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Fragility in money market funds:

sponsor support and regulation.

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Abstract

I develop a model of money market funds (MMFs) to study the ability of sponsor support to provide stability to the industry. I find that strategic complementarities in the sponsors' support decisions can make MMFs vulnerable to runs different from the canonical bank-runs: it may lead to runs of intermediaries on each other through firesales in the money market. I then use the model to analyze the effects of imposing a floating net asset value and capital requirements on MMFs. I find that general equilibrium effects, which are mostly ignored in the policy discussion, can overturn conventional intuition.

JEL: $G.01, G.21, G.28$

Keywords: Money Market Funds, Support, Fire Sales, Regulation, Runs

Non-technical Summary

Money market funds (MMFs) are financial intermediaries key to the financial system. With assets under management of over 2.6 trillion dollars in the US and around 1 trillion euros in Europe they are among the most important suppliers of short term financing, especially to financial institutions. Because of the size of the MMF industry and its importance as a supplier of liquidity to financial institutions, fluctuations in flows into and out of MMFs can affect the whole financial system. In fact, large outflows from the MMF industry were associated with the freezing of the short-term funding market in 2008 and with the liquidity crisis in Europe in 2011.

Due to the systemic importance of MMFs, their regulation has been at the center of the policy discussion both in the U.S. and in Europe. On July 23, 2014, after several years of debate, the Securities and Exchange Commission (SEC) voted to impose new regulations on the MMF industry. The European Commission is also in favor of a MMF reform. The mutual fund industry opposes these changes and argues that further regulations would make the MMF industry less profitable and reduce the availability of short-term funding.

The regulations that are being considered target some key characteristics of MMFs, mainly the stable net asset value (NAV) and the support offered by sponsors to the funds. In the U.S., the SEC voted to impose redemption fees on investors in retail MMFs, and to force institutional MMFs to abandon the stable NAV in favor of a floating NAV. Redemption fees will increase investors' costs of redeeming shares and, therefore, reduce their incentives to run on the MMFs. By adopting a floating NAV, MMFs would not be subject to liquidation after breaking the buck. Alternatively, the European Commission proposed that stable NAV funds convert to variable NAV funds or hold a 3 percent capital buffer, and that sponsor support be prohibited unless approved by the appropriate regulator. These proposed regulations aim to make MMFs more similar to other financial intermediaries: more like regular mutual funds in the case of adopting a floating NAV and more like banks in the case of a capital buffer.

Before one can start analyzing how to regulate MMFs it is important to understand the sources of fragility in the MMF industry. In this paper, I show that sponsor support, which is a characteristic particular to MMFs, can also be a source of fragility and make the MMF industry vulnerable to runs that are different from the canonical bank runs: these runs are not runs of investors on the intermediaries but of intermediaries on each other through the fire sales in the money market.

One of the key features of the MMF industry in the US has been the stability of the NAV. MMFs aim to maintain a stable NAV, usually of 1 dollar. If the NAV goes below 1 dollar, which is known as "breaking-the-buck", the fund is eventually liquidated, generating costs both to investors and to the company that sponsors the fund. To prevent this costly liquidation, sponsors can offer support to their funds to keep the share value at 1 dollar. In fact, one of the main ways to achieve the stability of the NAV is through sponsor support.

Since MMFs are very important as suppliers of liquidity in the money market, if the demand for money

market instruments by MMFs decreases, the price of these instruments will decrease and MMFs will be at risk of breaking-the-buck. If the sponsor of a MMF expects these asset prices, and thus the NAV, to be low, he will also expect a high cost of offering support, and may choose not to keep the fund open, and sell all the fund's assets. If all sponsors share these beliefs, the demand for assets will be low, the NAV will be low, and the MMF industry will experience large outflows due to the liquidation of the funds. These large outflows can be thought of as a run of the MMFs on the money market. On the other hand, if a sponsor expects the NAV to be high, he has incentives to offer support and keep the fund open. If all sponsors behave in this way, the need for support will be lower and the funds will remain open. By preventing the liquidation of a fund, sponsor support keeps the demand for assets high, and, through the calculation of the NAV, decreases other sponsors' need to offer support. The self-fulfilling nature of the runs described here can be a source of fragility in the MMF industry different from that in other financial intermediaries.

The adoption of a floating NAV would decrease the incentives of sponsors to offer support. This, in turn, would decrease the amount of insurance that investors in MMFs get through these transfers. Everything else equal, risk-averse investors would respond to this loss of insurance by reducing their exposure to MMFs. Thus -as argued by the industry- the supply of liquidity in the asset market would decrease. But, the new regulation would also change asset prices and, hence, the risk and the return of investing in a MMF beyond this loss of insurance. A floating NAV would also be accompanied with fewer fund liquidations, and therefore, would lead to higher liquidation prices for assets and to lower fees. Both of these effects, in contrast to the loss of insurance mentioned above, would lead to higher returns of investing in a MMF.

Imposing a capital buffer on MMFs also has countervailing effects on the return received by investors. On one hand, it increases the cost for managers to offer intermediation services and, therefore, increases the fees charged by MMFs. On the other hand, it forces managers to offer support up to the size of the capital buffer which increases the stability of the NAV and the expected return of investing in a MMF.

The overall importance of the general equilibrium effects will depend on whether the increase in the stability of the return offsets the increase in the intermediation fees. In numerical examples, I show that the general equilibrium effects described above which are mostly ignored in the policy discussion can overturn conventional intuition. I find that the total supply of liquidity can increase after the adoption a floating NAV and after the adoption of a capital buffer.

1 Introduction

Money market funds (MMFs) are financial intermediaries key to the financial system. By the end of 2013, U.S. MMFs managed over 2.6 trillion dollars in assets, almost a quarter of all U.S. mutual fund assets, and over 10 percent of mutual fund assets worldwide. In Europe, MMFs represent 15 percent of the European fund industry with assets under management of roughly 1 trillion euros.[1]

MMFs are also important suppliers of short term financing, especially for financial institutions. In December of 2011, MMFs owned over 40 per cent of U.S. dollar-denominated financial commercial paper, around a third of dollar-denominated negotiable certificates of deposit, and they were among the biggest category of repo lenders, with an estimated $460 billion in repos.[2] Given the size of the MMF industry, and its importance as a supplier of liquidity to financial institutions, flows into and out of MMFs can affect the whole financial system. In fact, the large outflows experienced by the MMF industry after Reserve Primary Fund "broke the buck" in 2008 contributed largely to the freezing of the short-term funding market. Later, in 2011, the heavy exposures of MMFs to European financial institutions put the MMF industry at risk of transmitting distress from Europe to the U.S. short-term funding market and the outflows from the MMF industry worsened the situation of the Eurozone banks.[3]

On top of the importance of MMFs in the money market, MMFs have some peculiar institutional features that make them unique financial institutions.[4] As all open-ended mutual funds, MMFs issue demandable shares, *i.e.*, they provide "same day" liquidity, allowing investors to redeem their shares at any time at the net asset value (NAV) of the shares. What makes these institutions - all MMFs in the U.S. and constant NAV MMFs in Europe- special is that they seek to maintain a stable NAV, usually of $1. If there are positive returns on the MMFs investments, these are paid out entirely as dividends. If a fund "breaks the buck", that is if the NAV drops below $1, it is eventually liquidated.

One of the main ways in which MMFs are able to maintain a stable NAV is voluntary sponsor support. Breaking the buck is costly for MMFs' sponsors: there are forgone future profits from operating the MMF and, possibly, negative spillovers to other activities in which the sponsor participates. Therefore, the sponsors may choose to offer support to their MMFs to prevent the funds from being liquidated. Brady et al. (2012) find that at least 21 MMFs would have broken the buck if they had not received sponsor support during the last financial crisis. Between 2007 and 2011, 78 MMFs (out of a total of 341 MMFs) received sponsor support in 123 instances for a total amount of at least $4.4 billion. In fact, as figure 1 shows, sponsor support has been a common feature throughout the history of the MMF industry even prior to the recent financial crisis.

Though sponsor support had been successful in avoiding the liquidation of funds before 2008,[5] even

[1] See ICI (2012) and European Commission (2013).
[2] Source: McCabe et al. (2012), Mackenzie, Financial Times May 3, 2011.
[3] See Board (2009), SEC (2009) , PWG (2010) , FSOC (2011) and Chernenko and Sunderam (2012).
[4] See the appendix for a more detail account of the institutional details of U.S. MMFs.
[5] With the exception Community Bankers Money fund, a small institutional MMF fund that broke the buck in 1994.

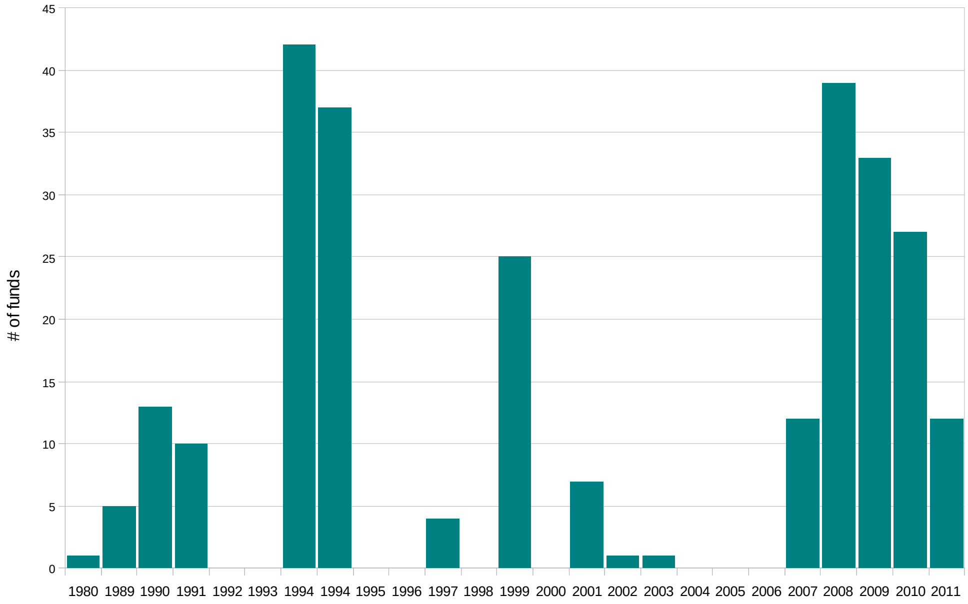

Figure 1: Number of Funds Receiving Support: 1980-2011. Sources: Brady et al. (2012) and Moody's (2010).

throughout the Asset-Backed Commercial Paper (ABCP) crisis of 2007, it was not enough to stop share redemptions after Reserve Primary broke the buck. The outflows only stopped after the U.S. government guaranteed the "deposits" in MMFs and created the "ABCP Money Market Mutual Fund Liquidity Facility" which was aimed at stopping the ongoing fire sales in the ABCP market.

In this paper I build an equilibrium model to study the ability of sponsor support to provide stability to the MMF industry. I find that, even though sponsor support seems to be good at providing stability after idiosyncratic shocks, it may become useless in the presence of systemic shocks. Moreover, I show that the ability of sponsors to offer voluntary support may lead to coordination failures that would amplify systemic shocks. I then use the model to analyze the trade-offs involved in the adoption of the main regulations that are being considered to change the MMF industry: abandoning the stable NAV in the US and a imposing a capital buffer in Europe.

The model is a 3-period model of financial intermediation that incorporates features that make MMFs unique financial institutions. There are two types of agents: risk averse investors and risk neutral fund managers. A short-term safe asset and a long-term risky asset are traded in competitive markets. Managers are the only agents who can access the risky asset market directly. Investors may only access the risky asset through the managers. The intermediation contract between investors and managers captures the demandable nature of the shares held by investors in MMFs, the eventual liquidation after a fund breaks the buck, and, through the possibility of voluntary sponsor support, the stable NAV. The asset structure incorporates the fluctuation in asset prices faced by MMFs: at the time of purchase there is uncertainty about the quality of the risky asset and its default risk. Asset prices are determined in equilibrium, which captures the size and relevance of the MMF industry in the market for short-term financing.

The managers' support decisions determine the demand for the risky asset in the interim period and its liquidation price. At the same time, the liquidation price of the risky asset in the interim period determines the costs and benefits for managers of offering support. This interdependence may give rise to strategic complementarities in the managers' support decisions and may make the model vulnerable to runs that are different in nature from the canonical bank runs described in Diamond and Dybvig (1983): these runs are not runs of investors on the financial intermediaries but runs of financial intermediaries on the asset market.

The model developed in this paper is consistent with several stylized facts documented in the literature on MMFs. In particular, the model captures the responsiveness of MMFs' inflows to the funds' past return (positive performance-flow relation), and the lower risk-taking by fund with sponsors that also offered non-money market mutual funds and other financial services.

Finally, I analyze how the equilibrium changes with the adoption of a floating NAV and with the adoption of a capital buffer. I find that, in the model, the adoption of a floating NAV would be successful in decreasing the fragility of the MMF industry. However, in equilibrium, the regulation would also affect intermediation fees, support decisions, and asset prices. All these, in turn, could have countervailing effects on the risk and the return of intermediation for investors and MMF managers, and, hence, on the provision of liquidity and on welfare.

When going from a stable to a floating NAV system, investors in MMFs lose the insurance provided by sponsor support. Everything else equal, risk averse investors would reduce their exposure to MMFs in response to this loss of insurance. Thus -as argued by the industry- the supply of liquidity in the asset market would decrease. However, in equilibrium, adopting a floating NAV for the whole industry also changes asset prices and the fees charged for intermediation. Thus, the risk and the return of investing in a MMF change beyond the loss of insurance. In numerical examples, I find that the total supply of liquidity increases when adopting a floating NAV.

Similarly, the adoption of a capital buffer in the MMF industry would affect the return of investing in MMFs directly, by absorbing losses and increasing the stability of the NAV, and indirectly, by increasing the fees charged by MMFs and changing the equilibrium in the asset market. These mechanisms would have countervailing effects on the return and risk of investing in MMFs. In numerical examples I find that the liquidity supply increases even though the fees charged for intermediation are higher after the adoption of a capital buffer.

The results in the numerical examples depend mainly on the elasticity of the investors demand of intermediation, i.e., on their attitude towards risk, and on the elasticity of the supply of the risky asset. This last elasticity will depend on whether the money market can rely on other market participants other than MMF to supply liquidity. The higher the relative importance of MMFs as suppliers of liquidity the more important the general equilibrium effects mentioned above would be.

1.1 Related Literature

To the best of my knowledge, this paper is the first to develop a model of MMFs to focus on sponsor support in the MMF industry. The regulations that are being considered for the MMF industry target particular institutional features of MMFs that would affect the provision of sponsor support and, thus, the way in which the whole industry operates. Therefore, a model that captures these peculiar features which make MMFs different from banks and other mutual funds is needed to analyze the impact of these regulations.

The MMF industry first appeared in the 1970's as an instrument to offer liquidity to investors while making a small return on their money. During this period, banks were subject to regulations that imposed caps on the interest rate that could be paid by checking accounts. This regulation only applied to banks, which made MMFs a very convenient substitute to bank deposits.[6] Though the regulations that gave rise to the MMF industry have been lifted, there are other restrictions faced by banks, such as capital requirements, that may give MMFs a role from a regulatory arbitrage point of view. Throughout the paper I take the function of MMFs, and the contract they offer, as given: I assume that the role of MMFs is to create safe assets for investors while providing them access to the money market.[7]

Though a model of MMFs has not been developed in the literature, several papers have analyzed the MMF industry empirically. Chen et al. (2010) document the presence of strategic complementarities in the redemption behavior of investors in all mutual funds, including MMFs. These strategic complementarities, which are not the focus of my paper, make MMFs prone to self-fulfilling runs like those considered by Diamond and Dybvig (1983) and analyzed extensively in the banking literature. These run-like events, and their relation to the risk taken by the funds, have been documented in McCabe (2010), Kacperczyk and Schnabl (2013) and Schmidt et al. (2013). Chernenko and Sunderam (2012) document large outflows from MMFs with exposures to Eurozone banks in 2011 and analyze the spillover effects of these runs on the provision of liquidity from MMFs to non-European firms.

McCabe et al. (2012) focus on how to prevent investor runs in MMFs. They propose a minimum balance at risk rule to reduce the investors' incentives to redeem MMF shares quickly when a fund is in distress and, therefore, mitigate the vulnerability of MMFs to investor runs. Hanson et al. (2012) provide an extensive analysis of the reform proposals for MMFs focusing mainly on the their ability to stop investor runs. This paper offers a complementary analysis of the proposed policies: I focus on sponsor support, a central feature of the MMF industry, and on the general equilibrium implications of the adoption of these policies. I shift the focus of the analysis from investor runs on the intermediaries to intermediary runs on the money market.

The frequency and magnitude of sponsor support in the MMF industry is documented by Moody's (2010) and Brady et al. (2012). McCabe (2010) and Kacperczyk and Schnabl (2013) show evidence that differences in the sponsor's ability and incentives to offer support affect the portfolio choices of MMFs. McCabe

[6] For a discussion on the origin of MMFs see chapter 1 in Carnell et al. (2008) and Rosen and Katz (1983).

[7] Ennis (2012) analyzes MMFs as providers of liquidity (as banks) and as investment managers (as mutual funds).

(2010) finds that sponsor support was more likely for riskier MMFs and for funds with bank-affiliated sponsors. Kacperczyk and Schnabl (2013) show that, between 2007 and 2010, funds sponsored by financial intermediaries with more money market funds business took on more risk. Though the risk of investor runs on MMFs is closely related to the stability of the NAV, Gordon and Gandia (2013) find, consistently with Chen et al. (2010), that both constant and floating NAV MMFs in Europe experienced investor runs in 2008 following Lehman Brother's bankruptcy and that the sponsor's capacity to support the fund was relevant in explaining the outflows in both types of funds. The evidence in these papers suggests that sponsor support plays a very important role in the way in which MMFs work ex-ante, by influencing the portfolio choices made by the funds, and ex-post, as a mechanism to prevent their liquidation. Understanding these links, and their implications for the stability of the MMF industry, is the main goal of my paper.

The next section introduces the model. Section 3 defines and characterizes the equilibrium. Section 4 discusses the forces at play in the model. Sections 5 contains the policy analysis and section 6 concludes. The appendix contains some more detailed institutional features of MMFs and all the proofs.

2 Model

The model has three-periods, $t = 0, 1, 2$, and one good. There are two types of agents in this economy: investors and managers. There is a continuum of measure 1 of each type of agent. Investors are risk averse with preferences given by $\mathbb{E}\left[\log\left(W_2^I\right)\right]$ where W_2^I is the investor's wealth at $t = 2$, and they are endowed with W_0^I units of the good at $t = 0$. Managers are risk neutral with preferences given by $\mathbb{E}\left[W_2^M\right]$ where W_2^M is the manager's wealth at $t = 2$. Managers are endowed with W_0^M units of the good at $t = 0$ and $E > 0$ at $t = 1$. There are two types of assets: a short-term, safe asset, and a long-term, risky asset. The safe asset is a one-period bond supplied perfectly elastically at price q_t in periods $t = 0, 1$. One unit of the safe asset bought at t pays 1 unit of the good at $t+1$. The risky asset is a two-period asset. A unit of the asset bought at $t = 0$ has a random payoff d at $t = 2$. The payoff structure is as follows:

$$d = \begin{cases} \bar{d} > 0 & \text{with probability } \pi \\ 0 & \text{with probability } (1-\pi) \end{cases} .$$

The probability π is a random variable whose realization is observed by everyone at $t = 1$. π is uniformly distributed over $[\underline{\pi}, \bar{\pi}]$ with $0 \leq \underline{\pi} < \bar{\pi} < 1$. The probability π can be interpreted as the quality of the risky asset.

The risky asset is traded in competitive markets at $t = 0, 1$, at prices p_0 and $p_1(\pi)$ respectively. At $t = 0$, the supply of the risky asset is given by $S(p_0)$, where $S'(p_0) > 0$. The supply of the asset in period 1 is fixed and equal to the amount of the risky asset traded at $t = 0$. Consistently with the regulation of MMFs, I assume that no short-selling is allowed.

To introduce motives for intermediation, I follow the limited market participation literature and assume that investors can only invest directly in the safe asset. As in He and Krishnamurthy (2012) I allow managers to invest in the risky asset on behalf of investors and act as financial intermediaries.

Managers can choose whether to become intermediaries. If they choose not to become intermediaries, they manage their own wealth. If they choose to offer intermediation services, they can manage assets on behalf of one investor, at most, by opening a fund. The intermediation relation is long-term: once an investor chooses a manager with whom to invest at $t = 0$, he cannot invest with other managers at $t = 1$. Managers incur in a fixed cost $C > 0$ if they manage funds for an investor. Each period t, a fund consists of the manager's wealth, W_t^M, and the amount the manager is managing for the investor, A_t^I. The manager makes all portfolio decisions on behalf of the fund and these decisions are non-contractible. For simplicity, I assume that the manager has to invest his own wealth in the same way in which he invests the investor's wealth, *i.e.*, managers can only manage one portfolio at a time.

The intermediation contract between a manager and his investor is the following. Managers charge a fraction f of the assets under management each period in return for intermediation services. For example, if a manager manages A_0^I for an investor in period 0 and A_1^I in period 1, he will collect fees fA_0^I at $t = 0$ and fA_1^I at $t = 1$. By investing with a manager, an investor becomes a shareholder of the fund run by the manager owning $(1 - f) A_t^I$ shares out of a total of $\left(W_t^M + A_t^I\right)$ outstanding shares of the fund. At $t = 1$, managers and investors can readjust their portfolios. A manager will choose how much of his assets under management to invest in the risky and safe assets. An investor will choose how much of his wealth, including his shares in the fund, to invest with his manager and how much to invest in the safe asset. The value of a share, or net asset value (NAV), at $t = 1$ is the value of the fund's assets at $t = 1$ divided the total number of outstanding shares.

To capture the "breaking the buck" feature of MMFs, I assume that if the value of the investor's share goes below x in period 1, the fund is liquidated, where $x < 1/q_0$. Upon early liquidation of the fund, the manager suffers a "spillover" loss $B\pi$, $B \geq 0$, and he cannot offer intermediation services at $t = 1$. Since liquidation is costly for managers, both in terms of forgone fees and spillover losses $B\pi$, a manager can choose to offer support to his investor and keep the value of the investor's shares at x.

Finally, I assume that the intermediation industry is competitive and that all the surplus of a match between an investor and a manger goes to the investor.

The timing of the model is as follows. At $t = 0$, each manager chooses whether to offer intermediation services. If a manager chooses to become an intermediary, he chooses the fund's portfolio. Otherwise he chooses how to invest his own wealth. Each manager chooses which fraction of his assets under management to invest in the risky asset, a_0^M, and which to invest in the safe asset, $\left(1 - a_0^M\right)$. At the same time, investors make their portfolio decision: they choose a fraction a_0^I of their wealth to invest with their manager and a fraction $\left(1 - a_0^I\right)$ to invest in the safe asset. Simultaneously, the price of the risky asset, p_0, is determined in a centralized, competitive market.

At $t = 1$, the probability of success of the risky asset, π, is realized. After observing this probability, portfolio and support decisions are made simultaneously and, at the same time, the price of the risky asset

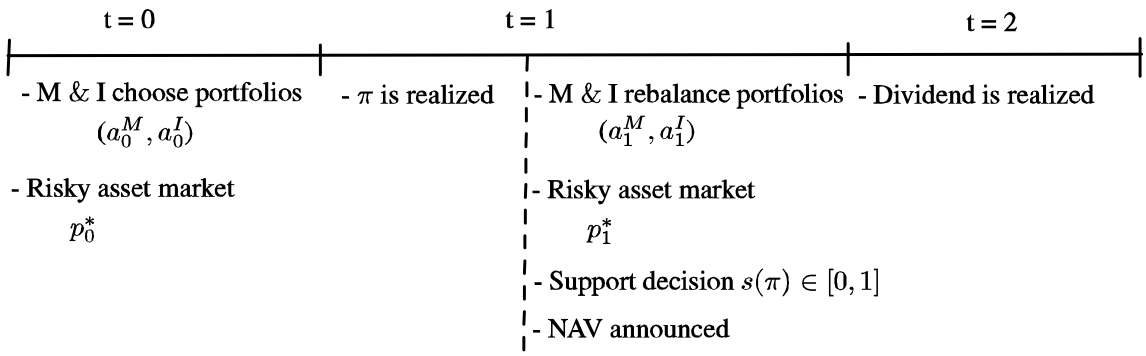

Figure 2: Timeline.

is determined in a competitive market. Each manager chooses a fraction a_1^M of his fund's assets to invest in the risky asset, a fraction $(1 - a_1^M)$ to invest in the safe asset, and the probability with which to offer support to investors, $s(\pi) \in [0,1]$. If a fund is not liquidated early, the investor chooses a fraction $a_1^I$of his wealth to invest with his manager and a fraction $(1 - a_1^I)$ to invest in the safe asset . Finally, at $t = 2$, the payoff of the risky asset is realized and all funds are liquidated. Figure 2 depicts this timeline.

2.1 Discussion of assumptions

The main difference between the model presented here and other banking and delegated portfolio models lies in the intermediation contract assumed, which incorporates key features of MMFs.[8] In particular, the demandable nature of shares and the open-endedness of MMFs are captured by the ability of investors to adjust their portfolio in the interim period. The liquidation threshold, x, introduced as part of the intermediation contract, gives incentives to managers to keep the net asset value stable and captures the eventual liquidation of a fund after a "breaking the buck" event. The cost of early liquidation of a fund imposed on managers, $B\pi$, is meant to capture the spillover losses a sponsor may suffer if one of his MMFs breaks the buck. Finally, the possibility of sponsor support is introduced by allowing the managers, who fulfill both the role of fund managers and sponsor companies in the model, to transfer resources to investors to prevent the fund from being liquidated.

There are costs for investors associated with the liquidation of a MMF, mainly due to the delay in the availability of their funds. For example, after the liquidation of Reserve Primary Fund, it took investors up to 18 months to get their money out of the fund. During this time, the fraction of the investors' wealth that was held up in the fund earned almost no return. The exclusion of an investor from the risky asset market after the liquidation of the fund is meant to capture this delay. This extreme assumption is made for ease of exposition. Qualitatively, the results in the model would still go through if investors were allowed to switch

[8] Given that the MMF industry appeared as a tool for regulatory arbitrage, the model developed in this paper takes the intermediation contract as given abstracting from optimal compensation structures in portfolio management relationships such as the ones analyzed by a large literature started by Bhattacharya and Pfleiderer (1985).

managers at $t = 1$ by incurring a positive cost.

The asset structure is meant to capture the maturity mismatch problem inherent to MMFs and the trade-off between maturity, risk and return. The short-term safe asset can be interpreted as a treasury bill while the long-term, risky asset can be though of as commercial paper. The uncertainty structure embedded in the payoff of the risky asset implies that there are two sources of risk for commercial paper. On one hand, the quality of commercial paper is subject to shocks before the maturity date (downgrades). On the other, at maturity, commercial paper is subject to default. The assumption of no short-selling is consistent with the regulation of MMFs.

The extra endowment E managers get at time 1 represents the aggregate liquidity in the market. For high values of E, there will be enough demand for the risky asset in the interim period so that the liquidation cost of the asset is equal to its expected return. For lower values of E, the liquidity in the asset market at $t = 1$ will be low and the liquidation price of the risky asset will involve a fire sale discount. In this case, there will be cash in the market pricing and the liquidation price of the asset will depend on whether funds remain open or not. In the rest of the paper I will focus on the case in which liquidity is scarce and fire sales occur.

The functional form of the utility functions assumed makes the model tractable. The managers' risk neutrality makes the calculation of the support decision independent of wealth levels other than through the managers' ability to offer support. In the same way, investors having constant relative risk aversion implies that the portfolio choice in the interim period depends only on the quality of the risky asset, π, and on its price. This investment decision is independent of the investors' wealth and, therefore, of the support decision and the choices at $t = 1$. These simplifications make it possible to have closed form solutions for prices and support thresholds at $t = 1$.

The assumption that $x < 1/q_0$ implies that a manager can always choose a risky portfolio which is still conservative enough to always avoid breaking the buck. If the only way for the manager to avoid breaking the buck in all states π was to invest in a safe portfolio, managers would have to choose between breaking the buck with positive probability or not opening a fund at all. Since investors can invest in the safe asset on their own, they would not pay fees to a manager that chose a safe portfolio.

3 Equilibrium

The return of a unit invested with a manager depends on the asset prices, on the manager's portfolio choice and on the manager's support decision. Hence, to decide how much to invest with a manager, investors need to anticipate not only the equilibrium asset prices, but also their managers' actions. In the same way, a manager's payoffs depend on the amount of resources he manages for investors. Therefore, managers need to anticipate asset prices and their investors' actions when making choices. This is captured in the definition of equilibrium.

Definition 1 *A symmetric equilibrium is a set of price functions* $\{p_0^*, p_1^*(\pi)\}$, *functions for portfolio choices* $\left\{a_0^{M*}, a_0^{I*}, a_1^{M*}(\pi), a_1^{I*}(\pi)\right\}$, *a support probability function* $s^*(\pi) \in [0,1]$, *and a fee* f^* *such that:* $\left(a_0^{I*}, a_1^{I*}(\pi)\right)$ *solve each investors' problem taking prices and his manager's decisions as given,* $\left(a_0^{M*}, a_1^{M*}(\pi), s^*(\pi)\right)$ *solve each manager's problem taking prices and his investor's decisions as given, the risky asset market clears at* $t = 0, 1$, *and managers are indifferent between managing an investor's funds and not doing so.*

The equilibrium can be computed by backwards induction. I will divide the characterization of equilibrium in two parts: time 0 and time 1. In the following subsection I characterize the agents' problems at $t = 1$ for a given realization of π, and given portfolio choices a_0^M and a_0^I and fees f. Then, I move to the decisions at $t = 0$ which take into account the equilibrium choices at $t = 1$.

3.1 t=1

All decisions at $t = 1$ depend on the choices and equilibrium objects determined at $t = 0$, that is, on a_0^M, a_0^I, f and p_0. To simplify the notation, I will mostly ignore this dependence when writing down equilibrium objects in the subgame at $t = 1$.

3.1.1 Manager's portfolio choice at t=1

At $t = 1$, a manager with wealth W_1^M who receives fees fA_1^I chooses a fraction a_1^M of his wealth to invest in the risky asset to maximize his expected wealth. He solves

$$\max_{a_{1,i}^M \in [0,1]} \mathbb{E}_d \left[\left(a_1^M \frac{d}{p_1} + \left(1 - a_1^M\right) \frac{1}{q_1} \right) \left(W_1^M + fA_1^I\right) \middle| \pi \right].$$

The manager's risk neutrality implies that his portfolio choice depends only on the expected returns of the assets in which he can invest. If the expected return of the risky asset is larger than that of the safe asset, if $\pi\bar{d}/p_1 > 1/q_1$, he will invest everything in the risky asset, i.e., $a_{1,i}^{M*} = 1$. Analogously, if the return of the safe asset is larger than the expected return of the risky asset, i.e., if $\pi\bar{d}/p_1 < 1/q_1$, a manager will invest everything in the safe asset, i.e., $a_{1,i}^{M*} = 0$. If the safe and risky assets pay the same in expectation, i.e., if $\pi\bar{d}/p_1 = 1/q_1$, the manager will be indifferent between any portfolio. Notice that the manager's portfolio choice at $t = 1$ is independent of his own wealth and of the amount investors invest with him.

3.1.2 Investor's problem at t=1

At $t = 1$, an investor will choose a fraction $a_{1,i}^I$ of his wealth to invest with his manager i to maximize his own expected wealth at $t = 2$. The investor's portfolio decision will depends on the behavior he anticipates for his manager, which, in turn, depends only on the observable state π. Therefore, an investor with wealth W_1^I who anticipates his manager will choose to invest a fraction $a_{1,i}^M$ in the risky asset solves

$$\max_{a_{1,i}^I \in [0,1]} \mathbb{E}_d \left[\log \left(\left(a_{1,i}^I (1-f) \left(a_{1,i}^M \frac{d}{p_1} + \left(1 - a_{1,i}^M\right) \frac{1}{q_1} \right) + \left(1 - a_{1,i}^I\right) \frac{1}{q_1} \right) W_1^I \right) \middle| \pi \right]$$

An investor would never pay fees for something he could do himself. Therefore, investors will choose not to invest with their manager if $\pi\bar{d}/p_1 \leq 1/q_1$. Furthermore, they will only invest with their manager if the expected return of doing so is greater than the return of the safe asset, *i.e.*, if $\pi(1-f)\bar{d}/p_1 > 1/q_1$. Therefore, the optimal portfolio choice for investors is

$$a^I_{1,i}(p_1,\pi) = \frac{\max\left\{\pi(1-f)\frac{\bar{d}}{p_1} - \frac{1}{q_1}, 0\right\}}{(1-f)\frac{\bar{d}}{p_1} - \frac{1}{q_1}}.$$

As it is usual with CRRA preferences, the share of wealth invested with the manager is independent of the wealth level. As I mentioned above, this assumption makes the problem easier to track. To keep notation simple, I will define $A^i_{0,i} \equiv a^I_{0,i}W^I_0$ and $A^I_{1,i}(p_1,\pi) \equiv a^I_{1,i}(p_1,\pi)W^I_{1,i}(p_1)$.

3.1.3 Demand for risky assets at t=1

The demand for the risky asset at $t = 1$ of an individual manager i with wealth $W^M_{1,i}(p_1)$ is

$$D^i(p_1,\pi) = \begin{cases} a^M_{1,i}(p_1,\pi)\left(A^I_{1,i}(p_1,\pi) + W^M_{1,i}(p_1)\right)/p_1 & \text{if the fund continues} \\ a^M_{1,i}(p_1,\pi)W^M_{1,i}(p_1)/p_1 & \text{if the fund is liquidated} \end{cases}$$

where $W^I_{1,i}(p_1)$ is the wealth level for manager $i's$ investor when the price is p_1.[9] If the fund continues at $t = 1$, the manager will manage his own wealth and whatever his investor gives him, $A^I_1(p_1,\pi)$. If the fund is liquidated, a manger will only manage his own wealth.

Given these individual demands for the risky asset, the equilibrium liquidation price $p^*_1(\pi)$ will be such that

$$p^*_1(\pi)\int\left[D^i(p^*_1(\pi),\pi) - a^M_{0,\,i}\left(a^I_{0,i}W^I_0 + W^M_0\right)/p_0\right]di = 0$$

where the last term inside brackets is the supply of the risky asset which is fixed and equal to the total amount traded at $t = 0$.

The wealth levels of managers and investors will depend, not only on whether the fund continues, but also on whether sponsor support is offered. Therefore, to compute the equilibrium price as a function π one needs to consider four possible cases: π such that no sponsor support is needed, π such that all sponsors offer support if needed, π such that sponsors offer support with probability $s \in (0,1)$ when it is needed, and π such that sponsor support is needed to prevent the liquidation of the funds, but it is not provided. Then, the equilibrium liquidation price of the risky asset will be

$$p^*_1(\pi) = \begin{cases} p^{NS}_1(\pi) & \text{if no support is needed} \\ p^S_1(\pi) & \text{if sponsors offer support always } s^*(\pi) = 1 \\ p^{SS}_1(\pi) & \text{if sponsors offer support with probability } s^*(\pi) \in (0,1) \\ p^L_1(\pi) & \text{if support is needed but not offered} \end{cases}$$

The characterizations of the price functions $p^{NS}_1(\pi)$, $p^S_1(\pi)$, $p^{SS}_1(\pi)$, and $p^L_1(\pi)$ can be found in the appendix.

[9] Expressions for these wealth levels can be found in the appendix.

3.1.4 Equilibrium Support Decision

In a symmetric equilibrium, all managers will choose the same portfolios and support strategies. Therefore, if a manager does not need to offer support the relevant price is p_1^{NS}, if he strictly prefers to offer support it is p^S, if he is indifferent between offering support and liquidating the fund it is $p_1^{SS}(\pi)$, and if the fund is liquidated it is p^L.

A manager's support decision will depend on the equilibrium price through the market value of the shares or NAV. The net asset value, $n\left(p_1, a_0^M\right)$, is the return on the portfolio of a manager that chose to invest a fraction a_0^M of the fund in the risky asset when the liquidation price of the risky asset is p_1, *i.e.*, $n\left(p_1, a_0^M\right) = a_0^M p_1/p_0 + \left(1 - a_0^M\right)/q_0$. As long as $n\left(p_1, a_0^M\right) \geq x$, the manager will not have any need to offer support for his investor. This will be the case, whenever the risky asset's quality is high enough.

Proposition 1 *Given a portfolio choice for managers at $t = 0$ a_0^M, there exists a threshold $\pi_x\left(a_0^M\right)$ such that in a symmetric equilibrium of the subgame at $t = 1$ managers will not need to offer support to investors if $\pi \geq \pi_x\left(a_0^M\right)$.*

The proof of this proposition follows from the fact that the relevant price function when no support is offered is $p_1^{NS}(\pi)$ and from monotonicity of $p_1^{NS}(\pi)$ in π.

If support is needed for a fund to continue, *i.e.*, if $\pi < \pi_x\left(a_0^M\right)$, given p_1 a manager will choose choose his support decision to solve

$$\max_{s\in[0,1]} s\frac{\pi\bar{d}}{p_1}\left(n\left(p_1, a_0^M\right)\left(W_0^M + fA_0^I\right) + E + fA_1^I\left(p_1,\pi\right) - \left(x - n\left(p_1, a_0^M\right)\right)(1-f)A_0^I\right)$$
$$+(1-s)\left(\frac{\pi\bar{d}}{p_1}\left(n\left(p_1, a_0^M\right)\left(W_0^M + fA_0^I\right) + E\right) - B\pi\right)$$

s.t.

$$n\left(p_1, a_0^M\right)\left(W_0^M + fA_0^I\right) + E + fA_1^I\left(p_1,\pi\right) \geq \left(x - n\left(p_1, a_0^M\right)\right)(1-f)A_0^I, \tag{1}$$

where $s(\pi)$ is the probability with which a manager chooses to offer support in state π. Since $p_1(\pi) \leq \pi\bar{d}/q_1$, the expected return of the managers' investment at $t = 1$ is $\pi\bar{d}/p_1(\pi)$. If the manager offers support in state π, i.e. if $s = 1$, he will manage a portfolio comprised of his shares in the fund, $n\left(p_1, a_0^M\right)\left(W_0^M + fA_0^I\right)$, his endowment in period 1, E, the incoming fees from keeping the fund open, fA_1^I, minus the cost of offering support, $\left(x - n\left(p_1, a_0^M\right)\right)(1-f)A_0^I$. If he does not offer support when support is needed to keep the fund open he will invest his shares of the fund and his endowment but he will suffer a loss $B\pi$ from liquidating the fund early. Moreover, the manager's support decision does not only depend on his willingness to offer support. The amount of support a manager is able to offer is determined by the amount of resources he has available at $t = 1$. This is captured by constraint (1).

The maximization presented above implies that the manager will choose to offer support to the investors if the benefit of offering support offsets the costs of doing so. That is if support is needed and

$$\begin{aligned}&\frac{\pi\bar{d}}{p_1(\pi)}\left(\left(x-n\left(p_1(\pi),a_0^M\right)\right)(1-f)A_0^I-fA_1^I\left(p_1(\pi),\pi\right)\right)\\ \leq\quad&\min\left\{B\pi,\frac{\pi\bar{d}}{p_1(\pi)}\left(n\left(p_1\left(\pi,a_0^M\right),a_0^M\right)\left(W_0^M+fA_0^I\right)+E\right)\right\}.\end{aligned}\tag{2}$$

The left hand side of this expression represents the opportunity cost of offering support to the investor, net of incoming fees. The right hand side includes both the manager's willingness and ability to offer support. The first term inside the curly brackets captures the manager's willingness to offer support. Since the manager loses $B\pi$ if the fund is liquidated early, he will be willing to offer support, on top of the incoming fees, to up to an amount equal to his loss, *i.e.*, $B\pi$. The second term, captures the manager's ability to offer support. The manager will only be able to offer support up to the amount of resources he owns. To offer support, the manager has to be both willing and able to offer support. This is captured by the minimum operator.

As one can see from 2, the manager's support decision will depend on the liquidation price of the risky asset and, through it, on the other managers' support decisions.

Proposition 2 *Given a portfolio choice for managers at $t=0$ a_0^M, there exist a thresholds $\pi^*\left(a_0^M\right)$ and $\pi^{**}\left(a_0^M\right)$ such that:*

(i) if $\pi^*\left(a_0^{M*}\right)\leq\pi<\pi_x\left(a_0^{M*}\right)$ there is a symmetric equilibrium of the subgame at $t=1$ in which all managers offer support

(ii) if $\pi\in\left[\pi_x\left(a_0^{M*}\right),\bar{\pi}\right]\cup\left[\underline{\pi},\pi^{**}\left(a_0^M\right)\right)$ there is a symmetric equilibrium of the subgame at $t=1$ in which all managers choose not to offer support

The proof of this proposition relies on the monotonicity and affinity of $p_1^S(\pi)$ and $p_1^L(\pi)$ in π. The appendix characterizes the thresholds $\pi_x\left(a_0^M\right)$, $\pi^*\left(a_0^M\right)$, and $\pi^{**}\left(a_0^M\right)$, and using these characterizations, the following proposition can be shown.

Corollary 1 *If $\pi^*\left(a_0^M\right)<\pi^{**}\left(a_0^M\right)$ there are multiple equilibria in the subgame at $t=1$ when $\pi\in\left[\pi^*\left(a_0^M\right),\pi^{**}\left(a_0^M\right)\right]$. In particular, there is an equilibrium in which all managers offer support, one in which all managers choose not to offer support, and one in which managers choose to offer support with probability $s\in(0,1)$.*

The corollary follows from proposition 2 and it highlights the strategic complementarities that can be present in the managers' support decisions. If all managers choose (not) to offer support, the net demand for the risky asset is high (low) and the equilibrium liquidation price for the risky asset is high (low). Given this high (low) price, the cost of offering support is low (high) for an individual managers and, thus, he has more (less) incentives to offer support. This strategic complementarity between the managers support decision through the equilibrium liquidation price of the asset is only present in the model because support is voluntary. For the remainder of this paper, I will assume that whenever there are multiple equilibria, managers will coordinate in the equilibrium in which all managers offer support.

Proposition 3 *If $\pi^{**}\left(a_0^M\right) < \pi^*\left(a_0^M\right)$ there is a unique equilibrium in the subgame at $t = 1$ for each value of π and $s(\pi) \in (0,1)$ for $\pi \in \left(\pi^{**}\left(a_0^M\right), \pi^*\left(a_0^M\right)\right)$.*

If there is a unique equilibrium for all values of π, i.e., if $\pi^{**} \leq \pi^*$, managers may choose to offer support with probability $s \in (0,1)$. For $\pi \in (\pi^{**}, \pi^*)$ an individual manager chooses to offer support when he expects no other manager to offer support and he chooses not to offer support when he expects all other managers to offer support. Therefore, there is no symmetric equilibrium in pure strategies for the support decision. Nevertheless, there is an equilibrium for these values of π in which managers are indifferent between offering support and not doing so if they expect all other managers to offer support with positive probability but not for sure and, therefore, he finds it optimal to do follow the same strategy other managers follow.

Runs on the risky asset market The managers' support decisions determine the demand for the risky asset in the interim period and, through it, the liquidation price of the risky asset. If a manager chooses not to offer support when support is needed to keep the fund open, his fund breaks the buck and is liquidated. Upon liquidation, the manager cannot offer intermediation services at $t = 1$ and his investor is excluded from the risky asset market. This implies that the manager's demand for the risky asset is going to be lower if he chooses not to offer support than if he keeps the fund open and manages funds for his investor.[10] If all managers choose not to offer support, the total demand for the risky asset in the interim period and, therefore, the liquidation price of the risky asset will be lower than if all funds remained open.

Moreover, the managers' support decisions depend on the liquidation price of the risky asset. This liquidation price will determine the costs and the benefits of offering support (as one can see from 2). To illustrate this mechanism, suppose there are no spillover losses, *i.e.*, $B = 0$. On the one hand, a low liquidation price of the risky asset translates into a low NAV and a high cost of offering support. On the other hand, a low price of the risky asset implies a high expected return of investing in the risky asset and increases investors' incentives to invest with managers. The higher volume of intermediation increases the fees earned by the managers and the benefits of offering support and avoiding the liquidation of the fund. If the increase in the cost dominates the increase in the benefit of offering support, there may be strategic complementarities in the managers' support decisions and multiple equilibria may arise. As seen from corollary 1, these complementarities arise when $\pi^* < \pi^{**}$.

When $\pi^* < \pi^{**}$, if an individual manager expects all other managers (not) to offer support and liquidate their funds, he expects a high (low) demand for the risky asset in the interim period, a high (low) liquidation price of the risky asset, and a low (high) cost of offering support which will increase (decrease) the manager's incentives to offer support. This source of complementarity gives rise to self fulfilling equilibria that may lead to runs on the risky asset market. These runs are different from the canonical bank runs: they are not runs of investors on intermediaries but runs of intermediaries on each other through fire sales in the risky

[10] Assuming that the amount of support does not exceed the total amount investors invest with their managers if they receive support.

asset market.[11]

3.1.5 Equilibrium price

Given the equilibrium support decision for the managers and the portfolio choice a_0^M one can compute the equilibrium liquidation price of the risky asset.

Assumption B is such that the amount of support offered does not exceed the total amount investors invest with their manager when they receive support, i.e.,

$$A_1^I < fA_1^I + \min\left\{B\frac{p_1^L}{\bar{d}}, n\left(p_1^L, a_0^M\right)\left(W_0^M + fA_0^I\right) + E\right\}.$$

This assumption holds for $B = 0$ and for values of B low enough. Moreover, it implies that the demand for the risky asset is lower when sponsors decide not to offer support than when they do and it guarantees the monotonicity of the price function.[12] For the remainder of the paper, I will assume this assumption holds.

Proposition 4 *Under assumption* 3.1.5*, the price function* $p_1^*\left(\pi, a_0^M\right)$ *is non-decreasing* π *for all* $\pi \in [\underline{\pi}, \bar{\pi}]$.

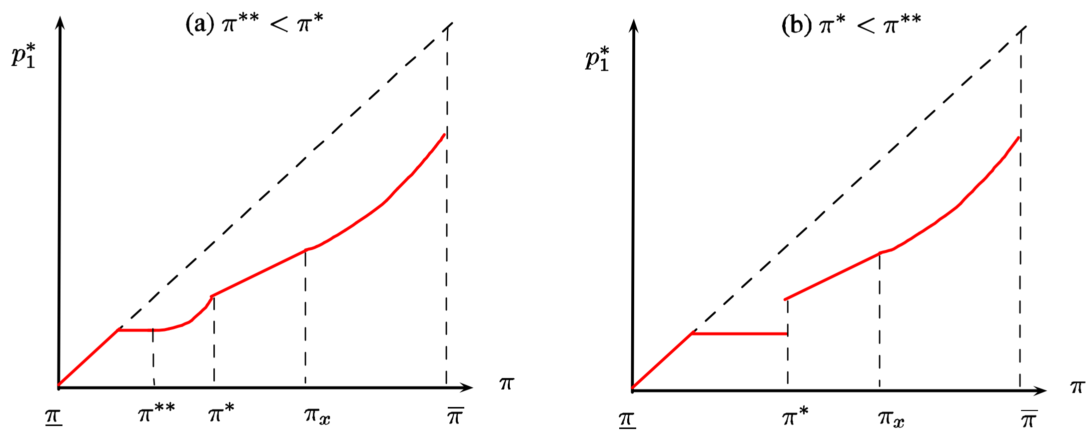

Figure 3: The red solid line is the liquidation price of the risky asset at $t = 1$ as a function of π. The black dotted line is the expected discounted dividend payed by the risky asset.

The liquidation price of the risky asset will be higher the higher the probability of success π. A higher realization of π implies a higher expected return of investing in the risky asset and a higher demand for it. Since the supply of the risky asset at $t = 1$ is fixed, the equilibrium price has to increase with π for the market to clear.

[11] The possibility of fire sales arises due to the *cash in the market pricing* in the risky asset market as in Allen and Gale (1994). See Shelifer and Vishny (2011) for a review of fire sales in the finance and macroeconomics literature.

[12] This assumption doesn't seem unrealistic. Between 2007 and 2010, the maximum amount of support offered by a sponsor accounted for 3% of the fund.

Figure 3 depicts the price function when (a) $\pi^{**} < \pi^{*}$ and when (b) $\pi^{*} < \pi^{**}$ (assuming that managers coordinate on the support equilibrium). When all funds are liquidated, *i.e.*, $\pi < \min\{\pi^{**}, \pi^{*}\}$, only managers participate in the risky asset market. Since managers are risk neutral, they will either invest everything they have in the risky asset or they will be indifferent between investing in the risky asset and in the safe asset. If the demand for the risky asset is determined by the cash held by managers, *i.e.*, by $\left(1 - a_0^M\right)\left(W_0^M + fA_0^I\right) + E$, the market clearing price will be given by p_1^L, such that

$$\frac{\left(1 - a_0^M\right)\left(W_0^M + fA_0^I\right) + E}{p_1^L} = (1 - f)\, a_0^M A_0^I \tag{3}$$

where the right hand side is the amount of risky asset held by investors indirectly through their shares. Moreover, in equilibrium $p_1^*(\pi) \leq \bar{d}\pi q_1$.Otherwise, since the supply of the risky asset at $t = 1$ is fixed and positive, the demand for the risky asset at $t = 1$ would be 0 and the market would not clear. Thus, if the funds are liquidated, the equilibrium liquidation price will be $p_1^*(\pi) = \min\left\{\bar{d}\pi q_1, p_1^L\right\}$. If $p_1^*(\pi) = \bar{d}\pi q_1$ when liquidation occurs, $a_1^M(p_1, \pi)$ adjusts to clear the market.

If at least some funds continue operating at $t = 1$, *i.e.*, $\pi \geq \min\{\pi^{**}, \pi^{*}\}$, investors can invest in the risky asset, albeit indirectly. In this case, the demand for the risky asset also depends on the investors' propensity to invest with their managers, $a_1^I(p_1, \pi)$. Since this propensity is increasing in π, the demand for the risky asset is increasing in π, and so is the equilibrium liquidation price of the risky asset.

When $\pi^{*} < \pi^{**}$ there may be a jump in the price function at π^{*} which captures the sharp decrease in the demand for the risky asset: for $\pi > \pi^{*}$ all funds are open whereas for $\pi < \pi^{*}$ all funds are liquidated.

3.1.6 Individual support decisions

In order to characterize the manager's problem completely at $t = 0$, one needs to understand the individual manager's support decision and how this decision depends on the manager's portfolio choice in the initial period.

Proposition 5 *There exist unique thresholds* $\pi_{x,i}\left(a_{0,i}^M\right)$ *and* $\pi_i^*\left(a_{0,i}^M\right)$ *such that an individual managers will strictly prefer to offer support, iff* $\pi \in \left(\pi_i^*\left(a_{0,i}^M\right), \pi_{x,i}\left(a_{0,i}^M\right)\right)$.

This proof of this proposition follows from the monotonicity of the price function and can be found in the appendix. The intuition for these results is analogous to the one presented for the aggregate thresholds π_x and π^*.

3.2 t=0

3.2.1 Investor's problem

An investor will make his portfolio choice at $t = 0$ to maximize his expected utility anticipating the equilibrium that will be played at $t = 1$ for each realization of π, and taking his manager's portfolio choices and prices as given.

An investor with manager i solves

$$\max_{a_0^I\in[0,1]} \mathbb{E}_\pi\left[\log\left(W_2^I\left(a_0^I; a_{0i}^M, \pi_i^*, \pi_{x,i}, \bar{s}\left(\pi\right), \pi\right)\right)\right]$$

where $\bar{s}_i\left(\pi\right) \in \{0,1\}$ is a manager's realized support decision and $W_2^I\left(a_0^I;\cdot\right)$ is his wealth at $t = 2$. This wealth depends on the investor's portfolio choice at $t = 0$, on manager i's portfolio choice at $t = 0$ and support decisions at $t = 1$, and on the realized quality of the long term asset, π. The investor's problem is concave in a_0^I and has a unique solution.

3.2.2 Manager's problem

Since for $\pi < \pi^*\left(a_0^M\right)$ the manager either prefers not to offer support or is indifferent between offering support and not doing so, I can solve the manager's problem by assuming that he will liquidate the fund if $\pi < \pi^*\left(a_0^M\right)$. Then, manager i solves $V_0^M\left(W_0^M; f\right) =$

$$\max_{a_{0,i}^M\in[0,1]} \mathbb{E}_\pi\left(W_2^M\left(a_{0,i}^M; \pi_{x,i}\left(a_{0,i}^M\right), \pi_i^*\left(a_{0,i}^M\right), a_0^I, \pi\right)\right) - \mathbb{E}_\pi\left(\mathbf{1}\left\{\pi \le \pi_i^*\left(a_{0,i}^M\right)\right\} B\pi\right)$$

where $\mathbf{1}$ is the indicator function, and $W_2^M\left(a_{0,i}^M;\cdot\right)$ is the manager's wealth at $t = 2$. W_2^M depends on the manager's portfolio choice at $t = 0$ directly, and indirectly through the support thresholds. Moreover, it depends on the investor's portfolio choice at $t = 0$ and on the quality of the risky asset π. The manager makes his portfolio choice taking his investor's decisions and prices as given.

As it is usual in the presence of threshold decisions, the manager's objective function is not well-behaved. The liquidation rule and the possibility of offering support change the manager's attitude towards risk for different portfolio choices, $a_{0,i}^M$, from risk neutral to risk averse to risk lover. Nevertheless, the manager's problem can be fully characterized: there are two possible candidates at which the maximum would be attained: the highest $a_{0,i}^M$ such that there is no risk of liquidating the fund, and $a_0^M = 1$. To avoid dealing with cases, I will use numerical examples in the policy section to illustrate the mechanisms at play. The manager's objective function is characterized in the appendix as is W_2^M .

3.2.3 Risky asset market at t=0

The equilibrium price in the risky asset market at $t = 0$, p_0^*, is determined by

$$S\left(p_0^*\right) = \frac{a_0^M}{p_0^*}\left(W_0^M + A_0^I\right)$$

where the right hand side is the total amount invested in the risky asset at $t = 0$, *i.e.*, the fraction managers choose to invest in the risky asset, a_0^M, times the size of the funds, $W_0^M + a_0^I W_0^I$, divided by the price of the risky asset, p_0.

3.2.4 Intermediation Fees

Finally, to close the model, since investors have all the bargaining power, the equilibrium fees are determined by the indifference condition for managers. If a manager chooses to open a fund, he incurs in operating costs

$C > 0$ and gets utility $V_0^M\left(W_0^M; f^*\right)$. If he chooses not to open a fund, he invests his own endowment to maximize his expected wealth at $t = 2$. Therefore, in equilibrium, intermediation fees f^* are such that

$$V_0^M\left(W_0^M; f^*\right) - C = \max_{a\in[0,1]} \int_{\underline{\pi}}^{\bar{\pi}} \left(\frac{\pi \bar{d}}{p_1^*(\pi)}\left(\left(a\left(\frac{p_1^*(\pi)}{p_0} - \frac{1}{q_0}\right) + \frac{1}{q_0}\right) W_0^M + E\right)\right) d\pi.$$

4 Results

All the proofs of the propositions in this section are in the appendix.

Proposition 6 *If $B = 0$, $a_0^{M*} = 1$ in a symmetric equilibrium.*

Increasing the position in the risky asset has the following three effects for the manager: it increases the expected return on the manager's wealth, it increases the expected collected fees from intermediation at $t = 1$, and it also increases the probability of breaking the buck. Without any extra cost from liquidating the fund early other than the forgone fees, in equilibrium, the benefits from taking risk offset the costs, and managers are better off taking as much risk as they can. This is shown by proposition 6.

The model is consistent with several stylized facts documented in the literature. Chernenko and Sunderam (2012), Christoffersen and Musto (2002), and Kacperczyk and Schnabl (2013) document a strong performance-flow relation in the MMF industry: they show that MMFs inflows are highly responsive to the fund's past returns. As shown in the next proposition, the model developed in this paper is consistent with this finding.

Proposition 7 *The amount of assets managed for investors at $t = 1$, A_1^I, is increasing in the net asset value, $n\left(p_1, a_0^M\right)$.*

The total amount investors invest with managers at $t = 1$, A_1^I, is a stock and it is correlated 1 to 1 with the flow into the funds, which is given by $A_1^I(\pi) - A_0^I$. Moreover, the funds' performance is measured by the return on the investors' shares, which is $n\left(p_1, a_0^M\right)$ when no support is offered, and x otherwise. Therefore, the proposition above shows that the model captures the positive performance-flow relation present in the data.[13]

Moreover, Kacperczyk and Schnabl (2013), using data from 2007 to 2010, document that MMFs sponsored by companies that also offered non-money market mutual funds and other financial services took on less risk, and that funds sponsored by financial intermediaries with limited financial resources took on less risk. The next proposition shows that the model is consistent with this stylized fact.

Proposition 8 *Suppose $B = 0$ for all managers but for manager j. Then, in equilibrium, the risk taken by manager j at $t = 0$ will be decreasing in B_j and he will choose $a_{0,j}^M \in \left\{a_0^{MS}, 1\right\}$ where $a_0^{MS} = \max a_{0,i}^M$ s.t. $\pi_{x,i}\left(a_{0,i}^M\right) = \underline{\pi}$.*

[13] This recult comes from the CRRA utility assumption for investors.

Larger B_j implies a higher cost of liquidating the fund early for the manager. Investing in the risky asset has two effects on the manager's expected wealth. On one hand, a higher exposure to the risky asset increases the manager's portfolio expected return and, from proposition 7, the expected fees collected from managing the investor's funds at $t = 1$. On the other hand, a higher $a_{0,j}^M$ increases the probability of early liquidation of the fund which increases the expected losses suffered by the manager due to this early liquidation. When $B = 0$, the latter effect is not present and the manager is always better off by investing everything in the risky asset. If B_j is high enough, the loss from breaking the buck is too high and managers are better off choosing a portfolio such that the fund is never liquidated early. In this last case, managers prefers to forgo expected return to avoid losses from breaking the buck, which are too high.

5 Policy analysis

Because of the systemic importance of MMFs, their regulation has been at the center of the policy discussion both in the U.S. and in Europe. On July $23, 2014$, after several years of debate, the Securities and Exchange Commission (SEC) voted to impose new regulations on the MMF industry. The European Commission is also in favor of a MMF reform. The mutual fund industry opposes these changes and argues that further regulations would make the MMF industry less profitable and reduce the availability of short-term funding.[14]

The regulations that are being considered target some key characteristics of MMFs, mainly the stable NAV and the support offered by sponsors to the funds. In the U.S., the SEC voted to impose redemption fees on investors in retail MMFs, and to force institutional MMFs to abandon the stable NAV in favor of a floating NAV. Redemption fees will increase investors' costs of redeeming shares and, therefore, reduce their incentives to run on the MMFs. By adopting a floating NAV, MMFs would not be subject to liquidation after breaking the buck. Alternatively, the European Commission proposed that stable NAV funds convert to variable NAV funds or hold a 3 percent capital buffer, and that sponsor support be prohibited unless approved by the appropriate regulator. These proposed regulations aim to make MMFs more similar to other financial intermediaries: more like regular mutual funds in the case of adopting a floating NAV and more like banks in the case of a capital buffer.

5.1 Floating vs. Stable NAV

The main regulation proposed by the SEC is to abandon the stable NAV in favor of a floating one. Adopting a floating NAV would diminish the risk of sudden redemptions by making MMFs like any other mutual fund and eliminating the possibility of breaking the buck. In the context of the model developed in this paper, going from a stable to a floating NAV system is equivalent to going from $x > 0$ to $x = 0$. If $x = 0$, the funds

[14] See McCabe (2011), Mendelson and Hoerner (2011), Lacker (2011), Squam Lake Group (2011), Volcker (2011), Hanson et al. (2012), McCabe et al. (2012), and http://www.preservemoneymarketfunds.org/the-impact-on-you/. Last visited November 25, 2012.

managed by the managers become regular mutual funds and there is no breaking the buck rule. In this case, no support is ever offered: 1 unit transferred to investors represents $fa_1^I < 1$ worth of fees for the manager. The manager does not have any incentive to offer support.

The portfolio decisions at $t = 1$ presented in the benchmark model are independent on x, and, thus, remain unchanged when going from $x > 0$ to $x = 0$. In terms of the thresholds presented in the previous section, $\pi_{x,i}\left(a_{0,i}^M; a_0^M\right) = \pi_i^*\left(a_{0,i}^M; a_0^M\right) = 0$ for all $\left(a_{0,i}^M; a_0^M\right) \in [0,1]^2$ when $x = 0$. The manager's problem at $t = 1$ now becomes

$$\max_{a_{0,i}^M \in [0,1]} \mathbb{E}_\pi \left[\frac{\pi \bar{d}}{p_1^*(\pi)} \left(n\left(p_1(\pi), a_{0,i}^M\right) \left(W_0^M + f a_0^I W_0^I\right) + E \right) \right]$$
$$+\mathbb{E}_\pi \left[\frac{\pi \bar{d}}{p_1^*(\pi)} f a_1^{I*}(\pi) \left(n\left(p_1(\pi), a_{0,i}^M\right)(1-f) + \left(1 - a_0^{I*}\right) \frac{1}{q_0} \right) W_0^I \right]$$

This implies that there is no downside to taking risk for the risk neutral managers and, therefore, $a_0^{M*f} = 1$.

The payoff for investors from investing with the manager is affected in many different ways. If all decisions at $t = 0$ remained unchanged and the liquidation price of the risky asset was kept fixed, investors would lose insurance going from a stable NAV system to a floating one, *i.e.*, they would lose the transfer they were receiving from their managers in all the states in which support was offered. Figure 4 illustrates this argument. However, when all funds go from a stable to a floating NAV system, the liquidation price of the risky asset changes. In particular, keeping all decisions at $t = 0$ unchanged, the equilibrium liquidation price of the risky asset is (weakly) higher when $x = 0$. To see this, first note that the demand for the risky asset increases in those states in which some support was offered when $x > 0$. When support is offered, for each unit managers transfer to investors, only a fraction a_1^I goes to the risky asset market, whereas if managers kept this for themselves they would invest it all in the risky asset. Moreover, for those states in which the fund was liquidated but investors would still have liked to invest in the risky asset via the managers, demand also goes up. When $x > 0$, they were prevented from investing because the funds were closed, but now they can do so. These effects drive the demand for the risky asset up, and increase the equilibrium liquidation price. This change is illustrated in figure 5.

Therefore, when one considers the change in the liquidation price of the risky asset, one can see that investors prefer $x > 0$ in those states in which support is offered with high probability but are better off when $x = 0$ in the states in which the fund is very likely to be liquidated. Figure 6 illustrates this trade-off.

So far, this analysis keeps all decisions at $t = 0$ fixed. Figure 7 compares the equilibria for different values of W_0^M for $x > 0$ and $x = 0$. When going from a stable to a floating NAV system, managers need to be compensated less to offer intermediation services because they don't risk facing losses B. Putting this together with the partial equilibrium analysis presented above, one can see that there are countervailing effects on the incentives for investors to invest with managers. In the example computed here, both the risk and the expected return of investing with a manager decrease for investors. What happens with the total amount intermediated depends on which effect is larger. In the example shown here, the intermediation level, and thus the supply of liquidity, increase when going from a stable to a floating NAV.

Finally, welfare levels for managers and investors measured in consumption equivalence. In this example managers are worse off when adopting a floating NAV while investors are better off. This is consistent with the industry opposing abandoning the stable NAV and the SEC arguing for it.

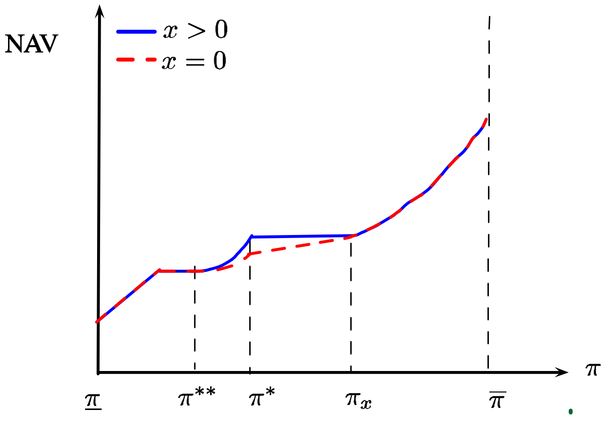

Figure 4: Investor's payoff: The solid (dotted) line represents the payoff for an investor who invests with a manager that offers (does not offer) support when everyone else is offering support. Suport is offered for all values of π between the vertical dotted lines.

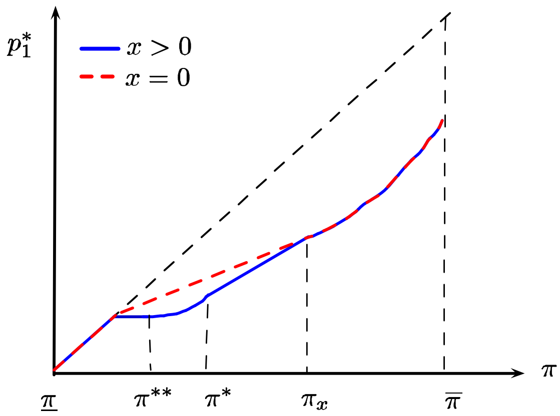

Figure 5: Liquidation price of the risky asset when going from a stable NAV ($x > 0$) to a floating NAV ($x = 0$) fixing all decisions at $t = 0$.

5.2 Capital Buffer

One of the main policies which is being considered by the European Commission is to impose a capital buffer on stable NAV MMFs. In the context of the model, a capital buffer requirement is equivalent to assuming that managers are required to hold capital equal to a fraction F of the funds they manage for investors to cover potential losses at $t = 1$. In this case a fund will consist of the manager's initial wealth minus the

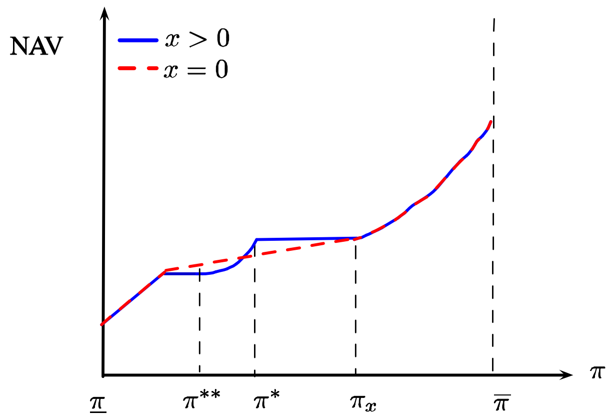

Figure 6: Payoff for investors from investing with a manager when going from a stable NAV ($x > 0$) to a floating NAV ($x = 0$) fixing all decisions at $t = 0$.

capital requirement plus the amount the investor chooses to invest with the manager, i.e, $W_0^M + (1 - Fq_0)\, A_0^I$.

The initial investment Fq_0 will generate a capital buffer equal to F at $t = 1$ which the sponsor would be forced to use to cover any decrease of the NAV below x. Once this buffer has been exhausted, managers may still choose to keep the fund running at $t = 1$ by offering support beyond the amount required by the new regulation. The characterization of the new equilibrium is analogous to the one shown for the benchmark model and it can be found in the appendix.

Imposing a capital buffer on MMFs has countervailing effects on the return received by investors. On one hand, it decreases the demand for the risky asset at $t = 0$ and, therefore, ceteris paribus, increases the equilibrium price p_0 and decreases the return of investing in the risky asset. Without any changes in the liquidation price of the risky asset, this implies a lower net asset value for all realizations of π, a higher cost of offering support for sponsors and, therefore, lower incentives for the fund to offer voluntary support. On the other hand, the capital buffer forces managers to offer support at least up to the amount of the buffer which, everything else equal, increases the expected return and decreases the risk of intermediation. The effect on the level of intermediation will depend on whether the increase in the return and the decrease in the risk of investing with a manager offsets the increase in the intermediation fees. The effect on the economy's fragility will depend on the increase in incentives to offer support for sponsor's due to higher fees and increase in the cost of offering support due to a lower return of the risky asset at $t = 1$. All of these forces have countervailing effects.

Figure 8 shows the equilibrium fees, the level of intermediation, the expected return and risk of intermediation and the managers' and investors' welfare for numerical examples with different values of spillover losses B for the benchmark economy in which there is no capital buffer (blue solid line) and for the case in which a capital buffer of 3% is imposed on the funds (red dotted line). In these examples, capital requirements are successful in decreasing the fragility of the economy and they increase the level of intermediation period

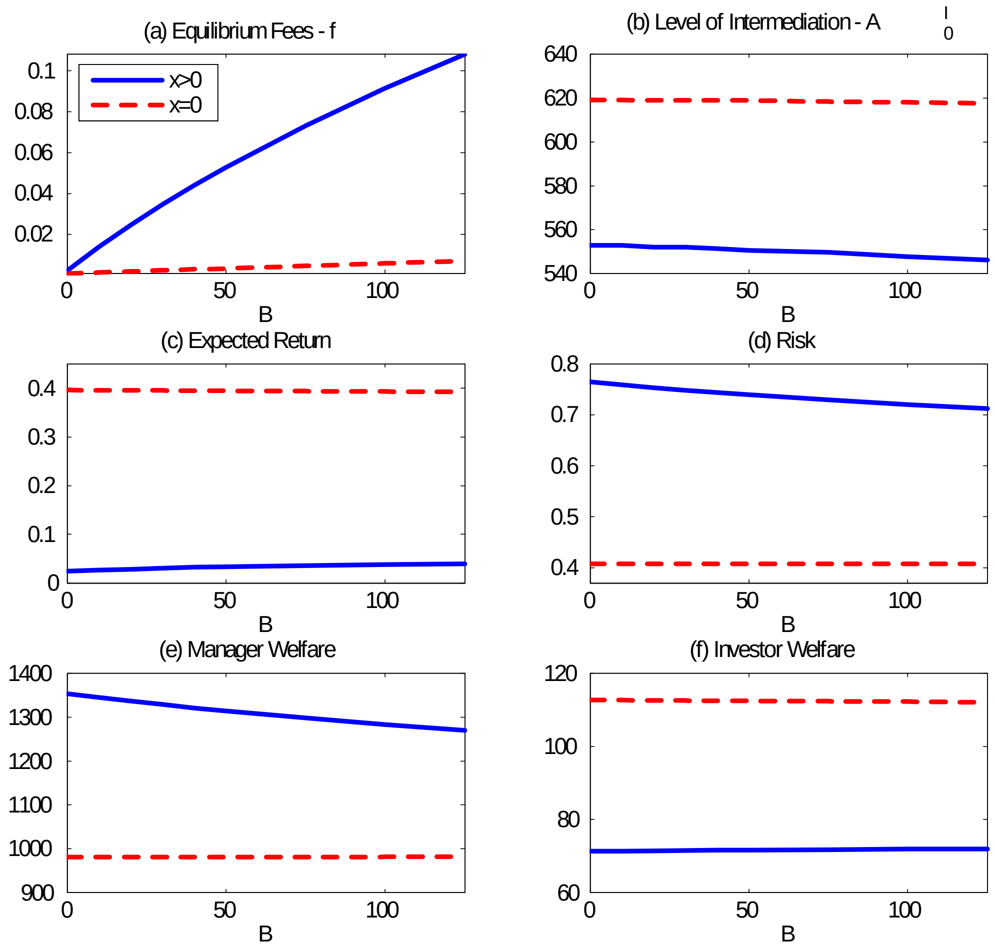

Figure 7: Floating vs Stable NAV. The blue solid line represents the equilibrium in the benchmark economy with a sable NAV. The red dotted line represents the equilibirum in an economy with a floating NAV and no breaking the buck liquidation rule.

0. Moreover, when capital requirements are imposed managers need to be compensated for having to hold the capital buffer and intermediation fees are higher. In equilibrium, the changes in the liquidation price of the asset and in the support decision of managers imply higher expected return and lower risk for investors investing with a manager. These effects offset the increase in fees and lead to a higher level of intermediation and higher investor welfare. The increase in the level of intermediation and in the intermediation fees are not enough to compensate the managers for having to hold the capital buffer and therefore their welfare is less when a capital buffer is in place.

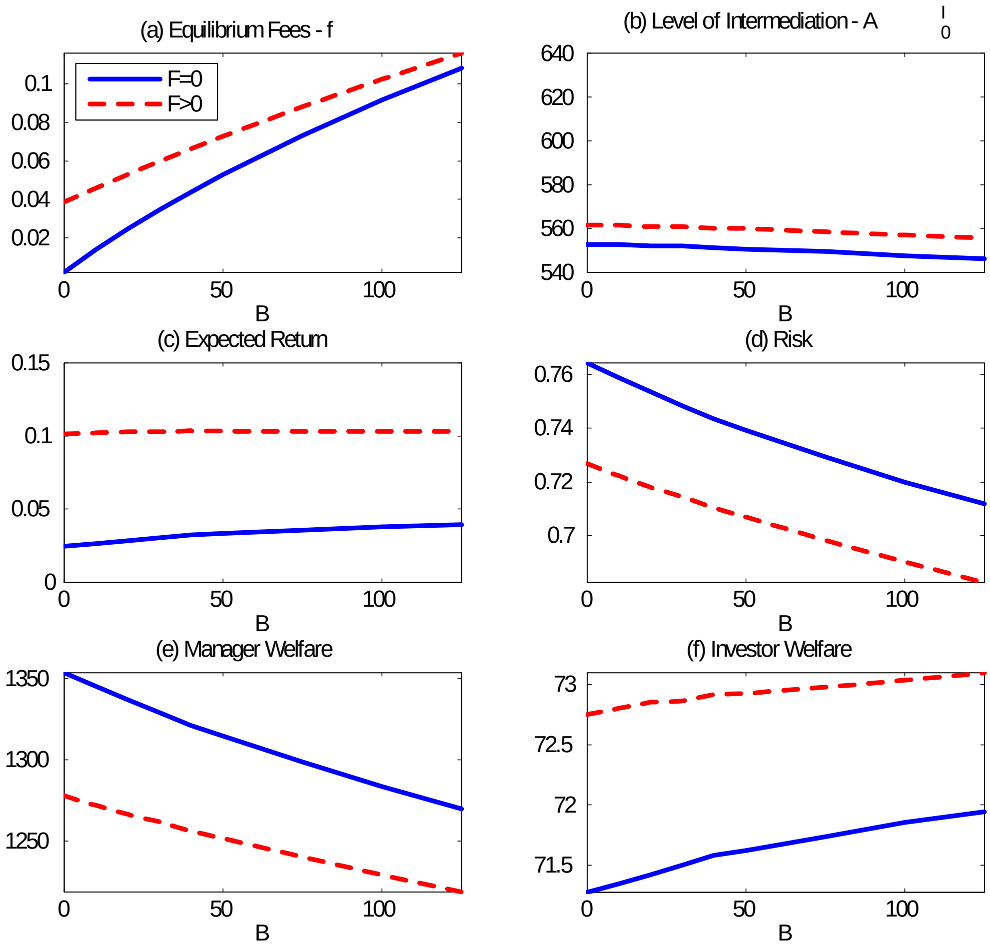

Figure 8: Capital Buffer vs. No Capital Buffer: The blue solid lines represents the equilibrium for the benchmark economy in which there is a stable NAV and no capital buffer. The red dotted line represents the equilibrium in an economy with a stable NAV and a capital buffer of 3%.

5.3 Discussion of numerical results

The numerical results illustrated above are chosen to illustrate how ignoring the general equilibrium effects of the policies can, in principle, lead to wrong conclusions.[15] The strength of these effects will depend mainly on two elasticities: the elasticity of the demand for intermediation and the elasticity of the supply of the risky asset.

The demand for intermediation is given by the investors' attitude towards risk. The higher the risk aversion of investors, the higher the demand for stability and, thus, the stronger the response of the level of intermediation to changes in the return and risk of investing with managers.

The elasticity of the supply of the risky asset can be interpreted as a measure of the importance of MMFs as supplies of liquidity in the money market. If there are other potential suppliers of liquidity other than MMFs the price will not react much to changes in the amount of the risky asset demanded by the MMF industry. Alternatively, if the MMFs are the sole suppliers of liquidity in the MMF one would expect the price of risky assets to react a lot to changes in the MMF industry. The more inelastic the supply of the risky asset, the more important the changes in the equilibrium prices will be in determining the overall effect of the policy analyzed.

6 Conclusion

In this paper, I developed a novel model of MMFs to analyze the role of sponsor support in the industry's stability. The model incorporates several features that are characteristic of MMFs: the investors' ability to redeem their shares on demand, the stability of the NAV, the liquidation of the funds after breaking the buck, and, most importantly, the provision of voluntary sponsor support. The fluctuation in the value of the funds' assets is captured by shocks to the quality of risky assets which affect the equilibrium prices.

The model shows that, even in the absence of investor runs, the MMF industry may be fragile. MMFs may subject to a source of fragility that differs from the canonical bank runs: there may be strategic complementarities in the sponsors' support decisions that may give rise to multiple equilibria and to runs of the MMFs on the asset market. Therefore, sponsor support, which is instrumental in providing stability to the MMFs after idiosyncratic shocks, may be not so effective when the shocks are systemic and it may even amplify them.

This model is consistent with stylized facts documented in the literature on MMFs. It captures the positive performance-flow sensitivity in the MMF industry and the difference in incentives for risk taking for funds with different sponsors. In particular, it is consistent with the fact that MMFs sponsored by companies that also offer non-money market mutual funds and other financial services tend to take on less risk.

I then use the model to analyze the trade-offs involved in the adoption of a floating NAV and of a capital buffer for MMFs. The consequences of the regulations depend on the interaction between potentially

[15] See the appendix for the parameters values.

countervailing effects. Changing the institutional setup of the MMF industry would affect the risks and returns of intermediation for investors and MMF managers not only directly, but also through the change in equilibrium outcomes such as intermediation fees, the sponsors' support decision, and asset prices. The model allows me to take into account these general equilibrium effects, which seem particularly important given the relative size of the MMF industry in the market for short term financing. One of the key determinants of the overall effect of the policies is the elasticity of the supply of assets faced by MMFs. In light of this, the model suggests that a crucial piece in the policy analysis is whether other market participant would be able and willing to offer liquidity in the money market if MMFs were not there.

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7 Appendix

7.1 Money Market Funds: Institutional Features

MMFs are open-ended mutual funds that offer individuals, corporations, and governments access to money market instruments, such as US Treasury bills and commercial paper. MMFs act as intermediaries between investors and borrowers who seek short-term financing. As required by regulation, all mutual funds, including MMFs, issue demandable shares, *i.e.*, they provide "same day" liquidity, allowing investors to redeem their shares at any time at the net asset value of the shares (NAV).[16] What makes these institutions - all MMFs in the U.S. and stable NAV MMFs in Europe- special is that they seek to maintain a stable NAV, usually of $1. To prevent the NAV from going above $1, all positive investment returns are paid out entirely as dividends, with no capital gains or losses to track. If the NAV drops below $1, it is said that the fund "broke the buck". In the event of a fund breaking the buck, U.S. regulation states that "the board of directors shall promptly consider what action, if any, should be initiated by the board of directors." (Rule 2a-7(b), 17 C.F.R. § 270.2a-7(b)) In practice, in the only two cases in which a U.S. MMF's NAV dropped below $1, the fund were eventually liquidated.[17]

Even though a MMF could become a floating NAV fund after breaking the buck, adopting a floating NAV does not reduce investors incentives to redeem their shares and it decreases the NAV even more. Since a fund would likely sell the more liquid assets to meet excess redemptions, investors who chose not to redeem their shares would be left holding a portfolio of less-liquid, longer-dated securities. This increases the incentives of investors to withdraw quickly, even at a reduced NAV, and drives the NAV even lower.[18] This downward spiral mechanism makes it impossible for a MMF to transition to a floating NAV fund and makes liquidation the only viable option after breaking the buck. For example, on September 16, 2008, a day after Lehman Brothers declared bankruptcy, the Reserve Primary Fund (RPF) delayed share redemptions for up to seven days, abandoned the stable NAV, and became a floating NAV fund. On September 18, 2008, RPF suspended the redemption of shares and started the orderly liquidation of the fund's assets after experiencing massive share redemptions. RPF, which had approximately $62 billion in assets under management on September 15, 2008, experienced redemptions of $60 billion between September 15 and September 18, 2008.[19]

In order to maintain a stable NAV, MMFs rely on two mechanisms: the computation of the NAV and sponsor support. The SEC, via Rule $2a - 7$, allows MMFs to use amortized cost valuation and penny-rounding pricing to compute the NAV.[20] European stable NAV funds are also allowed to use amortized cost accounting. Many of the assets held by MMFs lack market price quotations. This makes it difficult for

[16] Title 17 - Commodity and Securities Exchanges - 17 CFR § 270.22c-1 (b)(c)

[17] See Fisch and Roiter (2011) for a detailed description of the current regulation of MMFs.

[18] See Hanson et al. (2012).

[19] See http://www.primary-yieldplus-inliquidation.com/pdf/PressRelease2008_0916.pdf and http://www.primary-yieldplus-inliquidation.com/pdf/PressReleasePrimGovt2008_0919.pdf . Last accessed February 15, 2013.

[20] Rule 2a-7 allows MMFs to use amortized cost valuation "only so long as the [fund] board of directors believes that it fairly reflects the market-based net asset value per share".

MMFs to price their assets accurately. Amortized cost valuation allows MMFs to value their assets as if held to maturity. Penny-rounding pricing allows MMFs to report a NAV of $1 as long as the calculated value is between $0.995 and $1.005. Using amortized cost valuation and penny rounding makes MMFs prone to shareholder runs. For example, if the NAV is just below $1, shareholders who redeem their shares first get $1 and by doing so reduce the value of the fund's assets, imposing costs on non-redeeming shareholders who might not get $1 for their shares. Analogously, if the NAV calculated using amortized cost valuation differs from the market value of the asset, investors might be better off redeeming their shares. Since the liquidation value of the assets differs from the NAV computed by the MMF, investors who do not redeem their shares bear the cost of paying redeeming investors $1 for something which is worth less than $1 in the market. These phenomena resemble the mechanism behind the canonical bank runs described by Diamond and Dybvig (1983), and dealt with in a very large literature.

In my model, I abstract from the possibility of runs on MMFs by assuming that all assets are traded in frictionless competitive markets, and that the shares in MMFs are priced-to-market. In my model, there is no need to use amortized cost accounting since price quotations are always available.

The features described above make MMF unique financial institutions. In particular, as illustrated in figure 9, the payoff received by investors in MMFs can be seen as a hybrid between that received by investors in other mutual funds, and that received by depositors in banks. If no sponsor support is offered, investors in MMFs are true shareholders and, as for investors in other mutual funds, the value of their shares coincides with the market NAV. If sponsor support is offered, the value of the shares for investors in MMFs is the same independently of the value of the fund's assets. This flat portion of the investors' payoff makes shares in MMFs resemble debt. Nevertheless, sponsor support is voluntary, and, though it is anticipated by investors it is not mandated by the intermediation contract. The intermediation contract assumed in the model developed in the next section captures these features.

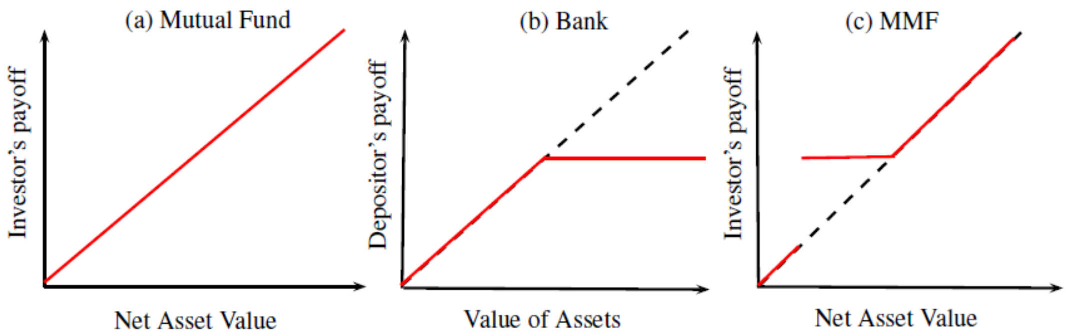

Figure 9: The net asset value for mutual funds and MMFs is calculated as the total value of the fund's assets divided the total number of outstandig shares. The asset value of assets for banks is calculated as the total value of the bank's assets divided the total amount of deposits.

7.2 Equilibrium Price

When the return on the manager's portfolio is above x, no support is needed to keep the fund operating. In this case, if a_1^{I*} is interior, the equilibrium price is

$$p_1^{NS}(\pi) = \min\left\{ \frac{(1-f)\bar{d}\left(\pi\left(\left(1-a_0^M\right)(1-f)a_0^I+\left(1-a_0^I\right)\right)\frac{1}{q_0}W_0^I+\left(1-a_0^M\right)\frac{1}{q_0}\left(W_0^M+fa_0^IW_0^I\right)+E\right)}{\frac{a_0^M}{p_0}a_0^I(1-f)^2(1-\pi)\bar{d}W_0^I+\frac{1}{q_1}\frac{1}{q_0}\left(\left(1-a_0^I\right)W_0^I+\left(1-a_0^M\right)\left(W_0^M+a_0^IW_0^I\right)\right)+\frac{E}{q_1}}, \bar{d}\pi q_1 \right\}.$$

If support is provided by the manager, and a_1^{I*} is interior, the equilibrium price, $p_1^S(\pi)$, is determined by

$$\left(x(1-f)a_0^IW_0^I - \frac{\left(1-a_0^M\right)}{q_0}\left(W_0^M + a_0^IW_0^I\right) - E\right) = a_1^{I*}(p_1(\pi),\pi)\left(x(1-f)a_0^I + \frac{\left(1-a_0^I\right)}{q_0}\right)W_0^I.$$

If $x(1-f)a_0^IW_0^I - \left(1-a_0^M\right)\frac{1}{q_0}\left(W_0^M + a_0^IW_0^I\right) - E < 0$, there is an excess demand for the risky asset at every price $p_1(\pi) < \bar{d}\pi q_1$. Therefore, in this case, $p_1(\pi) = \bar{d}\pi q_1$, $a_1^{I*}(p_1(\pi),\pi) = 0$ and the share invested by managers in the risky asset, $\bar{a}_1^M(\pi)$, is given by

$$\left(1-\bar{a}_1^M(\pi)\right)\frac{\bar{d}\pi q_1}{p_0}a_0^M\left(W_0^M + a_0^IW_0^I\right) + \bar{a}_1^M(\pi)\left(x(1-f)a_0^IW_0^I - \frac{\left(1-a_0^M\right)}{q_0}\left(W_0^M + a_0^IW_0^I\right) - E\right) = 0.$$

If $x(1-f)a_0^IW_0^I - \left(1-a_0^M\right)\frac{1}{q_0}\left(W_0^M + a_0^IW_0^I\right) - E > 0$ the equilibrium price is given by

$$a_1^{I*}(p_1(\pi),\pi) = \frac{\left(x(1-f)a_0^IW_0^I - \frac{\left(1-a_0^M\right)}{q_0}\left(W_0^M + a_0^IW_0^I\right) - E\right)}{\left(x(1-f)a_0^I + \frac{\left(1-a_0^I\right)}{q_0}\right)W_0^I} =: a_1^{IS}.$$

This condition implies an affine equation in $\bar{d}/p_1^S(\pi)$, which gives the following equilibrium price

$$p_1^S(\pi) = \min\left\{\max\left\{\frac{\left(\pi - a_1^{IS}\right)}{\frac{1}{q_1}\left(1-a_1^{IS}\right)}(1-f)\bar{d}, 0\right\}, \bar{d}\pi q_1\right\}.$$

Note that, when there is support, the share investors choose to invest with the manager in period 1 is independent of π.

If sponsors choose to offer support with probability s, assuming the law of large numbers holds, a mass s of managers will offer support and keep the fund open while a mass $(1-s)$ will choose not to offer support and liquidate the fund early. Then the net demand for the risky asset will be

$$\begin{aligned} D(p_1,\pi) \quad &= \quad s\left(\frac{a_1^M(p_1,\pi)\left(A_1^I(p_1,\pi) + W_1^M(p_1, s=1)\right)}{p_1} - \frac{a_0^M\left(a_0^IW_0^I + W_0^M\right)}{p_0}\right) \\ &\quad + (1-s)\left(\frac{a_1^M(p_1,\pi)W_1^M(p_1,s=0)}{p_1} - \frac{a_0^M\left(a_0^IW_0^I + W_0^M\right)}{p_0}\right). \end{aligned}$$

In this case, the equilibrium price if $A_1^I(p_1,\pi)>0$ will be given by

$$\begin{aligned} 0 &= s\left(\left(\pi\bar{d}(1-f)-\frac{p_1}{q_1}\right)\frac{(1-a_0^I)}{q_0}-(1-\pi)\bar{d}(1-f)x(1-f)a_0^I\right)W_0^I \\ &+\left(\frac{(1-a_0^M)}{q_0}\left(W_0^M+(f+(1-f)s)a_0^IW_0^I\right)+E\right)\left(\bar{d}(1-f)-\frac{p_1}{q_1}\right) \\ &-\frac{p_1}{p_0}a_0^M(1-f)(1-s)a_0^IW_0^I\left(\bar{d}(1-f)-\frac{p_1}{q_1}\right) \\ 0 &= \bar{d}(1-f)\left(s\left(\pi\frac{(1-a_0^I)}{q_0}-(1-\pi)x(1-f)a_0^I\right)W_0^I+\frac{(1-a_0^M)\left(W_0^M+(f+(1-f)s)a_0^IW_0^I\right)}{q_0}+E\right) \\ &-\frac{p_1}{q_1}\left(s\frac{(1-a_0^I)}{q_0}W_0^I+\frac{(1-a_0^M)\left(W_0^M+(f+(1-f)s)a_0^IW_0^I\right)}{q_0}+E+\frac{q_1\bar{d}(1-f)\left(a_0^M(1-f)(1-s)a_0^IW_0^I\right)}{p_0}\right) \\ &+p_1^2\frac{a_0^M(1-f)(1-s)a_0^IW_0^I}{p_0q_1} \end{aligned}$$

This quadratic function is negative at $p_1=\bar{d}(1-f)q_1$. Therefore the equilibrium price is given by the smallest root

$$p_1^{SS}(\pi,s)=\frac{-c_1(\pi,s)-\sqrt{c_1(\pi,s)^2-4c_0(\pi,s)c_2(\pi,s)}}{2c_2(\pi,s)}.$$

if $s<1$, where

$$\begin{aligned} c_0(\pi,s) &= \bar{d}(1-f)\left(s\left(\pi\frac{(1-a_0^I)}{q_0}-(1-\pi)x(1-f)a_0^I\right)W_0^I+\frac{(1-a_0^M)\left(W_0^M+(f+(1-f)s)a_0^IW_0^I\right)}{q_0}+E\right) \\ c_1(\pi,s) &= -\frac{1}{q_1}\left(s\frac{(1-a_0^I)}{q_0}W_0^I+\frac{(1-a_0^M)\left(W_0^M+(f+(1-f)s)a_0^IW_0^I\right)}{q_0}+E+\frac{q_1\bar{d}(1-f)a_0^M(1-f)(1-s)a_0^IW_0^I}{p_0}\right) \\ c_2(\pi,s) &= \frac{a_0^M(1-f)(1-s)a_0^IW_0^I}{p_0q_1} \end{aligned}$$

Otherwise, $A_1^I(p_1,\pi)$ would equal 0. Moreover, since the quadratic function is decreasing and positive at $p_1=0$, $p_1^{SS}>0$.

Finally, if the fund is liquidated the equilibrium price, $p_1^L(\pi)$, is given by

$$p_1^L(\pi)=\min\left\{\frac{\left(1-a_0^M\right)\frac{1}{q_0}\left(W_0^M+fa_0^IW_0^I\right)+E}{\frac{a_0^M}{p_0}(1-f)a_0^IW_0^I},\bar{d}\pi q_1\right\}$$

and it is determined by the amount of resources managers have that don't come from their holdings of the risky asset, *i.e.*, by the cash in the market.[21]

7.3 Equilibrium threshold characterization

This section finds closed form solutions for the thresholds $\pi_x\left(a_0^M\right)$ and $\pi^*\left(a_0^M\right)$.

$\pi_x\left(a_0^M\right)$ is given by

$$\frac{p_1^{NS}\left(\pi_x\left(a_0^M\right)\right)}{p_0}a_0^M+\left(1-a_0^M\right)\frac{1}{q_0}=x.$$

[21] See Allen and Gale (1994) and Allen and Gale (2005) for more on cash-in-the-market pricing.

using that $p_1^{NS}\left(\pi_x\left(a_0^M\right)\right) = p_1^S\left(\pi_x\left(a_0^M\right)\right)$ and the definition of $p_1^S(\pi)$,

$$\pi_x\left(a_0^M\right) = \begin{cases} \left(\left(x - \frac{1}{q_0}\right) + a_0^M \frac{1}{q_0}\right) \frac{p_0}{a_0^M} \frac{\left(1-a_1^{IS}\right)}{\bar{d}(1-f)q_1} + a_1^{IS} & a_1^{IS} > 0 \\ \left(\left(x - \frac{1}{q_0}\right) + a_0^M \frac{1}{q_0}\right) \frac{p_0}{a_0^M} \frac{1}{\bar{d}q_1} & \text{else} \end{cases}.$$

The equilibrium support threshold is determined by

$$\begin{aligned} & \frac{\pi^* \bar{d}}{p_1^S(\pi^*)}\left(\left(x - n\left(p_1^S(\pi^*), a_0^M\right)\right)(1-f) A_0^I - f A_1^I\left(p_1^S(\pi^*), \pi^*\right)\right) \\ = \quad & \min\left\{B\pi^*, \frac{\pi^* \bar{d}}{p_1^S(\pi^*)} n\left(p_1^S(\pi^*), a_0^M\right)\left(W_0^M + f A_0^I\right) + E\right\}. \end{aligned}$$

Then, $\pi^*\left(a_0^M\right)$ is determined by

$$p_1^S\left(\pi^*\left(a_0^M\right)\right) = \begin{cases} \dfrac{\left(\left(1-fa_1^{IS}\right)x(1-f)a_0^I - fa_1^{IS}\frac{\left(1-a_0^I\right)}{q_0} - \frac{\left(1-a_0^M\right)}{q_0}(1-f)a_0^I\right)W_0^I}{\frac{B}{\bar{d}} + \frac{a_0^M}{P_0}(1-f)a_0^I W_0^I} & \chi\left(a_0^M\right) = 1 \\ \dfrac{\left(\left(1-fa_1^{IS}\right)x(1-f)a_0^I - fa_1^{IS}\frac{\left(1-a_0^I\right)}{q_0}\right)W_0^I - \frac{\left(1-a_0^M\right)}{q_0}\left(W_0^M + a_0^I W_0^I\right) - E}{\frac{a_0^M}{P_0}\left(W_0^M + a_0^I W_0^I\right)} & \chi\left(a_0^M\right) = 0 \end{cases}$$

where $a_1^{IS} = 0$ if $p_1^S\left(\pi^*\left(a_0^M\right)\right) = \bar{d}\pi^*\left(a_0^M\right) q_1$ and

$$\chi\left(a_0^M\right) = \begin{cases} 1 & \frac{B}{\bar{d}} p_1^S\left(\pi^*\left(a_0^M\right), a_0^M\right) < \left(n\left(p_1^S\left(\pi^*\left(a_0^M\right), a_0^M\right), a_0^M\right)\left(W_0^M + f a_0^{I*} W_0^I\right) + E\right) \\ 0 & \frac{B}{\bar{d}} p_1^S\left(\pi^*\left(a_0^M\right), a_0^M\right) \geq \left(n\left(p_1^S\left(\pi^*\left(a_0^M\right), a_0^M\right), a_0^M\right)\left(W_0^M + f a_0^{I*} W_0^I\right) + E\right) \end{cases}.$$

Finally, $\pi^{**}\left(a_0^M\right)$ is given by

$$\begin{aligned} & \frac{\pi^{**} \bar{d}}{p_1^L(\pi^{**})}\left(\left(x - n\left(p_1^L(\pi^{**}), a_0^M\right)\right)(1-f) A_0^I - f A_1^I\left(p_1^L(\pi^{**}), \pi^{**}\right)\right) \\ = \quad & \min\left\{B\pi^{**}, \frac{\pi^{**} \bar{d}}{p_1^L(\pi^{**})} n\left(p_1^L(\pi^{**}), a_0^M\right)\left(W_0^M + f A_0^I\right) + E\right\}. \end{aligned}$$

Proposition 9 *The thresholds* $\pi_x\left(a_0^M\right)$, $\pi^*\left(a_0^M\right)$, *and* $\pi^{**}\left(a_0^M\right)$ *are increasing in* a_0^M.

The net asset value, $n\left(p_1, a_0^M\right)$, is decreasing in a_0^M for all $p_1 < p_0/q_0$. Since $x < \frac{1}{q_0}$ by assumption, $p_1^S(\pi) < p_0/q_0$ for all $\pi < \pi_x\left(a_0^M\right)$, *i.e.*, when support is needed. Therefore, the higher the exposure to the risky asset at $t = 0$, the lower the net asset value when support is needed for any given price. This implies that a higher a_0^M requires a higher liquidation price for the manager not to need to offer support. Since the price function is increasing in π for $\pi \geq \pi_x\left(a_0^M\right)$, higher a_0^M requires higher realizations of π not to need to offer support, *i.e.*, a higher π_x. The same argument can be applied to the thresholds π^* and π^{**}.

The following proposition characterizes the support region, and shows that managers will offer support in some states π as long as they have something to lose from closing the fund, either from forgone fees of from spillover losses.

Proposition 10 $\pi^*\left(a_0^M\right) \leq \pi_x\left(a_0^M\right)$ *and* $\pi^*\left(a_0^M\right) < \pi_x\left(a_0^M\right)$ *iff* $A_1^I\left(p_1\left(\pi_x\left(a_0^M\right)\right), \pi_x\left(a_0^M\right)\right) > 0$ *or* $B > 0$.

Proof. From the definition of $\pi_x\left(a_0^M\right)$ it is easy to see that (2) always holds for $\pi_x\left(a_0^M\right)$. Therefore $\pi_x\left(a_0^M\right)$ $\geq \pi^*\left(a_0^M\right)$. Moreover, if $A_1^I\left(p_1\left(\pi_x\left(a_0^M\right)\right), \pi_x\left(a_0^M\right)\right)$ or $B > 0$, (2) holds with strict inequality at $\pi_x\left(a_0^M\right)$ which implies that $\pi_x\left(a_0^M\right) > \pi^*\left(a_0^M\right)$. ■

7.4 Equilibrium price function

Given these thresholds, the equilibrium price function can be further characterized.

Proposition 11 *Under assumption* 3.1.5 *the equilibrium price function* $p_1^*\left(\pi, a_0^M\right)$ *is continuous and non-decreasing in* π *for all* $\pi \in \left[0, \pi^*\left(a_0^M\right)\right) \cup \left(\pi^*\left(a_0^M\right), 1\right]$ *for all* a_0^M.

Proof. To see this note that the expressions for $p_1^{NS}\left(\pi, a_0^M\right)$, $p_1^{SS}\left(\pi, a_0^M\right)$, $p_1^S\left(\pi, a_0^M\right)$ and $p_1^L\left(\pi, a_0^M\right)$ are non-decreasing in π when 3.1.5 holds, $p_1^L\left(\pi^{**}\left(a_0^M\right), a_0^M\right) = p_1^{SS}\left(\pi^{**}\left(a_0^M\right), a_0^M\right)$, and $p_1^{NS}\left(\pi_x, a_0^M\right) = p_1^S\left(\pi_x, a_0^M\right)$. ■

From the definition of $\pi_x\left(a_0^M\right)$ and $\pi^{**}\left(a_0^M\right)$, it is easy to see that p

$$_1^L\left(\pi^{**}\left(a_0^M\right), a_0^M\right) = p_1^{SS}\left(\pi^{**}\left(a_0^M\right), a_0^M\right),$$

and that $p_1^{NS}\left(\pi_x, a_0^M\right) = p_1^S\left(\pi_x, a_0^M\right)$. This follows from the continuity of the demand for the risky asset at $\pi_x\left(a_0^M\right)$ and $\pi^{**}\left(a_0^M\right)$. However, the demand for the risky asset may be discontinuous at $\pi^*\left(a_0^M\right)$. For $\pi \geq \pi^*\left(a_0^M\right)$, the demand for the risky asset is equal to the funds' size which include the managers' wealth and the investors' investment with their managers. For $\pi < \pi^*\left(a_0^M\right)$, all funds may be liquidated and the demand for the risky asset is conformed by the managers' wealth only. Then, if all funds are liquidated for $\pi < \pi^*\left(a_0\right)$, *i.e.*, $\pi^*\left(a_0^M\right) < \pi^{**}\left(a_0^M\right)$ the demand function is discontinuous at $\pi^*\left(a_0^M\right)$ provided investors choose to invest with the manager when the realized quality of the risky asset is $\pi^*\left(a_0^M\right)$. This, in turn, implies that the equilibrium price may be discontinuous at $\pi^*\left(a_0^M\right)$.

Proposition 4 Under assumption 3.1.5, the price function $p_1^*\left(\pi, a_0^M\right)$ is non-decreasing π for all $\pi \in [\underline{\pi}, \bar{\pi}]$.

Proof. From proposition 11 the price function is non-decreasing when it is continuous. Therefore, to prove the proposition it is enough to show that

$$\lim_{\pi \to \pi^*\left(a_0^{M*}\right)^-} p_1^*\left(\pi, a_0^M\right) < \lim_{\pi \to \pi^*\left(a_0^{M*}\right)^+} p_1^*\left(\pi, a_0^M\right)$$

when the price function is discontinuous. The price is discontinuous only if $\pi^*\left(a_0^M\right) < \pi^{**}\left(a_0^M\right)$. In this case,

$$\lim_{\pi \to \pi^*\left(a_0^{M*}\right)^-} p_1^*\left(\pi, a_0^M\right) = p_L\left(\pi^*\left(a_0^{M*}\right), a_0^M\right)$$

and

$$\lim_{\pi \to \pi^*\left(a_0^{M*}\right)^+} p_1^*\left(\pi, a_0^M\right) = p_1^S\left(\pi^*\left(a_0^{M*}\right), a_0^M\right).$$

Suppose by contradiction that

$$p_L\left(\pi^*\left(a_0^{M*}\right), a_0^M\right) > p_1^S\left(\pi^*\left(a_0^{M*}\right), a_0^M\right).$$

Then, from the definition of $\pi^*\left(a_0^M\right)$ it follows that

$$\begin{aligned} &\left(x - n\left(p_1^L\left(\pi^*\left(a_0^M\right), a_0^M\right), a_0^M\right)\right)(1-f)A_0^I - fA_1^I\left(p_1^S\left(\pi^*\left(a_0^M\right), a_0^M\right), \pi^*\left(a_0^M\right)\right) \\ < \;& \min\left\{\frac{Bp_1^L\left(\pi^*\left(a_0^M\right), a_0^M\right)}{\bar{d}}, n\left(p_1^L\left(\pi^*\left(a_0^M\right), a_0^M\right), a_0^M\right)\left(W_0^M + fA_0^I\right) + E\right\}. \end{aligned}$$

Moreover, using that $\pi^*\left(a_0^M\right) < \pi^{**}\left(a_0^M\right)$ and the definition of $\pi^{**}\left(a_0^M\right)$ one gets

$$\begin{aligned} fA_1^I\left(p_1^L\left(\pi^{**}\left(a_0^M\right),a_0^M\right),\pi^{**}\left(a_0^M\right)\right) - fA_1^I\left(p_1^S\left(\pi^{*}\left(a_0^M\right),a_0^M\right),\pi^{*}\left(a_0^M\right)\right) &< 0 \\ fA_1^I\left(p_1^S\left(\pi^{**}\left(a_0^M\right),a_0^M\right),\pi^{**}\left(a_0^M\right)\right) - fA_1^I\left(p_1^S\left(\pi^{*}\left(a_0^M\right),a_0^M\right),\pi^{*}\left(a_0^M\right)\right) &< 0 \end{aligned}$$

But this implies $\pi^*\left(a_0^M\right) > \pi^{**}\left(a_0^M\right)$ since $A_1^I\left(p_1^S\left(\pi, a_0^M\right),\pi\right)$ is increasing in π which is a contradiction. ■

7.5 Individual threshold characterization

An individual manager who invested a fraction $a_{0,i}^M$ in the risky asset will not need to offer support if

$$n\left(p_1^*\left(\pi\right), a_{0,i}^M\right) > x. \tag{4}$$

Proposition 12 *There exists a unique threshold $\pi_{x,i}\left(a_{0,i}^M\right)$ such that support is not needed if $\pi \geq \pi_{x,i}\left(a_{0,i}^M\right)$.*

Proof. The proof is straightforward using that the price function $p_1^*\left(\pi\right)$ is non-decreasing in π. The left hand side of this expression is always increasing in π.The right hand side is constant in π. Therefore, there is a unique threshold $\pi_{x,i}\left(a_{0,i}^M\right)$ such that for all $\pi \geq \pi_{x,i}\left(a_{0,i}^M\right)$ (4) holds. ■

As the previous proposition shows, if the realized probability of success of the risky project is high enough, the manager will not need to offer support to his investor to keep the fund open. In this case, the realized net asset value will be above the liquidation threshold x. Moreover, since the net asset value depends on the manager's portfolio choice at $t = 0$, how high a realization of π is needed not to need support will depend on this portfolio choice. The following proposition shows that the higher the risk incurred by the manager in the initial period, the higher the realization of π needed not to need to offer support. A full characterization of the threshold $\pi_{x,i}$ is provided in the appendix and shows that $\boldsymbol{\pi}_{x,i}$ is discontinuous in $a_{0,i}^M$ if the price function is constant for an interval $[\pi_a, \pi_b]$ where $\pi_a < \pi_b$.

Proposition 13 *$\pi_{x,i}\left(a_{0,i}^M\right)$ is increasing in $a_{0,i}^M$*

Proof. The left hand side of (4) is constant in $a_{0,i}^M$ whereas the right hand side is increasing in $a_{0,i}^M$ since $x \leq \frac{1}{q_0}$. Therefore, $\pi_{x,i}$ is increasing in $a_{0,i}^M$. ■

If $\pi < \pi_{x,i}$, the manager cannot continue operating the fund unless he supports his investor. A manager that chose to invest $a_{0,i}^M$ in the risky asset in period 0, will choose to offer support if

$$H\left(\pi; a_{0,i}^M\right) \geq 0$$

where

$$\begin{aligned} H\left(\pi; a_{0,i}^M\right) \quad : \quad &= \left(fa_1^{I*}\left(p_1^*\left(\pi\right),\pi\right)\left(x\left(1-f\right)a_0^I + \frac{\left(1-a_0^I\right)}{q_0}\right) - \left(x - n\left(p_1^*\left(\pi\right), a_{0,i}^M\right)\right)a_0^{I*}\left(1-f\right)\right)W_0^I \\ &+ \min\left\{\frac{Bp_1^*(\pi)}{\bar{d}}, n\left(p_1^*\left(\pi\right), a_{0,i}^M\right)\left(W_0^M + fa_0^{I*}W_0^I\right) + E\right\}. \end{aligned}$$

This condition is analogous to the one presented to compute the aggregate support threshold π^*. The following two propositions characterize the support decision for an individual manager.

Proposition 14 *Characterization of* $\pi_{x,i}$

$$\pi_{x,i}\left(a_{0.i}^{M};a_{\ 0}^{M}\right)\begin{cases} =\underline{\pi} & \textit{for } a_{0,i}^{M}\leq\widehat{a}_{0,i}^{M} \\ =\left(\frac{\left(x-\frac{1}{q_0}\right)}{a_{0.i}^{M}}+\frac{1}{q_0}\right)\frac{p_0}{\bar{d}q_1} & \textit{for } \widehat{a}_{0,i}^{M}<a_{0,i}^{M}\leq\underline{a}_{0,i}^{M}\left(a_0^M\right) \\ =\pi^*\left(a_0^M\right) & \textit{for } \underline{a}_{0,i}^{M}\left(a_0^M\right)<a_{0,i}^{M}\leq\bar{a}_{0,i}^{M}\left(a_0^M\right) \\ >\pi^*\left(a_0^M\right) & \textit{for } a_{0,i}^{M}\geq\bar{a}_{0,i}^{M}\left(a_0^M\right)\end{cases}$$

where

$$\widehat{a}_{0,i}^{M}=\frac{\left(x-\frac{1}{q_0}\right)}{\left(\frac{p_1^*(\underline{\pi})}{p_0}-\frac{1}{q_0}\right)},$$

$$\pi_1\left(a_{\ 0}^{M}\right)=\frac{\left(1-a_0^M\right)\frac{1}{q_0}\left(W_0^M+fa_0^IW_0^I\right)+E}{\frac{a_0^M}{p_0}\left(1-f\right)a_0^IW_0^I\bar{d}q_1},$$

$$\underline{a}_{0,i}^{M}\left(a_0^M\right)=\frac{\left(x-\frac{1}{q_0}\right)a_0^M\left(1-f\right)a_0^IW_0^I}{\left(\left(1-a_0^M\right)\left(W_0^M+fa_0^IW_0^I\right)-a_0^M\left(1-f\right)a_0^IW_0^I\right)\frac{1}{q_0}+E},$$

and

$$\bar{a}_{0,i}^{M}\left(a_0^M\right)=\frac{\left(x-\frac{1}{q_0}\right)}{\bar{d}\left(1-f\right)\frac{\pi^*\left(a_0^M\right)-a_1^{IS}}{1-a_1^{IS}}-q_0}$$

Proof. $\pi_{x,i}\left(a_{0.i}^{M};a_{\ 0}^{M}\right)$ is given by the minimum π such that

$$\frac{p_1^*\left(\pi;a_0^M\right)}{p_0}\geq\frac{\left(x-\frac{1}{q_0}\right)}{a_{0.i}^{M}}+\frac{1}{q_0}. \tag{5}$$

If $a_{0,i}^{M}\leq\widehat{a}_{0,i}^{M}$, the right hand side of this expression is ≤ 0 which implies that 5 holds for all $\pi\geq\underline{\pi}$ and therefore $\pi_{x,i}\left(a_{0.i}^{M};a_{\ 0}^{M}\right)=\underline{\pi}$. If $\underline{\pi}<\pi_1\left(a_{\ 0}^{M}\right)$, for $\pi\in\left[\underline{\pi},\pi_1\left(a_{\ 0}^{M}\right)\right]$, $p_1^*\left(\pi;a_0^M\right)=\pi\bar{d}q_1$ and $\pi_{x.i}$ is given by

$$\frac{\pi_{x.i}\bar{d}q_1}{p_0}=\frac{\left(x-\frac{1}{q_0}\right)}{a_{0.i}^{M}}+\frac{1}{q_0}.$$

But $\pi_{x.i}\left(a_{0.i}^{M};a_{\ 0}^{M}\right)\leq\pi_1\left(a_{\ 0}^{M}\right)$ iff $a_{0,i}^{M}\leq\underline{a}_{0,i}^{M}\left(a_0^M\right)$. For $\pi\in\left(\pi_1\left(a_{\ 0}^{M}\right),\pi^*\left(a_0^M\right)\right)$, $p_1^*\left(\pi\right)=p_L$. Therefore, $\pi_{x.i}\notin\left(\underline{\pi}\left(a_{\ 0}^{M}\right),\pi^*\left(a_0^M\right)\right)$ since if 5 holds for one $\pi\in\left(\pi_1\left(a_{\ 0}^{M}\right),\pi^*\left(a_0^M\right)\right)$, it holds for all $\pi_0\in\left[\pi_1\left(a_{\ 0}^{M}\right),\pi^*\left(a_0^M\right)\right)$. Moreover, $p_L=\pi_1\left(a_{\ 0}^{M}\right)\bar{d}q_1$. Thus, $\pi_{x.i}\left(a_{0.i}^{M};a_{\ 0}^{M}\right)\geq\pi^*\left(a_0^M\right)$ for all $a_{0,i}^{M}>\underline{a}_{0,i}^{M}\left(a_0^M\right)$. Finally, for $\underline{a}_{0,i}^{M}\left(a_0^M\right)<a_{0,i}^{M}\leq\bar{a}_{0,i}^{M}\left(a_0^M\right)$

$$\frac{p_1^*\left(\pi^*;a_0^M\right)}{p_0}\geq\frac{\left(x-\frac{1}{q_0}\right)}{a_{0.i}^{M}}+\frac{1}{q_0}$$

and

$$\frac{\lim_{\pi\to\pi^*}p_1^*\left(\pi;a_0^M\right)}{p_0}<\frac{\left(x-\frac{1}{q_0}\right)}{a_{0.i}^{M}}+\frac{1}{q_0}$$

which implies $\pi_{x,i}\left(a_{0.i}^{M};a_{\ 0}^{M}\right)=\pi^*\left(a_0^M\right)$. ■

Lemma $H\left(\pi;a_{0,i}^{M}\right)$ is increasing in π whenever it is continuous.

Proof. If the equilibrium price function is continuous in π, $H\left(\pi; a_{0,i}^M\right)$ is always increasing in π. To see this note that $a_1^{I*}\left(p_1^*\left(\pi\right),\pi\right)$ is always increasing in π if the equilibrium price is continuous in π. ■

Proposition 5 Given the equilibrium price, there exists a unique $\pi_i^*\left(a_{0,i}^M\right)$ such that for all $\pi_i^*\left(a_{0,i}^M\right) < \pi < \pi_{x,i}\left(a_{0,i}^M\right)$ there is an equilibrium in which all individual managers strictly prefer to offer support.

Proof. An individual manager will offer support in all states π such that $H\left(\pi; a_{0,i}^M\right) \geq 0$. If the equilibrium price is continuous in π or if the equilibrium price is discontinuous at $\pi^*\left(a_0^M\right)$ but $\lim_{\pi\to\pi^*} H\left(\pi; a_{0,i}^M\right) < H\left(\pi^*; a_{0,i}^M\right)$, the proof follows from monotonicity of $H\left(\pi; a_{0,i}^M\right)$ in π, using lemma 7.5. If $\lim_{\pi\to\pi^*} H\left(\pi; a_{0,i}^M\right) > H\left(\pi^*; a_{0,i}^M\right)$ and $\lim_{\pi\to\pi^*} H\left(\pi; a_{0,i}^M\right) > H\left(\pi^*; a_{0,i}^M\right) > 0$ or $0 > \lim_{\pi\to\pi^*} H\left(\pi; a_{0,i}^M\right) > H\left(\pi^*; a_{0,i}^M\right)$, $H\left(\pi; a_{0,i}^M\right)$ crosses 0 only once and the statement of the proposition holds.

Suppose that $\lim_{\pi\to\pi^*} H\left(\pi; a_{0,i}^M\right) > 0 > H\left(\pi^*; a_{0,i}^M\right)$. Then, the set of realizations of π for which the manager offers support is given by $\left[\underline{\pi}_i^*\left(a_{0,i}^M\right),\pi^*\right] \cup \left[\bar{\pi}_i^*\left(a_{0,i}^M\right),1\right]$ where

$$\begin{aligned} 0 &= \left(fa_1^{I*}\left(p_1^L\left(\underline{\pi}_i^*\right),\underline{\pi}_i^*\right)\left(x\left(1-f\right)a_0^I + \tfrac{\left(1-a_0^I\right)}{q_0}\right) - \left(x - n\left(p_1^L\left(\underline{\pi}_i^*\right),a_{0,i}^M\right)\right)a_0^{I*}\left(1-f\right)\right)W_0^I \\ &\quad + \min\left\{\frac{Bp_1^L\left(\underline{\pi}_i^*\right)}{\bar{d}}, n\left(p_1^L\left(\underline{\pi}_i^*\right),a_{0,i}^M\right)\left(W_0^M + fa_0^{I*}W_0^I\right)\right\}, \end{aligned}$$

or $\underline{\pi}_i^*\left(a_{0,i}^M\right) = 0$ and

$$\begin{aligned} 0 &= \left(fa_1^{I*}\left(p_1^S\left(\bar{\pi}_i^*\right),\bar{\pi}_i^*\right)\left(x\left(1-f\right)a_0^I + \tfrac{\left(1-a_0^I\right)}{q_0}\right) - \left(x - n\left(p_1^S\left(\bar{\pi}_i^*\right),a_{0,i}^M\right)\right)a_0^{I*}\left(1-f\right)\right)W_0^I \\ &\quad + \min\left\{\frac{Bp_1^S\left(\bar{\pi}_i^*\right)}{\bar{d}}, n\left(p_1^S\left(\bar{\pi}_i^*\right),a_{0,i}^M\right)\left(W_0^M + fa_0^{I*}W_0^I\right)\right\}. \end{aligned}$$

Note that $\underline{\pi}_i^*\left(a_{0,i}^M\right) \leq \pi^*\left(a_0^M\right) \leq \bar{\pi}_i^*\left(a_{0,i}^M\right)$, and therefore, the LHS of the expressions above is decreasing in $a_{0,i}^M$, since $p_1^L\left(\underline{\pi}_i^*\right) < p_1^S\left(\bar{\pi}_i^*\left(a_{0,i}^M\right)\right) \leq \frac{1}{q_0}$ (if $\bar{\pi}_i^*\left(a_{0,i}^M\right) > \pi_x\left(a_{0,i}^M\right)$, LHS>0). Moreover, from the definition of $\pi^*\left(a_0^M\right)$ and using that $H\left(\pi; a_{0,i}^M\right)$ is increasing in π in $\left(\pi^*\left(a_0^M\right),1\right]$,

$$\begin{aligned} 0 &< \left(fa_1^{I*}\left(p_1^*\left(\bar{\pi}_i^*\right),\bar{\pi}_i^*\right)\left(x\left(1-f\right)a_0^I + \tfrac{\left(1-a_0^I\right)}{q_0}\right) - \left(x - n\left(p_1^*\left(\bar{\pi}_i^*\right),a_0^M\right)\right)a_0^{I*}\left(1-f\right)\right)W_0^I \\ &\quad + \min\left\{\frac{Bp_1^*\left(\bar{\pi}_i^*\right)}{\bar{d}}, n\left(p_1^*\left(\bar{\pi}_i^*\right),a_0^M\right)\left(W_0^M + fa_0^{I*}W_0^I\right)\right\} \end{aligned}$$

which implies that $a_{0,i}^M \geq a_0^M$. From the definition of $\pi^*\left(a_0^M\right)$,

$$\begin{aligned} 0 &> \left(\begin{array}{c} fa_1^{I*}\left(p_1^L\left(\pi^*\left(a_0^M\right)\right),\pi^*\left(a_0^M\right)\right)\left(x\left(1-f\right)a_0^I + \tfrac{\left(1-a_0^I\right)}{q_0}\right) \\ -\left(x - n\left(p_1^L\left(\pi^*\left(a_0^M\right)\right),a_0^M\right)\right)a_0^{I*}\left(1-f\right) \end{array}\right)W_0^I \\ &\quad + \min\left\{\frac{Bp_1^L\left(\pi^*\left(a_0^M\right)\right)}{\bar{d}}, n\left(p_1^L\left(\pi^*\left(a_0^M\right)\right),a_0^M\right)\left(W_0^M + fa_0^{I*}W_0^I\right)\right\} \end{aligned}$$

and since the LHS is decreasing in a_0^M, this implies

$$\begin{aligned} 0 &> \left(fa_1^{I*}\left(p_1^L\left(\underline{\pi}_i^*\right),\underline{\pi}_i^*\right)\left(x\left(1-f\right)a_0^I + \tfrac{\left(1-a_0^I\right)}{q_0}\right) - \left(x - n\left(p_1^L\left(\underline{\pi}_i^*\right),a_{0,i}^M\right)\right)a_0^{I*}\left(1-f\right)\right)W_0^I \\ &\quad + \min\left\{\frac{Bp_1^L\left(\underline{\pi}_i^*\right)}{\bar{d}}, n\left(p_1^L\left(\underline{\pi}_i^*\right),a_{0,i}^M\right)\left(W_0^M + fa_0^{I*}W_0^I\right)\right\} \end{aligned}$$

which is a contradiction.

Therefore, $H\left(\pi, a_{0,i}^{M}\right)$ crosses 0 at most once and the proposition holds. ■

Proposition $\pi_i^*\left(a_{0.i}^{M}; a\,_0^{M}\right)$ is increasing in $a_{0,i}^{M}$.

Proof. Follows from the definition of H and the fact that $p_1^*\left(\pi\right) < \frac{1}{q_0} < x$ for all $\pi \leq \pi_{x,i}\left(a_{0,i}^{M}\right)$. ■

7.6 Investor's problem

The $t = 2$ wealth of an investor with manager i is given by

$$W_2^I\left(a_0^I; a_{0i}^M, \pi_i^{**}, \pi_i^*, \pi_{x,i}, \bar{s}\left(\pi\right), \pi\right)$$

$$= \begin{cases} \left(a_1^{I*}\left(p_1\left(\pi\right), \pi\right)\left(\left(1-f\right)d - \frac{1}{q_1}\right) + \frac{1}{q_1}\right)W_1^I & \text{if} \quad \pi \geq \pi_i^* \\ \\ \frac{1}{q_1}W_1^I\left(\bar{s}\left(\pi\right)a_1^{I*}\left(p_1\left(\pi\right), \pi\right)\left(\left(1-f\right)d - \frac{1}{q_1}\right) + \frac{1}{q_1}\right) & \text{if} \quad \pi < \pi_i^* \end{cases},$$

where $W_1 = W_1\left(a_0^I; a_{0i}^M, \pi_i^{**}, \pi_i^*, \pi_{x,i}, \bar{s}\left(\pi\right), \pi\right)$

$$W_1^I\left(a_0^I; a_{0i}^M, \pi_i^{**}, \pi_i^*, \pi_{x,i}, \bar{s}\left(\pi\right), \pi\right)$$

$$= \begin{cases} \left(x\left(1-f\right)a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right)W_0^I & \text{if } \pi_{x,i} > \pi \geq \pi_i^* \\ \\ \left(\bar{s}\left(\pi\right)x + \left(1-\bar{s}\left(\pi\right)\right)n\left(p_1\left(\pi\right), a_{0,i}^M\right)\right)\left(1-f\right)A_0^I + \left(1-a_0^I\right)\frac{W_0^I}{q_0} & \text{otherwise} \end{cases}.$$

Because of log utility, the investor's problem can be rewritten as

$$\begin{aligned} &\max_{a_0^I \in [0,1]} \mathbb{E}_\pi s^*\left(\pi\right)\log W_1^I\left(a_0^I; a_{0i}^M, \pi_i^{**}, \pi_i^*, \pi_{x,i}, \bar{s}\left(\pi\right), \pi\right) \\ &+\mathbb{E}_\pi\left(1-s^*\left(\pi\right)\right)\log W_1^I\left(a_0^I; a_{0i}^M, \pi_i^{**}, \pi_i^*, \pi_{x,i}, \bar{s}\left(\pi\right), \pi\right). \end{aligned}$$

The first order condition for an interior solution is

$$\begin{aligned} 0 \quad = \quad & \int_{\pi_x\left(a_0^M\right)}^{\bar{\pi}} \frac{n\left(p_1\left(\pi\right), a_0^M\right)\left(1-f\right) - \frac{1}{q_0}}{\left(n\left(p_1\left(\pi\right), a_0^M\right)\left(1-f\right)a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right)}dG\left(\pi\right) \\ & + \int_{\pi^*\left(a_0^M\right)}^{\pi_x\left(a_0^M\right)} \frac{x\left(1-f\right) - \frac{1}{q_0}}{x\left(1-f\right)a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}}dG\left(\pi\right) \\ & + \int_0^{\pi^*\left(a_0^M\right)} \frac{n\left(p_1\left(\pi\right), a_0^M\right)\left(1-f\right) - \frac{1}{q_0}}{n\left(p_1\left(\pi\right), a_0^M\right)\left(1-f\right)a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}}dG\left(\pi\right) \end{aligned}$$

The S.O.C is negative.

$$
\begin{aligned}
0 \quad > \quad & -\int_{\pi_x\left(a_0^M\right)}^{\bar{\pi}} \left(\frac{n\left(p_1(\pi), a_0^M\right)(1-f) - \frac{1}{q_0}}{n\left(p_1(\pi), a_0^M\right)(1-f)\,a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}} \right)^2 dG(\pi) \\
& -\int_{\pi^*\left(a_0^M\right)}^{\pi_x\left(a_0^M\right)} \left(\frac{x(1-f) - \frac{1}{q_0}}{x(1-f)\,a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}} \right)^2 dG(\pi) \\
& -\int_{0}^{\pi^*\left(a_0^M\right)} \left(\frac{\lambda n\left(p_1(\pi), a_0^M\right)(1-f) - \frac{1}{q_0}}{\lambda n\left(p_1(\pi), a_0^M\right)(1-f)\,a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}} \right)^2 dG(\pi)
\end{aligned}
$$

7.7 Manager's Objective Function

Manager i's wealth at $t = 2$ is given by $\frac{\pi \bar{d}}{p_1} W_1^M$ where $W_1^M = W_1^M\left(a_{0,i}^M; \pi_{x,i}\left(a_{0,i}^M\right), \pi_i^*\left(a_{0,i}^M\right), a_0^I, \pi\right)$. If no support is needed for the fund to continue at $t = 1$,

$$W_1^M = n\left(p_1^*(\pi), a_{0,i}^M\right)\left(W_0^M + f a_0^I W_0^I\right) + E + f A_1^{I*}(\pi).$$

If the manager offers support to his investor at $t = 1$,

$$
\begin{aligned}
W_1^M \quad = \quad & n\left(p_1^*(\pi), a_{0,i}^M\right)\left(W_0^M + f a_0^I W_0^I\right) + E + f A_1^{I*}(\pi) \\
& -\left(x - n\left(p_1^*(\pi), a_{0,i}^M\right)\right) a_0^{I*}(1-f)\, W_0^I.
\end{aligned}
$$

Finally, if the fund is liquidated

$$W_1^M = n\left(p_1^*(\pi), a_{0,i}^M\right)\left(W_0^M + f a_0^I W_0^I\right) + E.$$

7.7.1 Continuity

Since $\pi_{x,i}\left(a_{0,i}^M\right)$ can have a discontinuity at $\underline{a}_{0,i}^M\left(a_0^M\right)$, the objective function can be discontinuous at $\underline{a}_{0,i}^M\left(a_0^M\right)$ too.

If $\pi_{x,i}\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right) = \pi_i^*\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right)$,

$$
\begin{aligned}
& Obj\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right) - \lim_{a \to \underline{a}_{0,i}^M\left(a_0^M\right)^+} Obj(a) \\
= \quad & \int_{\pi_{x,i}\left(a_{0,i}^M\right)}^{\pi^*} \left(\frac{\pi \bar{d}}{p_1^*(\pi)} f a_1^{I*}(\pi) \left(a_0^{I*}(1-f)\left(a_{0,i}^M \frac{p_1^*(\pi)}{p_0} + \frac{\left(1-a_{0,i}^M\right)}{q_0} \right) + \frac{\left(1-a_0^{I*}\right)}{q_0} \right) W_0^I + B\pi \right) dG(\pi) \geq 0
\end{aligned}
$$

and the objective function jumps down and a maximum always exists.

When $B = 0$, $p_1^*\left(\pi_{x,i}\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right)\right) = \bar{d}\pi_{x,i}\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right) q_1$ which implies that

$$a_1^I\left(p_1^*\left(\pi_{x,i}\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right)\right), \pi_{x,i}\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right)\right) = 0$$

and, hence, that $\pi_{x,i}\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right) = \pi_i^*\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right)$.

If $\pi_{x,i}\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right) > \pi_i^*\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right)$

$$Obj\left(\underline{a}_{0,i}^{M}\left(a_0^M\right)\right) - \lim_{a\to \underline{a}_{0,i}^{M}\left(a_0^M\right)^+} Obj\left(a\right)$$

$$= \int_{\pi_{x,i}\left(a_{0,i}^M\right)}^{\pi^*} \frac{\pi \bar{d}}{p_1^*\left(\pi\right)}\left(f a_1^{I*}\left(\pi\right)-1\right)\left(\left(a_{0,i}^M\left(\frac{p_1^*\left(\pi\right)}{p_0}-\frac{1}{q_0}\right)+\frac{1}{q_0}\right)-x\right) a_0^{I*}\left(1-f\right)W_0^I dG\left(\pi\right)\leq 0$$

and the objective function jumps up at $\underline{a}_{0,i}^{M}\left(a_0^M\right)$ and I cannot guarantee the existence of a maximum. In a symmetric equilibrium, if $\pi_{x,i}\left(a_{0,i}^M\right)$ is discontinuous, $\underline{a}_{0,i}^{M}\left(a_0^M\right) < a_0^M$.

7.7.2 Differentiability

The first derivative of the manager's objective function with respect to $a_{0,i}^M$ when the objective function is differentiable is given by

$$\begin{aligned}
&\int_{\pi_i^{*E}\left(a_{0,i}^M\right)} \frac{\pi \bar{d}}{p_1^*\left(\pi\right)}\left(\frac{p_1^*\left(\pi\right)}{p_0}-\frac{1}{q_0}\right)\left(W_0^M + f a_0^I W_0^I\right) dG\left(\pi\right)\\
&+\int_{\pi_{x,i}\left(a_{0,i}^M\right)} \frac{\pi \bar{d}}{p_1^*\left(\pi\right)} f a_1^{I*}\left(\pi\right)\left(\frac{p_1^*\left(\pi\right)}{p_0}-\frac{1}{q_0}\right) a_0^{I*}\left(1-f\right)W_0^I dG\left(\pi\right)\\
&+\int_{\pi_i^*\left(a_{0,i}^M\right)}^{\pi_{x,i}\left(a_{0,i}^M\right)} \frac{\pi \bar{d}}{p_1^*\left(\pi\right)}\left(\frac{p_1^*\left(\pi\right)}{p_0}-\frac{1}{q_0}\right) a_0^{I*}\left(1-f\right)W_0^I dG\left(\pi\right)\\
&-\frac{\partial \pi_i^*\left(a_{0,i}^M\right)}{a_{0,i}^M}\frac{\pi_i^*\left(a_{0,i}^M\right)\bar{d}}{p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)}\left(\begin{array}{c}
n\left(p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right), a_{0,i}^M\right)\left(W_0^M + f a_0^I W_0^I\right)\\
+ f a_1^{I*}\left(\pi_i^*\left(a_{0,i}^M\right)\right)\left(a_0^{I*}\left(1-f\right)x + \left(1-a_0^{I*}\right)\frac{1}{q_0}\right)W_0^I\\
-\left(x - n\left(p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right), a_{0,i}^M\right)\right)a_0^{I*}\left(1-f\right)W_0^I\\
+\frac{B}{\bar{d}}p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)
\end{array}\right) dG
\end{aligned}$$

Using the definition of π_i^* and the fact that if (condition) holds with strict inequality then $\frac{\partial \pi_i^*\left(a_{0,i}^M\right)}{a_{0,i}^M} = 0$ (when it exists), the last term in the first derivative is equal to

$$\frac{\partial \pi_i^*\left(a_{0,i}^M\right)}{a_{0,i}^M}\frac{\pi_i^*\left(a_{0,i}^M\right)\bar{d}}{p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)}\min\left\{0, n\left(p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right), a_{0,i}^M\right)\left(W_0^M + f a_0^I W_0^I\right) + E - \frac{B}{\bar{d}}p_1^*\left(\pi_i^{*E}\left(a_{0,i}^M\right)\right)\right\} dG.$$

Therefore, the first derivative of the objective function with respect to $a_{0,i}^M$ is $\frac{\partial Obj}{\partial a_{0,i}^M} =$

$$\begin{aligned}
&\int_{\underline{\pi}}^{\overline{\pi}} \frac{\pi \bar{d}}{p_1^*\left(\pi\right)}\left(\frac{p_1^*\left(\pi\right)}{p_0}-\frac{1}{q_0}\right)\left(W_0^M + f a_0^I W_0^I\right) dG\left(\pi\right)\\
&+\int_{\pi_{x,i}\left(a_{0,i}^M\right)}^{\overline{\pi}} \frac{\pi \bar{d}}{p_1^*\left(\pi\right)} f a_1^{I*}\left(\pi\right)\left(\frac{p_1^*\left(\pi\right)}{p_0}-\frac{1}{q_0}\right) a_0^{I*}\left(1-f\right)W_0^I dG\left(\pi\right)\\
&+\int_{\pi_i^*\left(a_{0,i}^M\right)}^{\pi_{x,i}\left(a_{0,i}^M\right)} \frac{\pi \bar{d}}{p_1^*\left(\pi\right)}\left(\frac{p_1^*\left(\pi\right)}{p_0}-\frac{1}{q_0}\right) a_0^{I*}\left(1-f\right)W_0^I dG\left(\pi\right)\\
&+\frac{\partial \pi_i^*\left(a_{0,i}^M\right)}{\partial a_{0,i}^M}\frac{\pi_i^*\left(a_{0,i}^M\right)\bar{d}}{p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)}\min\left\{0, n\left(p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right), a_{0,i}^M\right)\left(W_0^M + f a_0^I W_0^I\right) + E - \frac{B}{\bar{d}}p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)\right\} dG.
\end{aligned}$$

When $B = 0$, this first derivative is continuous when the objective function is differentiable.

When $B > 0$, the first derivative is not defined at the discontinuity points of $\pi_i^*\left(a_{0,i}^M\right)$, at $\underline{a}_{0,i}^M\left(a_0^M\right)$, $\hat{a}_{0,i}^M\left(a_0^M\right)$, and $\bar{a}_{0,i}^M\left(a_0^M\right)$.

$$\frac{\partial \pi_i^*\left(a_{0,i}^M\right)}{a_{0,i}^M}\begin{cases} =0 & \text{if } a_{0,i}^M < \underline{a}_{0,i}^M\left(a_0^M\right) \\ >0 & \text{if } a_{0,i}^M \in \left(\underline{a}_{0,i}^M\left(a_0^M\right), \hat{a}_{0,i}^M\left(a_0^M\right)\right) \\ =0 & \text{if } a_{0,i}^M \in \left(\hat{a}_{0,i}^M\left(a_0^M\right), \bar{a}_{0,i}^M\left(a_0^M\right)\right) \\ >0 & \text{if } a_{0,i}^M > \bar{a}\left(a_0^M\right) \end{cases}$$

Therefore

$$\frac{\partial Obj}{\partial a_{0,i}^M}\left(\bar{a}\right) - \lim_{a \to \left(a_{0,i}^M\right)} \frac{\partial Obj}{\partial a_{0,i}^M}\left(a\right)\begin{cases} >0 & \text{if } \bar{a} = \underline{a}_{0,i}^M\left(a_0^M\right) \\ <0 & \text{if } \bar{a} = \hat{a}_{0,i}^M\left(a_0^M\right) \\ >0 & \text{if } \bar{a} = \bar{a}_{0,i}^M\left(a_0^M\right) \\ =0 & \text{else} \end{cases}.$$

Therefore, $\underline{a}_{0,i}^M\left(a_0^M\right)$ and $\bar{a}_{0,i}^M\left(a_0^M\right)$ are candidates for a maximum.

When the first derivative is differentiable, the second derivative of the objective function is given by $\frac{\partial^2 Obj}{\partial a_{0,i}^{M2}} =$

$$\left(1 - f a_1^{I*}\left(\pi_{x,i}\left(a_{0,i}^M\right)\right)\right) \frac{\pi_{x,i}\left(a_{0,i}^M\right)\bar{d}}{p_1^*\left(\pi_{x,i}\left(a_{0,i}^M\right)\right)} \left(\frac{p_1^*\left(\pi_{x,i}\left(a_{0,i}^M\right)\right)}{p_0} - \frac{1}{q_0}\right) a_0^{I*}\left(1-f\right) W_0^I dG \frac{\partial \pi_{x,i}\left(a_{0,i}^M\right)}{a_{0,i}^M}$$
$$+ \frac{\pi_i^*\left(a_{0,i}^M\right)\bar{d}}{p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)} \left(\frac{p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)}{p_0} - \frac{1}{q_0}\right) a_0^{I*}\left(1-f\right) W_0^I dG\left(\pi\right) \frac{\partial \pi_i^*\left(a_{0,i}^M\right)}{a_{0,i}^M}$$
$$+ \frac{\partial^2 \pi_i^*\left(a_{0,i}^M\right)}{\partial a_{0,i}^{M2}} \frac{\pi_i^*\left(a_{0,i}^M\right)\bar{d}}{p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)} \min\left\{0, n\left(p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right), a_{0,i}^M\right)\left(W_0^M + f a_0^I W_0^I\right) + E - \frac{B}{\bar{d}} p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)\right\} dG$$
$$+ \left(\frac{\partial \pi_i^*\left(a_{0,i}^M\right)}{\partial a_{0,i}^M}\right)^2 \frac{\partial}{\partial a_{0,i}^M} \left(\frac{\pi_i^*\left(a_{0,i}^M\right)\bar{d}}{p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)} \min\left\{0, n\left(p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right), a_{0,i}^M\right)\left(W_0^M + f a_0^I W_0^I\right) + E - \frac{B p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)}{\bar{d}}\right\}\right) dG.$$

When $B = 0$,

$$\frac{\partial^2 Obj}{\partial a_{0,i}^{M2}}\left(a_0^M\right) > 0$$

therefore, there cannot be a symmetric equilibrium in which a_0^M is interior.

7.8 Results

Proposition 6If $B = 0$, $a_0^{M*} = 1$ in a symmetric equilibrium.

Proof. Using the characterization of the objective function for the manager, one can see this function is differentiable and convex at $a_{0,i}^M = a_0^M$ when $B = 0$. Therefore, in a symmetric equilibrium a_0^M cannot be an interior solution. Assuming that the supply function is such that there is always some risky asset bough at $t = 0$, a_0^M has to be 1 in a symmetric equilibrium. ■

Proposition 7 The amount of assets managed for investors at $t = 1$, A_1^I, is increasing in the net asset value, $n\left(p_1, a_0^M\right)$.

Proof. From the definition of A_1^I, we have

$$A_1^I = \begin{cases} a_1^I\left(p_1, \pi\right) W_1 & \text{if the fund continues} \\ 0 & \text{if the fund is liquidated} \end{cases}$$

where a_1^I is independent of the return of the investor's shares bought at $t = 0$. Moreover, the investor's wealth is (weakly) increasing in the net asset value since

$$W_1 = \max\left\{x, n\left(p_1, a_0^M\right)\right\}(1-f)\, a_0^I W_0^I + \frac{1}{q_0}\left(1 - a_0^I\right) W_0^I.$$

■

Lemma 1 *When $B = 0$, the manager's support decision is a threshold decision for all $B_i > 0$ and all $W_{0,i}^M$ at the equilibrium price (given the equilibrium decisions for other managers).*

Proof. To show that the proposition holds it is enough to show that it cannot be the case that

$$\lim_{\pi \to \pi^*} H\left(\pi; a_{0,i}^M, B_i, W_{0,i}^M\right) \geq 0 > H\left(\pi^*; a_{0,i}^M, B_i, W_{0,i}^M\right)$$

Let $H\left(\pi; a_{0,i}^M, B_i, W_{0,i}^M\right) :=$

$$\left(f a_1^{I*}\left(p_1^*(\pi), \pi\right)\left(x(1-f)\, a_0^I + \left(1 - a_0^I\right)\frac{1}{q_0}\right) - \left(x - n\left(p_1^*(\pi), a_{0,i}^M\right)\right) a_0^{I*}(1-f)\right) W_0^I$$
$$+ \min\left\{\frac{B p_1^*(\pi)}{\bar{d}}, n\left(p_1^*(\pi), a_{0,i}^M\right)\left(W_{0,i}^M + f a_0^{I*} W_0^I\right) + E\right\}.$$

■

I know from the definition of the aggregate support decision threshold that

$$\begin{aligned} H(\pi^*; 1, 0, 0) &= f a_1^{I*}\left(p_1^S(\pi^*), \bar{\pi}_i^*\right)\left(x(1-f)\, a_0^I + \tfrac{\left(1-a_0^I\right)}{q_0}\right) W_0^I - \left(x - \tfrac{p_1^S(\pi^*)}{p_0}\right) a_0^{I*}(1-f)\, W_0^I \\ &= 0 \end{aligned}$$

and that

$$\begin{aligned} \lim_{\pi \to \pi^*} H(\pi; 1, 0, 0) &= f a_1^{I*}\left(p_1^L(\pi^*), \bar{\pi}_i^*\right)\left(x(1-f)\, a_0^I + \tfrac{\left(1-a_0^I\right)}{q_0}\right) W_0^I - \left(x - \tfrac{p_1^L(\pi^*)}{p_0}\right) a_0^{I*}(1-f)\, W_0^I \\ &\leq 0 \end{aligned}$$

$H\left(\pi; a_{0,i}^M, B_i\right)$ is decreasing in $a_{0,i}^M$ and increasing in B_i and $W_{0,i}^M$. Therefore, it cannot be the case that for some triplet $\left(a_{0,i}^M, B_i, W_{0,i}^M\right)$

$$\lim_{\pi \to \pi^*} H\left(\pi; a_{0,i}^M, B_i; W_{0,i}^M\right) \geq 0 > H\left(\pi^*; a_{0,i}^M, B_i, W_{0,i}^M\right)$$

since $H\left(\pi^*; a_{0,i}^M, B_i, W_{0,i}^M\right) \geq 0$ always.

Proposition 8 Suppose $B = 0$ for all managers but for manager j. Then, in equilibrium, the risk taken by manager j at $t = 0$ will be decreasing in B_j and he will choose $a_{0,j}^M \in \left\{a_0^{MS}, 1\right\}$ where $a_0^{MS} = \max a_{0,i}^M$ s.t. $\pi_{x,i}\left(a_{0,i}^M\right) = \underline{\pi}$.

Proof. Using lemma 1 and the characterization of the manager's objective function when $B = 0$, one can see that the two candidates for maxima are $V^M\left(W_0^M; a_0^{MS}\right)$ and $V^M\left(W_0^M; 1\right)$, where a_0^{MS} is the highest $a_{0,i}^M$ such that $\pi_{x,i}\left(a_{0,i}^M\right) = \underline{\pi}$. Then,

$$a_{0,j}^{M*} = \begin{cases} a_0^{MS} & \text{if } V^M\left(W_0^M; a_0^{MS}\right) - V^M\left(W_0^M; 1\right) > 0 \\ 1 & \text{if } V^M\left(W_0^M; a_0^{MS}\right) - V^M\left(W_0^M; 1\right) < 0 \end{cases}.$$

When $B_j = 0$, $V^M\left(W_0^M; a_0^{MS}\right) < V^M\left(W_0^M; 1\right)$ (follows from Proposition 6). Moreover,

$$\lim_{B\to\infty}\left(V^M\left(W_0^M; a_0^{MS}\right) - V^M\left(W_0^M; 1\right)\right) = \infty.$$

Let

$$\Theta = \mathbf{1}\left\{\frac{Bp_1^*\left(\pi^*\left(1\right)\right)}{\bar{d}} > \frac{p_1^*\left(\pi_i^*\left(1\right)\right)}{p_0}\left(W_{0,i}^M + fa_0^{I*}W_0^I\right) + E\right\}$$

Using the characterization of the objective function in the previous section,

$$\begin{aligned}
& \frac{\partial\left(V^M\left(W_0^M; a_0^{MS}\right) - V^M\left(W_0^M; 1\right)\right)}{\partial B_j} \\
\propto & \begin{cases} \int_{\underline{\pi}}^{\pi_i^*(1)} B_i \pi d\pi + \frac{\partial \pi_i^*}{\partial B_i}\left(1\right)\left(\frac{\pi_i^*(1)\bar{d}}{p_1^*(\pi)}\left(\frac{p_1^*(\pi_i^*(1))}{p_0}\left(W_{0,i}^M + fa_0^{I*}W_0^I\right) + E\right) - B_i\pi_i^*\left(1\right)\right) & \text{if } \Theta = 1 \\ \int_{\underline{\pi}}^{\pi_i^*(1)} B_i \pi d\pi & \text{if } \Theta = 0 \end{cases} \\
0 < & \frac{\partial\left(V^M\left(W_0^M; a_0^{MS}\right) - V^M\left(W_0^M; 1\right)\right)}{\partial B_j}
\end{aligned}$$

since $\frac{\partial \pi_i^*}{\partial B_i}\left(1\right) < 0$. ∎

7.9 Numerical Examples

The values used for the numerical examples are chosen to illustrate how taking into account general equilibrium effects can change the results of the policy analysis. The supply of the risky asset assumed is $S\left(p_0\right) = kp_0$. The parameter values are the following: $k = 987.4167$; $C = 0.845$; $q_0 = 1$; $q_1 = 0.8$; $x = 0.995$; $\bar{d} = 5$; $W_0^M = 200$; $W_0^I = 800$; $\bar{\pi} = 0.85$; $\underline{\pi} = 0.30$; $F = 0.03$.

7.10 Capital Buffer

7.10.1 Price Computation

Let $\hat{W}_0^M = W_0^M - q_0F\left(1-f\right)a_0^IW_0^I$. When a capital requirement F is in place, the demand for the risky asset at $t = 1$ is given by

$$n\left(p_1, a_0^M\right)\left(\hat{W}_0^M + fa_0^IW_0^I\right) + a_1^I\left(p_1, \pi\right)\left(n\left(p_1, a_0^M\right)\left(1-f\right)a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right)W_0^I + E + F\left(1-f\right)a_0^IW_0^I$$

if the capital buffer is not needed to keep the fund open at $t = 1$, by

$$n\left(p_1, a_0^M\right)\left(\hat{W}_0^M + a_0^IW_0^I\right) + a_1^I\left(p_1, \pi\right)\left(x\left(1-f\right)a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right)W_0^I + E + F\left(1-f\right)a_0^IW_0^I$$

if the capital buffer and additional support are used to keep the fund open at $t = 1$, and by

$$n\left(p_1, a_0^M\right)\left(\hat{W}_0^M + fa_0^IW_0^I\right) + E$$

if the fund is liquidated at $t = 1$. Then, if no support is needed at all the equilibrium price is given by

$$\hat{p}_1^{NS}(\pi) = \min\left\{ \frac{(1-f)\bar{d}\left(\pi\left(\left(1-a_0^M\right)(1-f)a_0^I+\left(1-a_0^I\right)\right)\frac{1}{q_0}W_0^I+\left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M+fa_0^IW_0^I\right)+E+F(1-f)a_0^IW_0^I\right)}{\frac{a_0^M}{p_0}a_0^I(1-f)^2(1-\pi)\bar{d}W_0^I+\frac{1}{q_1}\frac{1}{q_0}\left(\left(1-a_0^I\right)W_0^I+\left(1-a_0^M\right)\left(\hat{W}_0^M+a_0^IW_0^I\right)\right)+\frac{E+F(1-f)a_0^IW_0^I}{q_1}}, \bar{d}\pi q_1 \right\}.$$

If support is provided using the capital buffer or if additional support is offered by the manager, and a_1^{I*} is interior, the equilibrium price, $p_1^S(\pi)$, is determined by

$$\frac{\left(x(1-f)a_0^IW_0^I - \left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M + a_0^IW_0^I\right) - E - F(1-f)a_0^IW_0^I\right)}{\left(x(1-f)a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right)W_0^I} = a_1^{I*}(p_1(\pi), \pi).$$

If $x(1-f)a_0^IW_0^I - \left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M + a_0^IW_0^I\right) - E - F(1-f)a_0^IW_0^I < 0$, there is an excess demand for the risky asset at every price $p_1(\pi) < \bar{d}\pi q_1$. Therefore, in this case, $p_1(\pi) = \bar{d}\pi q_1$, $a_1^{I*}(p_1(\pi), \pi) = 0$ and the share invested by managers in the risky asset, $\bar{a}_1^M(\pi)$, is given by

$$\left(1 - \bar{a}_1^M(\pi)\right)\frac{\bar{d}\pi q_1}{p_0}a_0^M\left(\hat{W}_0^M + a_0^IW_0^I\right) + \bar{a}_1^M(\pi)\left(-\frac{\left(1-a_0^M\right)\left(\hat{W}_0^M+a_0^IW_0^I\right)}{q_0} - E - (F - x)(1-f)a_0^IW_0^I\right) = 0.$$

If $x(1-f)a_0^IW_0^I - \left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M + a_0^IW_0^I\right) - E - F(1-f)a_0^IW_0^I > 0$ the equilibrium price is given by

$$a_1^{I*}(p_1(\pi), \pi) = \frac{\left(x(1-f)a_0^IW_0^I-\left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M+a_0^IW_0^I\right)-E-F(1-f)a_0^IW_0^I\right)}{\left(x(1-f)a_0^I+\left(1-a_0^I\right)\frac{1}{q_0}\right)W_0^I} =: \hat{a}_1^{IS}\left(a_0^M\right).$$

This condition implies an affine equation in $\bar{d}/p_1^S(\varphi)$, which gives the following equilibrium price

$$\hat{p}_1^S(\pi) = \min\left\{\max\left\{\frac{\left(\pi - \hat{a}_1^{IS}\right)}{\frac{1}{q_1}\left(1 - \hat{a}_1^{IS}\right)}(1-f)\bar{d}, 0\right\}, \bar{d}\pi q_1\right\}.$$

Note that, as in the benchmark case, when there is support, the share investors choose to invest with the manager in period 1 is independent of π.

Finally, if all funds are liquidated the equilibrium price, $p_1^L(\pi)$, is given by

$$p_1^L(\pi) = \min\left\{\frac{\left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M + fa_0^IW_0^I\right) + E}{\frac{a_0^M}{p_0}(1-f)a_0^IW_0^I}, \bar{d}\pi q_1\right\}.$$

The payoff for investors at $t = 1$ from investing 1 unit with the manager at $t = 0$ is

$$\begin{array}{ll} n\left(p_1, a_0^M\right) & \text{if } n\left(p_1, a_0^M\right) > x \\ x & \text{if } n\left(p_1, a_0^M\right) + F > x \text{ or additional voluntary support is offered} \\ n\left(p_1, a_0^M\right) & \text{if the fund is liquidated.} \end{array}$$

Depending n the realization of π, there are 5 regimes: no support needed, $\pi \geq \hat{\pi}_x$; capital buffer is enough to cover losses, $\hat{\pi}_x > \pi \geq \hat{\pi}_B$; additional sponsor support is needed and offered by all funds, $\hat{\pi}_B > \pi \geq \hat{\pi}^*$; additional support is needed and funds offer support with positive probability but not for sure, $\hat{\pi}_B > \pi \geq \hat{\pi}^*$; and the fund is liquidated $\hat{\pi}^{**} > \pi$. This thresholds are determined by

$$\hat{p}_1^S(\hat{\pi}_x) = \left(\left(x - \frac{1}{q_0}\right)\frac{1}{a_0^M} + \frac{1}{q_0}\right)p_0,$$

$$\hat{p}_1^S\left(\hat{\pi}_{CB}\right)=\left(\left(x-F-\frac{1}{q_0}\right)\frac{1}{a_0^M}+\frac{1}{q_0}\right)p_0,$$

$$\left(f\hat{a}_1^{IS}\left(a_0^M\right)\left(x\left(1-f\right)a_0^I+\left(1-a_0^I\right)\frac{1}{q_0}\right)-\left(x-F-n\left(\hat{p}_1^S\left(\hat{\pi}^*\right),a_{0,i}^M\right)\right)a_0^{I*}\left(1-f\right)\right)W_0^I$$
$$+\min\left\{\frac{B\hat{p}_1^S\left(\hat{\pi}^*\right)}{\bar{d}},n\left(\hat{p}_1^S\left(\hat{\pi}^*\right),a_{0,i}^M\right)\left(\hat{W}_0^M+fa_0^{I*}W_0^I\right)+E\right\}=0,$$

$$\left(fa_1^{I*}\left(\hat{p}_1^L\left(\hat{\pi}^{**}\right),\pi\right)\left(x\left(1-f\right)a_0^I+\left(1-a_0^I\right)\frac{1}{q_0}\right)-\left(x-F-n\left(\hat{p}_1^L\left(\hat{\pi}^{**}\right),a_{0,i}^M\right)\right)a_0^{I*}\left(1-f\right)\right)W_0^I$$
$$+\min\left\{\frac{B\hat{p}_1^L\left(\hat{\pi}^{**}\right)}{\bar{d}},n\left(\hat{p}_1^L\left(\hat{\pi}^{**}\right),a_{0,i}^M\right)\left(\hat{W}_0^M+fa_0^{I*}W_0^I\right)+E\right\}=0.$$

The manager's utility when a capital requirement is in place is given by $V_0^{M,CR}\left(W_0^M;f\right)=$

$$\sup_{a_{0,i}^M\in[0,1]}\int_{\underline{\pi}}^{\bar{\pi}}\frac{\pi\bar{d}}{p_1^*\left(\pi\right)}\left[\left(a_{0,i}^M\left(\frac{p_1^*\left(\pi\right)}{p_0}-\frac{1}{q_0}\right)+\frac{1}{q_0}\right)\left(\hat{W}_0^M+fa_0^IW_0^I\right)+E+F\left(1-f\right)a_0^IW_0^I\right]d\pi$$
$$+\int_{\hat{\pi}_{x,i}\left(a_{0,i}^M\right)}^{\bar{\pi}}\left(\frac{\pi\bar{d}}{p_1^*\left(\pi\right)}fa_1^{I*}\left(\pi\right)\left(a_0^{I*}\left(1-f\right)\left(a_{0,i}^M\left(\frac{p_1^*\left(\pi\right)}{p_0}-\frac{1}{q_0}\right)+\frac{1}{q_0}\right)+\left(1-a_0^{I*}\right)\frac{1}{q_0}\right)W_0^I\right)d\pi$$
$$+\int_{\hat{\pi}_i^*\left(a_{0,i}^M\right)}^{\hat{\pi}_{x,i}\left(a_{0,i}^M\right)}\frac{\pi\bar{d}}{p_1^*\left(\pi\right)}fa_1^{I*}\left(\pi\right)\left(a_0^{I*}\left(1-f\right)x+\left(1-a_0^{I*}\right)\frac{1}{q_0}\right)W_0^Id\pi$$
$$-\int_{\hat{\pi}_i^*\left(a_{0,i}^M\right)}^{\hat{\pi}_{x,i}\left(a_{0,i}^M\right)}\frac{\pi\bar{d}}{p_1^*\left(\pi\right)}\left(x-\left(a_{0,i}^M\left(\frac{p_1^*\left(\pi\right)}{p_0}-\frac{1}{q_0}\right)+\frac{1}{q_0}\right)\right)a_0^{I*}\left(1-f\right)W_0^Id\pi-\int_{\underline{\pi}}^{\hat{\pi}_i^*\left(a_{0,i}^M\right)}B\pi d\pi.$$

The free entry condition is

$$V_0^{M,CR}\left(W_0^M;f\right)-C=\max_{a_{0,i}^M\in[0,1]}\int_0^{\bar{\pi}}\left(\frac{\pi\bar{d}}{p_1^*\left(\pi\right)}\left(\left(a_{0,i}^M\left(\frac{p_1^*\left(\pi\right)}{p_0}-\frac{1}{q_0}\right)+\frac{1}{q_0}\right)W_0^M+E\right)\right)d\pi.$$

7.10.2 Thresholds

If $p_1^S\left(\hat{\pi}_x\right)=\frac{\left(\hat{\pi}_x-\hat{a}_1^{IS}\right)}{\frac{1}{q_1}\left(1-\hat{a}_1^{IS}\right)}\left(1-f\right)\bar{d}$,

$$\hat{\pi}_x^I=\left(\left(x-\frac{1}{q_0}\right)\frac{1}{a_0^M}+\frac{1}{q_0}\right)p_0\frac{\left(\frac{1}{q_1}\left(1-\hat{a}_1^{IS}\right)\right)}{\left(1-f\right)\bar{d}}+\hat{a}_1^{IS}.$$

This will happen as long as $\left(x-\frac{1}{q_0}\right)\frac{1}{a_0^M}+\frac{1}{q_0}>0$ and $\hat{\pi}_x^I<\frac{\hat{a}_1^{IS}}{\left(\hat{a}_1^{IS}-f\right)}$.
If $\left(x-\frac{1}{q_0}\right)\frac{1}{a_0^M}+\frac{1}{q_0}<0$, $\hat{\pi}_x$ is not well defined. If $\hat{\pi}_x^I>\frac{\hat{a}_1^{IS}}{\left(\hat{a}_1^{IS}-f\right)}$

$$\hat{\pi}_x^C=\left(\left(x-\frac{1}{q_0}\right)\frac{1}{a_0^M}+\frac{1}{q_0}\right)\frac{p_0}{\bar{d}q_1}$$

Then,

$$\hat{\pi}_x=\begin{cases}\hat{\pi}_x^I & \text{if } \hat{\pi}_x^I<\frac{\hat{a}_1^{IS}}{\left(\hat{a}_1^{IS}-f\right)}\\ \hat{\pi}_x^C & \text{else}\end{cases}$$

π_{CB} is such that

$$\begin{aligned}
x - NAV &= F \\
x - \left(a_0^M \frac{p_1^S(\pi_{CB})}{p_0} + (1 - a_{0M})\frac{1}{q_0}\right) &= F \\
\left(x - F - (1 - a_{0M})\frac{1}{q_0}\right)\frac{p_0}{a_0^M} &= p_1^S(\pi_{CB})
\end{aligned}$$

Then, if $p_1^S(\pi_{CB}) = \frac{\left(\pi_{CB} - \hat{a}_1^{IS}\right)}{\frac{1}{q_1}\left(1 - \hat{a}_1^{IS}\right)}(1-f)\bar{d}$

$$\pi_{CB}^I = \left(x - F - (1 - a_{0M})\frac{1}{q_0}\right)\frac{p_0}{a_0^M}\frac{\frac{1}{q_1}\left(1 - \hat{a}_1^{IS}\right)}{(1-f)\bar{d}} + \hat{a}_1^{IS}$$

If $\pi_{CB}^I > \frac{\hat{a}_1^{IS}}{\left(\hat{a}_1^{IS} - f\right)}$

$$\pi_{CB}^C = \left(\left(x - F - \frac{1}{q_0}\right)\frac{1}{a_0^M} + \frac{1}{q_0}\right)\frac{p_0}{\bar{d}q_1}$$

Then,

$$\hat{\pi}_x = \begin{cases} \pi_{CB}^I & \text{if } \pi_{CB}^I < \frac{\hat{a}_1^{IS}}{\left(\hat{a}_1^{IS} - f\right)} \\ \pi_{CB}^C = \left(\left(x - F - \frac{1}{q_0}\right)\frac{1}{a_0^M} + \frac{1}{q_0}\right)\frac{p_0}{\bar{d}q_1} & \text{else} \end{cases}$$

π^* is such that

$$\begin{aligned}
&\frac{\pi^*\bar{d}}{p_1^S(\pi^*)}\left(\left(x - F - n\left(p_1^S(\pi^*), a_0^M\right)\right)(1-f)A_0^I - fA_1^I\left(p_1^S(\pi^*), \pi^*\right)\right) \\
= \quad &\min\left\{B\pi^*, \frac{\pi^*\bar{d}}{p_1^S(\pi^*)}\left(n\left(p_1^S(\pi^*), a_0^M\right)\left(W_0^M - F(1-f)a_0^I W_0^I q_0 + fA_0^I\right) + E\right)\right\}
\end{aligned}$$

If $A_1^I\left(p_1^S(\pi^*), \pi^*\right) > 0$, i.e, if

$$x(1-f)a_0^I W_0^I - \left(1 - a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M + a_0^I W_0^I\right) - E - F(1-f)a_0^I W_0^I > 0,$$

there are two possible cases.

If $p_1^S(\pi^*) = \frac{\left(\pi^* - \hat{a}_1^{IS}\right)}{\frac{1}{q_1}\left(1 - \hat{a}_1^{IS}\right)}(1-f)\bar{d}$ and

$$B\pi^* < \frac{\pi^*\bar{d}}{p_1^S(\pi^*)}\left(n\left(p_1^S(\pi^*), a_0^M\right)\left(W_0^M - F(1-f)a_0^I W_0^I q_0 + fA_0^I\right) + E\right)$$

$$\begin{aligned}
\bar{d}\left(\left(x - F - n\left(p_1^S(\pi^*), a_0^M\right)\right)(1-f)a_0^I W_0^I - fA_1^I\left(p_1^S(\pi^*), \pi^*\right)\right) &= Bp_1^S(\pi^*) \\
\left(\left(x - F - \left(\frac{p_1^S(\pi^*)}{p_0}a_0^M + \frac{\left(1 - a_0^M\right)}{q_0}\right)\right)(1-f)a_0^I W_0^I - f\hat{a}_1^{IS}W_0^I\left(x(1-f)a_0^I + \frac{\left(1 - a_0^I\right)}{q_0}\right)\right) &= B\frac{p_1^S(\pi^*)}{\bar{d}} \\
\frac{\left(\left(x - F - \frac{\left(1 - a_0^M\right)}{q_0}\right)(1-f)a_0^I W_0^I - f\hat{a}_1^{IS}W_0^I\left(x(1-f)a_0^I + \frac{\left(1 - a_0^I\right)}{q_0}\right)\right)}{\left(\frac{B}{\bar{d}} + \frac{a_0^M}{p_0}(1-f)a_0^I W_0^I\right)} &= p_1^S(\pi^*)
\end{aligned}$$

$$\pi_{I1}^* = \hat{a}_1^{IS} + \frac{\left(\left(x - F - \frac{\left(1 - a_0^M\right)}{q_0}\right)(1-f)a_0^I W_0^I - f\hat{a}_1^{IS}W_0^I\left(x(1-f)a_0^I + \frac{\left(1 - a_0^I\right)}{q_0}\right)\right)\frac{1}{q_1}\left(1 - \hat{a}_1^{IS}\right)}{\left(\frac{B}{\bar{d}} + \frac{a_0^M}{p_0}(1-f)a_0^I W_0^I\right)(1-f)\bar{d}}$$

If $p_1^S(\pi^*) = \frac{(\pi^* - \hat{a}_1^{IS})}{\frac{1}{q_1}(1-\hat{a}_1^{IS})}(1-f)\bar{d}$ and $B\pi^* > \frac{\pi^*\bar{d}}{p_1^S(\pi^*)} n\left(p_1^S(\pi^*), a_0^M\right)\left(W_0^M - F(1-f)a_0^I W_0^I q_0 + fA_0^I\right) + E$

$$\frac{(x-F)(1-f)a_0^I W_0^I - f\hat{a}_1^{IS}\left(x(1-f)a_0^I + \frac{(1-a_0^I)}{q_0}\right) - E}{\left(W_0^M - F(1-f)a_0^I W_0^I q_0 + a_0^I W_0^I\right)} = n\left(p_1^S(\pi^*), a_0^M\right)$$

$$\pi_{I2}^* = \left(\frac{(x-F)(1-f)a_0^I W_0^I - f\hat{a}_1^{IS}\left(x(1-f)a_0^I + \frac{(1-a_0^I)}{q_0}\right)W_0^I - E}{\left(W_0^M - F(1-f)a_0^I W_0^I q_0 + a_0^I W_0^I\right)} - \frac{\left(1-a_0^M\right)}{q_0}\right)\frac{p_0}{a_0^M}\frac{1}{q_1(1-f)\bar{d}}\left(1-\hat{a}_1^{IS}\right) + \hat{a}_1^{IS}$$

Finally, π^{**} is such that

$$\begin{aligned} & \frac{\pi^{**}\bar{d}}{p_1^L(\pi^{**})}\left(\left(x - F - n\left(p_1^L(\pi^{**}), a_0^M\right)\right)(1-f)A_0^I - fA_1^I\left(p_1^L(\pi^{**}), \pi^{**}\right)\right) \\ = \; & \min\left\{B\pi^{**}, \frac{\pi^{**}\bar{d}}{p_1^L(\pi^*)} n\left(p_1^L(\pi^*), a_0^M\right)\left(W_0^M - F(1-f)a_0^I W_0^I q_0 + fA_0^I\right) + E\right\} \end{aligned}$$

If $p_1^L(\pi^{**}) < \bar{d}\pi q_1$ and $B\pi^* < \frac{\pi^*\bar{d}}{p_1^S(\pi^*)} n\left(p_1^L(\pi^{**}), a_0^M\right)\left(W_0^M - F(1-f)a_0^I W_0^I q_0 + fA_0^I\right) + E$

$$A_1^I\left(p_1^L(\pi^{**}), \pi^{**}\right) = \frac{\pi\bar{d}\frac{a_0^M}{p_0}(1-f)^2 a_0^I W_0^I - \left(\left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M + fa_0^I W_0^I\right) + E\right)\frac{1}{q_1}}{\bar{d}\frac{a_0^M}{p_0}(1-f)^2 a_0^I W_0^I - \left(\left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M + fa_0^I W_0^I\right) + E\right)\frac{1}{q_1}}\left(x(1-f)a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right)W_0^I$$

$$\begin{aligned} \pi^{**} = \; & \frac{\left(\left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M + fa_0^I W_0^I\right) + E\right)\frac{1}{q_1}}{\bar{d}\frac{a_0^M}{p_0}(1-f)^2 a_0^I W_0^I} + \\ & \frac{\left(\left(x - F - \left(\frac{a_0^M}{p_0}p_1^L + \left(1-a_0^M\right)\frac{1}{q_0}\right)\right)(1-f)A_0^I - \frac{B}{\bar{d}}p_1^L\right)\left(\bar{d}\frac{a_0^M}{p_0}(1-f)^2 a_0^I W_0^I - \left(\left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M + fa_0^I W_0^I\right) + E\right)\frac{1}{q_1}\right)}{\left(x(1-f)a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right)fW_0^I\bar{d}\frac{a_0^M}{p_0}(1-f)^2 a_0^I W_0^I} \end{aligned}$$

If $\pi\bar{d}\frac{a_0^M}{p_0}(1-f)^2 a_0^I W_0^I - \left(\left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M + fa_0^I W_0^I\right) + E\right)\frac{1}{q_1} < 0$, then there is excess demand on behalf of the managers and $p_L(\pi) = \bar{d}\pi q_1$.

If $p_1^L(\pi^{**}) < \bar{d}\pi^{**}q_1$ and $Bp_1^L(\pi^{**})/\bar{d} > \left(n\left(p_1^L(\pi^{**}), a_0^M\right)\left(W_0^M - F(1-f)a_0^I W_0^I q_0 + fA_0^I\right) + E\right)$

$$\begin{aligned} \pi^{**} = \; & \frac{\left((x-F)(1-f)A_0^I - n\left(p_1^L(\pi^*), a_0^M\right)\left(\hat{W}_0^M + A_0^I\right) + E\right)}{\left(x(1-f)a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right)fW_0^I} \\ & \frac{\left(\left(\bar{d}\frac{a_0^M}{p_0}(1-f)^2 a_0^I W_0^I - \left(\frac{\left(1-a_0^M\right)}{q_0}\left(\hat{W}_0^M + fa_0^I W_0^I\right) + E\right)\frac{1}{q_1}\right) + \left(\left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M + fa_0^I W_0^I\right) + E\right)\frac{1}{q_1}\right)}{\bar{d}\frac{a_0^M}{p_0}(1-f)^2 a_0^I W_0^I} \end{aligned}$$

For the next two cases $\pi^* = \pi^{**}$

If $p_1^S(\pi^*) = \bar{d}\pi^* q_1$ and $B\pi^* < \frac{\pi^*\bar{d}}{p_1^S(\pi^*)} n\left(p_1^S(\pi^*), a_0^M\right)\left(W_0^M - F(1-f)a_0^I W_0^I q_0 + f A_0^I\right) + E$

$$\pi^* = \frac{\left(x - F - \frac{\left(1-a_0^M\right)}{q_0}\right)(1-f)\,a_0^I W_0^I}{\left(Bq_1 + \frac{\bar{d}q_1}{p_0} a_0^M (1-f)\,a_0^I W_0^I\right)}$$

If $p_1^S(\pi^*) = \bar{d}\pi q_1$ and $B\pi^* > \frac{\pi^*\bar{d}}{p_1^S(\pi^*)} n\left(p_1^S(\pi^*), a_0^M\right)\left(W_0^M - F(1-f)a_0^I W_0^I q_0 + f A_0^I\right) + E$

$$\pi^* = \left(\frac{(x-F)(1-f)\,a_0^I W_0^I - E}{\left(W_0^M - F(1-f)\,a_0^I W_0^I q_0 + a_0^I W_0^I\right)} - \frac{\left(1-a_0^M\right)}{q_0}\right)\frac{p_0}{\bar{d}a_0^M q_1}$$