbers a sample from a process following the error correction representation in (10) is obtained.

The sample sizes considered are 200 and 600. For each simulated sample, 200 initial observations have been discarded to minimise the effect of starting values. For each Monte Carlo experiment 10,000 replications have been carried out. Bias and Mean Square Error (MSE) statistics for these simulation exercises are shown in table 1. For illustration purposes, this table also reports simulation results for Johansen (1988) maximum eigenvalue test (JM) and also his trace test (JM), the procedure described in this paper is denoted by (CK).[3] Generally speaking the performance of the CK is satisfactory for most cases under study. The only exceptions are exercises run with samples of size 200, rank 2 and a pseudo-SETAR model. The test appears always best in terms of Bias and MSE for exercises of rank equal to 1, sample size equal to 600 and MA(1) errors. But for minor exceptions, the Johansen's procedures are always best for exercises conducted with normally distributed shocks.

6 Conclusion

This paper has formulated a rank determination procedure for the rank of the spectral density matrix at any frequency. The need for such techniques becomes apparent in areas such as multivariate factor models and cointegration. Phillips and Ouliaris (1988) suggested tests of the null of 'no cointegration' which amounted to a test of the hypothesis that the r smallest eigenvalues of the spectral density matrix of the innovation sequence at frequency zero are greater than zero. Phillips and Ouliaris (1990) expanded on the issue of choice of the null hypothesis in cointegration testing by pointing out that adopting the null hypothesis of cointegration may be more sensible from a methodological point of view given that cointegration is the focus of interest. However, it was also pointed out that standard test statistics based on the spectral density matrix provided inconsistent tests under the null hypothesis of no cointegration. This paper has described tests of the rank of the spectral density matrix which may serve, at frequency zero, as tests of the null of 'cointegration'. It is clear that, as long as a consistent estimate of the spectral density matrix of the innovation process exists and has an asymptotic complex normal distribution, the application of the test described will provide a consistent testing procedure for cointegration. The test of the rank of the spectral density matrix described in this paper is also relevant to identify a simplifying structure of a vector times series under the approach suggested by Pena and Box (1987), and to restrict the dimensionality of cyclical components at individual frequencies.

[3]GAUSS code to implement this test is available from the authors upon request.

The presentation of this paper has been focused in the particular case of the spectral density matrix. However, the test presented extends to any hermitian positive definite matrices. It is worth pointing that our method is also valid for testing the rank of a positive semidefinite Toeplitz matrix. The rank of this matrix conveys very relevant information in a number of signal processing applications, see, e.g., Pisarenko (1973) and Tryphou (2000).

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A Appendix

As $vec(\mathbf{\Lambda})$ is not analytic, it cannot be expanded as a Taylor series. We define instead for a hermitian complex matrix $\boldsymbol{A}$, a $2n \times 2n$ real symmetric matrix $\boldsymbol{A}^R$ which is an arrangement of the real and imaginary parts of the elements of $\boldsymbol{A}$. Details on $\boldsymbol{A}^R$ are given in Brillinger (1981, pp. 71). By Brillinger (1981, Lemma 3.7.1(i),(ii),(iv)), if $\mathbf{\Lambda} = \mathbf{\Sigma}_{22} - \mathbf{\Sigma}_{21}\mathbf{\Sigma}_{11}^{-1}\mathbf{\Sigma}_{12}$ then $\mathbf{\Lambda}^R = \mathbf{\Sigma}_{22}^R - \mathbf{\Sigma}_{21}^R\mathbf{\Sigma}_{11}^{R\,-1}\mathbf{\Sigma}_{12}^R$. Note that $(Re\ vec(\mathbf{\Sigma})', Im\ vec(\mathbf{\Sigma})')' \xrightarrow{d} N(\mathbf{0}, \mathbf{V}^r)$. Let $\boldsymbol{d}_{ij}$ be the vector of distinct elements of $\mathbf{\Sigma}_{ij}^R$. Define $\boldsymbol{J}_1$, $\boldsymbol{J}_2$, $\boldsymbol{J}_j^h$, $\boldsymbol{J}_{ij}^h$ and $\boldsymbol{D}_i$, $i, j = 1, 2$, as $\boldsymbol{s} \equiv \left(vec(\mathbf{\Sigma}_{11}^R)', vec(\mathbf{\Sigma}_{21}^R)', vec(\mathbf{\Sigma}_{12}^R)', vec(\mathbf{\Sigma}_{22}^R)'\right)' = \boldsymbol{J}_1\,(Re\ vec(\mathbf{\Sigma})', Im\ vec(\mathbf{\Sigma})')'$, $\boldsymbol{J}_2 vec(\mathbf{\Lambda}^R) = (Re\ vec(\mathbf{\Lambda})', Im\ vec(\mathbf{\Lambda})')'$, $\boldsymbol{J}_j^h \boldsymbol{d}_{jj} = vech(\mathbf{\Sigma}_{jj}^R)$, $\boldsymbol{J}_{ij}^h \boldsymbol{d}_{ij} = vec(\mathbf{\Sigma}_{ij}^R)$ and $vec(\mathbf{\Sigma}_{ii}^R) = \boldsymbol{D}_i vech(\mathbf{\Sigma}_{ii}^R)$. Then

$$\boldsymbol{R} \equiv \frac{\partial vec(\mathbf{\Lambda}^R)}{\partial \boldsymbol{s}} = \left[\frac{\partial vec(\mathbf{\Lambda}^R)}{\partial vec(\mathbf{\Sigma}_{11}^R)'}, \frac{\partial vec(\mathbf{\Lambda}^R)}{\partial vec(\mathbf{\Sigma}_{21}^R)'}, \frac{\partial vec(\mathbf{\Lambda}^R)}{\partial vec(\mathbf{\Sigma}_{12}^R)'}, \frac{\partial vec(\mathbf{\Lambda}^R)}{\partial vec(\mathbf{\Sigma}_{22}^R)'}\right]$$

Since $vec(\mathbf{\Sigma}_{21}^R\mathbf{\Sigma}_{11}^{R\,-1}\mathbf{\Sigma}_{12}^R) = \left(\mathbf{\Sigma}_{12}^{R\,\prime} \otimes \mathbf{\Sigma}_{21}^R\right) vec\left(\mathbf{\Sigma}_{11}^{R\,-1}\right)$, $\mathbf{\Sigma}_{11}^R$ and $\mathbf{\Sigma}_{22}^R$ are symmetric and $\mathbf{\Sigma}_{21}^R = \mathbf{\Sigma}_{12}^{R\,\prime}$, from Brillinger (1981, Lemma 3.7.1(v)), we have

$$\frac{\partial vec(\mathbf{\Lambda}^R)}{\partial vec(\mathbf{\Sigma}_{11}^R)'} = \left(\mathbf{\Sigma}_{12}^{R\,\prime} \otimes \mathbf{\Sigma}_{21}^R\right) \boldsymbol{D}_1 \boldsymbol{D}_1^+ \left(\mathbf{\Sigma}_{11}^{R\,-1} \otimes \mathbf{\Sigma}_{11}^{R\,-1}\right) \boldsymbol{D}_1 \boldsymbol{J}_1^h \boldsymbol{J}_1^{h\,+} \boldsymbol{D}_1^+ \qquad \text{(A-1)}$$

$$\frac{\partial vec(\mathbf{\Lambda}^R)}{\partial vec(\mathbf{\Sigma}_{21}^R)'} = -\left(\boldsymbol{I}_{4(n-r)^2} + \boldsymbol{K}_{2(n-r),2(n-r)}\right)\left(\mathbf{\Sigma}_{21}^R\mathbf{\Sigma}_{11}^{R\,-1} \otimes \boldsymbol{I}_{2(n-r)}\right) \boldsymbol{J}_{21}^h \boldsymbol{J}_{21}^{h\,+} \qquad \text{(A-2)}$$

$$\frac{\partial vec(\mathbf{\Lambda}^R)}{\partial vec(\mathbf{\Sigma}_{12}^R)'} = \frac{\partial vec(\mathbf{\Lambda}^R)}{\partial vec(\mathbf{\Sigma}_{21}^R)'}\boldsymbol{K}_{2r,2(n-r)}, \qquad \frac{\partial vec(\mathbf{\Lambda}^R)}{\partial vec(\mathbf{\Sigma}_{22}^R)'} = \boldsymbol{D}_2 \boldsymbol{J}_2^h \boldsymbol{J}_2^{h\,+} \boldsymbol{D}_2^+ \qquad \text{(A-3)}$$

where for a matrix $\boldsymbol{A}$, $\boldsymbol{A}^+ = (\boldsymbol{A}'\boldsymbol{A})^{-1}\boldsymbol{A}'$, $\boldsymbol{K}_{m,n}$ is a commutation matrix (see Lütkepohl (1996, Sec. 9.2)). (A-1), (A-2) and (A-3) follow from Lütkepohl (1996, 10.6(2) and 9.5.3(1)(ii)), Lütkepohl (1996, 10.5.1(7)) and Lütkepohl (1996, 10.4.1(1)(iii) and 9.5.3(1)(ii)) respectively. Then, $\sqrt{2M+1}\left(Re\ vec(\hat{\mathbf{\Lambda}})', Im\ vec(\hat{\mathbf{\Lambda}})'\right)' \xrightarrow{d} N(\mathbf{0}, \boldsymbol{W}^r)$ where $\boldsymbol{W}^r = \boldsymbol{J}\boldsymbol{V}^r\boldsymbol{J}'$ and $\boldsymbol{J} = \boldsymbol{J}_2\boldsymbol{R}\boldsymbol{J}_1$. Finally, $\sqrt{2M+1} vec(\hat{\mathbf{\Lambda}}) \xrightarrow{d} N^C(\mathbf{0}, \boldsymbol{W})$. An alternative to the above is the use of numerical derivatives.

Table 1: Bias and MSE of Estimated rank.[a]

Model	Noise	Test	rank	Bias 200	Bias 600	MSE 200	MSE 600
Linear	Normal	CK	1	0.191	0.134	0.206	0.135
			2	-0.366	-0.218	0.418	0.230
		JM	1	0.058	0.055	0.061	0.059
			2	0.059	0.060	0.059	0.060
		JT	1	0.060	0.056	0.073	0.070
			2	0.059	0.060	0.059	0.060
	MA(1)	CK	1	0.196	0.135	0.206	0.137
			2	-0.369	-0.206	0.425	0.218
		JM	1	0.158	0.162	0.183	0.187
			2	0.095	0.078	0.095	0.078
		JT	1	0.169	0.172	0.218	0.217
			2	0.095	0.078	0.095	0.078
STAR	Normal	CK	1	0.158	0.130	0.209	0.136
			2	-0.543	-0.307	0.670	0.341
		JM	1	0.055	0.056	0.059	0.059
			2	0.063	0.060	0.063	0.060
		JT	1	0.057	0.056	0.068	0.066
			2	0.063	0.060	0.063	0.060
	MA(1)	CK	1	0.173	0.125	0.206	0.129
			2	-0.492	-0.275	0.596	0.295
		JM	1	0.156	0.150	0.188	0.170
			2	0.093	0.081	0.093	0.081
		JT	1	0.145	0.160	0.229	0.196
			2	0.093	0.081	0.093	0.081
SETAR	Normal	CK	1	-0.125	0.081	0.342	0.156
			2	-1.019	-0.647	1.452	0.825
		JM	1	-0.115	0.055	0.208	0.058
			2	-0.041	0.060	0.177	0.060
		JT	1	-0.161	0.057	0.273	0.069
			2	-0.028	0.060	0.156	0.060
	MA(1)	CK	1	0.036	0.123	0.257	0.143
			2	-0.824	-0.466	1.110	0.558
		JM	1	-0.144	0.152	0.384	0.175
			2	-0.151	0.081	0.366	0.081
		JT	1	-0.166	0.159	0.489	0.202
			2	-0.055	0.081	0.236	0.081

[a]Sample sizes for Monte Carlo experiments are 200 and 600. CK denotes the Camba-Mendez and Kapetanios test, JM refers to Johansen's maximum eigenvalue test and JT to Johansen's trace test. rk denotes the cointegrating rank which is 1 or 2 for the different exercises conducted as described in the text.

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