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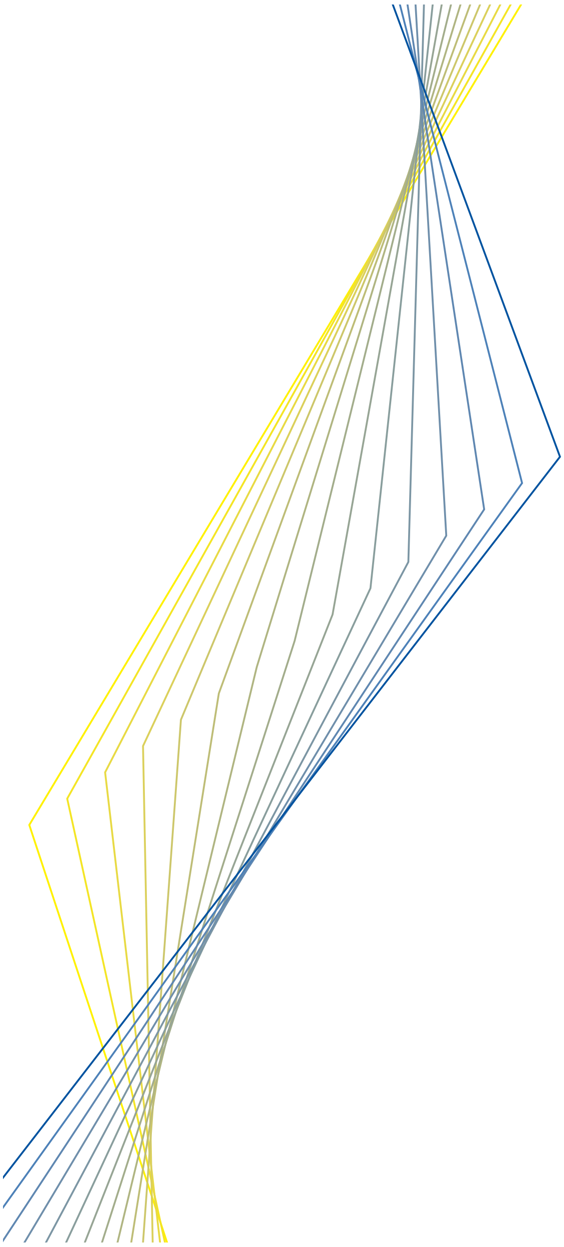

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Contents

Abstract

A simple model of the interaction between central bank liquidity management and the inter-bank overnight rate is suggested which helps understanding the effects of the publication of forecasts of liquidity factors by the European Central Bank adopted in June 2000. The paper argues that the main practical advantage of the publication of these forecasts is that it makes the signal extraction problem with regard to the central bank's operational intentions trivial and hence allows establishing a superior behavioural equilibrium between the central bank and the money market participants. In this equilibrium, the central bank can achieve a better steering of overnight rates than under private autonomous factor forecasts, depending of course also on the quality of liquidity forecasts. It is furthermore shown that the publication of an average of autonomous factors, such as adopted by the ECB, is, at least within the model presented, superior to the separate publication of autonomous factors for each single day.

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Monetary Policy instruments; money market; signal extraction

1. INTRODUCTION

In June 2000, the ECB took the decision to start publishing, together with the announcements of its weekly main refinancing operations, estimated liquidity needs of the banking system[2], joining for instance the Bank of Japan which has adopted a similar policy.

The liquidity needs of the banking system, i.e. the needs of the banking system regarding central bank money that have to be covered through monetary policy operations are composed of two main factors, namely reserve requirements and the so-called autonomous liquidity factors, such as banknotes in circulation and Government deposits. The role of liquidity forecasts in the central bank's liquidity management, independently of their publication, can be summarised as follows. The central bank attempts to provide liquidity through its open market operations in a way that, after taking into account its forecast effects of autonomous liquidity factors, counterparties can fulfil their reserve requirements on average over the reserve maintenance period without systematic recourse to the standing facilities (e.g. the deposit or marginal lending facilities in the case of the Eurosystem). If the central bank provides more (less) liquidity than this benchmark, counterparties will have to use at the end of the reserve maintenance period the standing facilities, which will push the overnight rate towards the relevant standing facility rate as soon as this liquidity imbalance becomes obvious. More precisely, in an efficient market, the overnight rate will correspond to the weighted rates of the standing facilities provided by the central bank, whereby the weights correspond to the respective probabilities that the market assigns to being short or long of liquidity at the end of the reserve maintenance period. Models based on this core relationship have been applied for instance by Angeloni and Prati [1996], Bartolini, Bertola and Prati [1998], Peres Quiros and Mendizabal [2000], and Bindseil and Seitz [2001].

The information policy of the central bank with regard to liquidity management is crucial since it affects expectations of counterparties of being short or long of liquidity at the end of the maintenance period, and hence the overnight rate, which normally plays an important role in the implementation of monetary policy since it constitutes the basic maturity in the yield curve. The following table summarises the information policy of major central banks with regard to liquidity management. It includes two columns for the ECB to allow representing both its old (i.e. pre-June 2000) and new information regime.

Table 1: The publication policy of central banks regarding liquidity management variables

	ECB old	*ECB new*	*FED*	*Bo Japan*
Autonomous factors (ex post)	Daily implicitly	Daily **explicitly**	Weekly averages	Daily
Autonomous factors (forecasts)	No	**Yes: forecast of average until next regular operation or end of reserve maintenance period**	No	Yes (one day horizon)
Open market operations (allotment volumes)	Yes, after allotment decisions	Yes, after allotment decision	Weekly averages	Yes
Standing facilities	Yes, daily	Yes, daily	No	Yes
Interest rate target	No	No	Yes	No

As indicated in the table, the Federal Reserve System publishes, in contrast to the ECB and the Bank of Japan, directly its overnight interest rate target. As will become clearer later in the paper, this can be interpreted as some kind of substitute for publishing autonomous factor forecasts.[3] A motivation of this approach adopted in 1995 is given for instance in Federal Reserve Bank of New York [2000, 46]. According to it, before 1995, market participants "closely watched the Desk's operations to detect policy signals. The use of open market operations to signal policy changes created, at times, considerable complications for the desk, especially when the funds rate and the reserve estimate gave conflicting signals… The recent disclosure procedures have essentially freed the desk from the risk that its normal technical or defensive operations would be misinterpreted as policy moves. Open market operations no longer convey any new information about changes in the stance of monetary policy." The move of the FED in 1995 was also the result of a longer debate on the pros and cons of secrecy of monetary policy, as represented for instance by Tabellini [1987] or Dotsey [1987]. In contrast to this literature, the present paper takes a more micro-economic, purely money-market oriented approach, focusing exclusively on the very short end of the implementation of monetary policy.[4]

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[2] See for instance ECB [2000a], [2000b]. An excerpt of the press release of the ECB of 16 June is reproduced in annex 1.

[3] This is the case because, basically, in a signal extraction problem with two unobserved variables, the publication of any of the two unobserved variables allows to also have perfect knowledge on the other one. However, it should also be noted that in the day-to-day implementation of monetary policy, the two alternatives (publication of an interest rate target vs. the publication of an autonomous factor forecast) also have various different practical implications, also depending on other aspects of the adopted operational framework. This paper will not go to the details of comparing the two approaches.

[4] However, section 4.2.2. will briefly put in perspective a result of this paper relative to a result of the previous, more macro-economic literature.

The aim of this paper is to provide an analytical framework to discuss, inter alia, the effects of the publication of forecast liquidity needs on the volatility and controllability of overnight rates in the context of the ECB, i.e. in terms of table 1, the implications of the switch from "ECB old" to "ECB new". At the same time, it provides a framework, which would be suitable to discuss all the potential decisions inherent in table 1. Furthermore, the paper analyses the role of the quality of the central bank's liquidity forecasts since, of course, the role of the publication of liquidity forecasts cannot be seen independently of their quality. Bindseil [2000] provided the intuition of the signal extraction solution for the case of a reserve maintenance period with 2 days. The present paper first assumes a one-day maintenance period, the simplicity of which allows deriving exact solutions for most of the problems and a series of propositions. It then also investigates again the case of a two-days maintenance period, whereby, again, exact solutions can be derived under some specific assumptions.

The rest of this paper proceeds as follows: Section 2 briefly introduces the model on which the subsequent sections are based on. Sections 3 and 4 consider the case of interest rate targeting, for the one and two days reserve maintenance period case, respectively. Section 5 restates the one day case in pure quantitative terms, circumventing the interest rate targeting by introducing instead a liquidity target. This allows to solve analytically the case of private autonomous factor forecasts, which had not been possible if the central bank target is expressed in the interest rate dimension. The analysis of each of the three cases (interest rate target for one and two days maintenance periods, quantitative target) will allow representing in the simplest way different aspects of the signal extraction problem banks face on the money market. Section 6 summarises the results and draws conclusions.

2. THE MODEL

Before proceeding, the basic framework of the model is briefly exposed, which is basically the same for all cases discussed subsequently.

The following **sequence of events** during the reserve maintenance period is assumed:

- First, the central bank conducts its open market operations. The allotment amount m_t (which is at the same time the amount of outstanding open market operations) is immediately published. If relevant, the central bank publishes its forecast of autonomous liquidity factors together with the tender announcement.
- Second, the inter-bank market on day 1 takes place and the overnight rate is fixed that clears the market.
- Third, the realisation of autonomous factors of day 1 takes place and is published.

- In case of a two days maintenance period, the preceding two steps take place again on the second day.
- Finally, the banks take recourse to standing facilities to cover the liquidity imbalance accumulated over the reserve maintenance period.

The limitation to the cases of a one or two days maintenance period, contrasting with actual maintenance periods of e.g. 14 days (FED) or one month (ECB) was made for the sake of simplicity.[5] The assumption that no open market operation takes place on the second day reflects the fact that central banks, which operate in a system with reserve requirements and averaging and regular open market operations, often do not have such an operation on the very last day of the maintenance period. For instance, the ECB (and previously the Bundesbank) has a weekly open market operation, such that the number of days between the last allotment decision of the reserve maintenance period and the end of the maintenance period can vary between five and one business days. The important fact captured by the model is that there is a period after the last open market operation of the maintenance period in which news on autonomous factor shocks are revealed and affect the perception of liquidity conditions by market participants.

Autonomous liquidity factors are assumed, for the sake of simplicity, to be white noise, i.e. $a_t = \varepsilon_t + \eta_t$, with ε_t, η_t being identically and independently normal distributed random variables with an expected value of zero and variances $\sigma^2_{\varepsilon 1} \in [0,1], \sigma^2_{\eta 1} = 1 - \sigma^2_{\varepsilon 1}$, $\sigma^2_{\varepsilon 2} \in [0,1], \sigma^2_{\eta 2} = 1 - \sigma^2_{\varepsilon 2}$ (in the two days case t = 1, 2; in the one-day case, t=1). Obviously, the total variance of autonomous factors per day is standardised to 1 in the model. The central bank is assumed to have perfect forecasts of ε, but it has no prior information on η. The higher $\sigma^2_{\varepsilon 1}$, the better is thus the quality of liquidity forecasts of the central bank for the autonomous factors on day 1. Note that it is assumed that autonomous factor shocks are not auto-correlated. This assumption simplifies calculus substantially, but should not affect the crucial conclusions of the note.[6] Banks are assumed to have no prior information on any of the two variables. This assumption seems to be in contrast to the Hayekian idea

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[5] The simplification is legitimate especially in so far as the period before the last open market operation of the reserve maintenance period is in general relatively "uninteresting" from the point of view of autonomous factor forecasting. For instance, the ECB has the reputation that it normally compensates any autonomous factor shocks that it had not anticipated in the preceding operation through its next allotment decisions, if any remains within the same reserve maintenance period. Hence, autonomous factor shocks before the last allotment decision of the maintenance period are normally of negligible influence on the overnight rate (see Bindseil and Seitz [2001]).

[6] In reality, autonomous factor shocks clearly exhibit some degree of auto-correlation.

of information as being dispersed among many individual actors within the economy. However, it appears in central bank practice that indeed, the central bank has practically always superior knowledge relative to market players with regard to anticipating autonomous liquidity factor flows[7], and that for instance it never obtains information that is valuable in terms of autonomous factor forecasting out of the bids submitted by banks in open market operations.[8] The assumptions of the model naturally also implies that information asymmetries between market participants are not relevant. Generally, market participants are assumed to operate under perfect competition, such that inter-bank interest rates reflect competitively the publicly available information.

In the following two sections (sections 3 and 4), it is assumed that the central bank has an **overnight interest rate target** that may change over time. From the point of view of the market, the interest rate target of the central bank contains some unpredictable elements. This is modelled by assuming that from the point of view of the market, the interest rate target i^* is symmetrically distributed around $i^* = 0.5$ with a density function $f_{i^*}(i^*)$ with $\forall i^* \notin]0,1[: f_{i^*}(i^*) = 0$. The value of i^* for the reserve maintenance period is drawn before the start of the reserve maintenance period. In contrast, Section 5 will assume that the central bank has, instead of an interest rate target, a direct quantitative (i.e. a liquidity) target denoted γ.

The following two assumptions are made solely for the purpose of a simpler representation, but have no relevance for the results obtained. Firstly, the rate of the deposit facility is set to zero and the rate of the marginal lending facility is set to one. Secondly, reserve requirements and the demand for excess reserves are zero. However, the averaging capacity provided by the reserve requirement system is unlimited (i.e. in the case of the two days reserve maintenance period, banks can go overdraft on the first day without having to take recourse to standing facilities)[9].

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[7] In the sense that the information available to market participants has no value added relative to the information available to the ECB.

[8] The conjecture that central banks extract valuable information held by market participants from the bids submitted by banks in open market operations has been put forward for instance by Nautz [1997].

[9] This assumption of unlimited averaging capacities is crucial to obtain, as in the present model, the martingale property of overnight interest rates in its pure form (see below). In reality, as has been shown by Peres Quiros and Mendizabal [2000] for the euro area, the martingale property is fulfilled to a large extent, but not perfectly. They explain this observation by limited averaging capacities at the level of the individual banks which imply that banks should have a preference for back-loading their reserve fulfilment within the reserve maintenance period.

The following simple theory of the relationship between liquidity and overnight rates forms the starting point of this paper. Assume for the sake of simplicity the one day case. Like many other markets, the market for reserves is interesting owing to its *uncertainty*. Assume for a moment that there is no uncertainty concerning either autonomous factors or the liquidity supply through open market operations in the remainder of the reserve maintenance period. In this setting, reserves are obviously either short or long in relation to reserve requirements, in which case the marginal utility of funds obtained in the inter-bank market, and therefore their price, either rises to the marginal lending rate, or drops to the deposit rate. The overnight rate would therefore correspond, under the assumption of perfect foresight with regard to liquidity conditions, to one of the standing facility rates relevant at the end of the maintenance period. One may call *(m-a)* the "non-borrowed" reserves, to use a term applied usually in the US (here and in the following, we drop, for the sake of simplicity, the index "1" in case of the one period model). The only interest rate elastic elements of the market equilibrium condition for bank deposits with the central bank, the standing facilities, have the following functional form, assuming perfect inter-bank markets (where L is the recourse to the marginal lending facility and D is the recourse to the deposit facility):

$$\begin{array}{ll} \forall i < 1: \; L(i) = 0 & \forall i > 0: \; D(i) = 0 \\ \forall i > 1: \; L(i) = \infty & \forall i < 0: \; D(i) = \infty \\ i = 1: \; L(i) = a - m & i = 0: \; D(i) = m - a \end{array} \tag{1}$$

The overnight interest rate that clears the market for central bank deposits is then determined by:

$$m + (L(i) - D(i)) - a = 0, \tag{2}$$

Now we shall consider the more interesting case in which the liquidity supply and the rates of the standing facilities are uncertain in the sense that the banking sector has a collective subjective density function for the relevant liquidity factors in its mind. The basic relationship between quantities and prices (overnight rates) under the assumptions made above (especially the one of perfect inter-bank markets and large reserve requirements) is then described by the following equation, in which f_{m-a} is the density function the money market participants assigns during the trading session to the random variable $m-a$:

$$i = 1P("short") + 0P("long") = P("short") = \int_{-\infty}^{0} f_{(m-a)}(z)dz \tag{3}$$

In words: the overnight rates on any day will correspond to the weighted expected rates of the two standing facilities, the weights being the respective probabilities that the market will be short or long at the end of the maintenance period before having recourse to standing facilities. It should be noted that this also implies the martingale property of the overnight interest rate within the reserve maintenance period, i.e. that the overnight rate on any day corresponds to the expected overnight rates on the following days of the same reserve maintenance period. This property holds under the assumptions of the model outlined above, but it should be kept in mind that some of these assumptions, and hence the martingale property, have also been questioned in the literature.[10] Since we set the deposit facility rate to zero and the marginal lending rate to one, the overnight rate will simply correspond to the likelihood of a shortage of non-borrowed funds. Expectations, i.e. the subjective density function f_{m-a} the banking sector assigns to the non-borrowed reserves, *m-a* will obviously be crucial.

Of course, all assumptions of the model constitute simplifications of reality. The maintenance period is much longer in the case of the ECB than one or two days. There are several operations in the maintenance period. Autonomous factors are not revealed suddenly at the end of the day, but more smoothly in the course of the day. Nevertheless, the model allows to represent the main elements which determine the relationship between liquidity management, information policy, and overnight rates. Therefore it allows improving our assessment of inevitable policy decisions such as to publish forecasts of liquidity needs or not, and to what extent a central bank should invest into the quality of its liquidity forecasts.

3. THE STEERING OF OVERNIGHT RATES BY THE CENTRAL BANK: THE ONE DAY CASE

Assume now the case of a one day maintenance period and that, when deciding on the open market operation volume, the central bank chooses

$$m = \arg\min\{\theta E(i-i^*)^2 + E(m-a)^2 \mid i^* = i_0^*, \varepsilon = \varepsilon_0\} \tag{4}$$

with $\theta \in \Re_+$ and i_0^*, ε_0 specific realisations of the random variables i_0, ε_0, respectively (in the rest of the paper, variables with a "0" index will always refer to specific realisations of random variables). In words: the central bank chooses an allotment volume m that minimises a loss function defined as the weighted sum of the expected squared differences of overnight market rates from its target rate and of the expected squared end of maintenance period liquidity imbalance. Normally (i.e. under non-perfect autonomous factor forecasts), the steering of interest rates will not be perfect as long as θ does not

[10] For a discussion of the martingale property of overnight rates, an empirical analysis for the US, and a tentative model to explain the observed deviations from it, see Hamilton [1996]. For an empirical analysis of the euro area, and a different, leaner explanatory model, see Peres Quiros and Mendizabal [2000].

tend to infinity, i.e. if the central bank aims independently at keeping the imbalance of liquidity at the end of the maintenance period limited.

As will be shown below, the second term in the objective function of the central bank, i.e. the one referring to the end of the reserve maintenance period liquidity imbalance, is necessary in order to motivate that the central bank does not, in the one period case, simply ignore its forecasts of autonomous factors. Why should liquidity at the end of the reserve maintenance period be relevant for the central bank independently from interest rates? Recourse to standing facilities is costly for counterparties, and as it normally does not hit all counterparties in the same way, the ones affected most may have to carry a substantive cost. This unavoidably raises criticism towards the central bank for not ensuring "orderly" market conditions. It could indeed be argued that large recourse to standing facilities at the end of the maintenance period either reflects miserable liquidity forecasts of the central bank, or that the central bank deliberately provided an amount of liquidity to the market that was inadequate and that imposed undue costs to the banking system.

In the two following sections, the cases of non-public and published autonomous factor forecasts will be treated subsequently.

3.1 If the central bank does <u>not</u> publish its autonomous factor estimates

In this case, the market equilibrium is characterised by the following pair of equations.

$$\begin{cases} m = \arg\min\{\theta E(i-i^*)^2 + E(m-a)^2 \mid i^* = i_0^*, \varepsilon = \varepsilon_0\} \\ i = P(m-a<0 \mid m = m_0) \end{cases} \tag{5}$$

In words: the central bank minimises its loss function knowing both its interest rate target and its autonomous factor forecast, while the market participants, who determine the actual overnight rates, only observe the amount allotted by the central bank.

Unfortunately, characterising the resulting equilibrium is not straightforward. The relationship between the variables which remain unobserved to the market (i_0^*, ε_0) and the observed ones (m_0), i.e. the signal extraction problem, is not linear. No simple analytical solution is available for this optimisation problem. Approaching the problem from the point of view of the theory of fixed-point theorems also does not provide easy help. By substituting, one obtains an equation $m = f(m)$ with $f: R \to R$ a function for which one has to find a fixed point. However, the function cannot be characterised sufficiently in order to allow the application of the standard fixed point theorems used in economics such as the one of Brouwer. Even if one would manage to describe some elements of the equilibrium point, its properties would be far from obvious, especially in a noisy environment such as the one of real money markets. Independently from this conclusion, it is clear that the central bank will not perfectly steer rates if it also cares independently about quantities and if it makes use of non-published autonomous factor forecasts.

3.2 If the central bank publishes its autonomous factors

The following proposition suggests that the publication of autonomous factors allows the central bank to steer interest rates in a precise way while making full use of its autonomous factor forecasts in order to minimise the end of maintenance period liquidity imbalance.

> **Observation 1:** *In the one day case, publishing autonomous factors allows the central bank to perfectly steer the overnight rate, independently of the quality of liquidity forecasts. At the same time, it allows the central bank to reduce the expected value of the squared end of maintenance period liquidity imbalance to the minimum that can be achieved with a perfect steering of interest rates, for a given quality of liquidity forecasts.*

If the central bank publishes its autonomous factors, interest rates are determined as follows:

$$i = P(m - a < 0 \mid m = m_0, \varepsilon = \varepsilon_0) = \int_{-\infty}^{0} f_{(m-a|m=m_0,\varepsilon=\varepsilon_0)}(x)dx = F_{(m-a|m=m_0,\varepsilon=\varepsilon_0)}(0) \tag{6}$$

where $f_{(m-a|m=m_0,\varepsilon=\varepsilon_0)}$ is the density function of $(m - a \mid m = m_0, \varepsilon = \varepsilon_0)$ and $F_{(m-a|m=m_0,\varepsilon=\varepsilon_0)}$ is its cumulative distribution.

To characterise the behavioural equilibrium between the market and the central bank in this case, assume that the central bank makes use of the following additive allotment strategy: $m_0 = \varepsilon_0 + \gamma(i^*)$, i.e. the allotted amount is composed of two additive components, one compensating the published expected value of autonomous factors, while the other maps the interest rate target into a liquidity target. The existence and exact shape of such a mapping allowing for a perfect interest rate steering will be shown in the annex 2, as well as the fact that there is no other allotment rule allowing also for a perfect steering of interest rates that allows achieving a smaller expected squared end of reserve maintenance period recourse to standing facilities. The proof there proceeds in three steps. *First*, it is shown that there exists a strategy of the type $m_0 = \varepsilon_0 + \gamma(i_0^*)$ that allows for a perfect steering of interest rates. Note that $F_{(m-a|m=m_0,\varepsilon=\varepsilon_0)}$ is the cumulative distribution function of an $N(m_0 - \varepsilon_0, 1 - \sigma_\varepsilon^2)$ distributed random variable since $E(m - a \mid m = m_0, \varepsilon = \varepsilon_0) = E(m_0 - (\varepsilon_0 + \eta)) = E(m_0 - \varepsilon_0))$ and $Var(m - a \mid m = m_0, \varepsilon = \varepsilon_0)$

$= Var(\eta) = 1-\sigma_\varepsilon^2$, and hence: $i = 1-\Phi\left(\frac{m_0-\varepsilon_0}{\sqrt{1-\sigma_\varepsilon^2}}\right) = 1-\Phi\left(\frac{\gamma}{\sqrt{1-\sigma_\varepsilon^2}}\right)$. This relationship can be used by the central bank to steer interest rates. The central bank has simply to choose an allotment volume corresponding to the sum of the expected autonomous factor and the i_0^* quantile of a normal distribution with variance $1-\sigma_\varepsilon^2$: $\gamma(i_0^*) = \sqrt{1-\sigma_\varepsilon^2}\,\Phi^{-1}(1-i^*)$. *Secondly*, we show that the set of alternative allotment strategies allowing for a perfect steering of interest rates is limited to the one in which the central bank deviates from the one proposed above by reducing the quality of its liquidity forecasts. *Finally*, it has to be shown that this allotment strategy minimises the expected squared variance of the recourse to standing facility at the end of the reserve maintenance period if the central bank makes full use of its autonomous factor forecasts.

It can be concluded that in the one-day case, the publication of autonomous factors allows the central bank to better achieve its aims than in the case of private autonomous factors. Specifically, it makes the signal extraction problem simple and allows hence the establishment of a transparent behavioural equilibrium between the central bank and counterparties in which the central bank can precisely steer market expectations and hence overnight rates, while at the same time minimising the expected recourse to standing facilities at the end of the reserve maintenance period.

It follows from the reasoning above that the central bank can of course also achieve a perfect steering of interest rates without a publication of autonomous factor forecasts if it simply ignores its private knowledge about them, which is equivalent to setting $\sigma_\varepsilon^2 = 0$. The cost of this strategy is that the variance of end of maintenance period imbalances is higher than in case of published and used forecasts, namely by the actual value of σ_ε^2. As expressed in the following proposition, this is a non-dominated strategy if the central bank only aims at steering interest rates.

> **Observation 2**. *In the one-day maintenance period case, the central bank can ignore its private autonomous factor forecast if it exclusively aims at targeting the overnight interest rate. Indeed, it can then achieve a perfect steering of interest rates.*

The proof of the observation follows immediately from observation 1, which stated that a perfect steering of the overnight rate is possible independently of the quality of liquidity forecasts. Ignoring autonomous factors is equivalent to setting their quality to zero. As will also be shown later, the result is specific to the assumed one-day reserve maintenance period. If another day would follow, the volatility of the liquidity situation would be translated into a volatility of overnight rates on subsequent days.

4. A TWO DAYS RESERVE MAINTENANCE PERIOD WITH AN OPEN MARKET OPERATION IN THE MORNING OF DAY 1

As in the previous section, the two cases of private and published autonomous factors are treated subsequently. Then, some ideas regarding a normative analysis of the results are provided. We assume that the central bank focuses in its optimisation primarily on the interest rate on the first day. We will then see that the quality of the steering on the second day depends on the quality and publication of liquidity forecasts. This approach is motivated in more detail in section 4.2 and it is shown that it is there equivalent to focus on both days simultaneously.

4.1 The case of private autonomous factor forecasts

Again, we consider first the case with private, but used forecasts of autonomous factors. Adopting the notation $m = (m_1 + m_2)/2; \quad a = (a_1 + a_2)/2$, the general problem of the central bank becomes:

$$\begin{cases} m = \arg\min\{E(i_1 - i^*)^2 \mid \varepsilon_1 = \varepsilon_{1,0}, \varepsilon_2 = \varepsilon_{2,0}, i^* = i_0^*\} \\ i_1 = P(m - a < 0 \mid m = m_0) \\ i_2 = P(m - a < 0 \mid m = m_0, a_1 = a_{1,0}) \end{cases} \tag{7}$$

As in the one-day case, it is not obvious to characterise the solution to this problem and to show the existence and uniqueness of equilibrium. Of course, some kind of equilibrium was observed when the ECB applied this policy. But it seems that we cannot easily describe this equilibrium in theoretical terms. We therefore limit again the detailed discussion to the case where the central bank publishes its autonomous factors.

4.2 If the central bank publishes its autonomous factor forecasts

It is now assumed that the central bank publishes its autonomous factor forecasts. We distinguish between two cases. In the first, the central bank publishes forecasts individually for the two days. In the second, the central bank publishes a forecast only for the average autonomous factors on the two days, which comes close to what the ECB has been doing since July 2000. We account for the possibility that the quality of liquidity forecasts declines, i.e. we distinguish explicitly between the qualities of forecasts at the two different time horizons, whereby we expect that the quality of liquidity forecasts may not increase when the horizon lengthens: $1 \geq \sigma_{\varepsilon 1}^2 \geq \sigma_{\varepsilon 2}^2 \geq 0$. In both cases, the analysis of the steering of interest rates will proceed as follows: first, it will be shown that the interest rate on the first day can be steered perfectly and it will be assumed that the central bank indeed follows the strategy to aim in the first place at perfectly steering the first day's interest rate. Then, the resulting variance of the interest rate on the second day will be quantified under both approaches. It will also be shown that the interest rate will follow in any case a martingale. This property allows us to indeed focus only on strategies of the central bank to steer in the first place the interest rate on the first day. Obviously, if the interest rate follows a martingale, the best steering of interest rates on the first day also minimises, independently of the way autonomous factor forecasts are published, the variance of the interest rate relative to the target on the second day.

4.2.1 The central bank publishes individual autonomous factor forecasts for each remaining day of the reserve maintenance period

Assuming, as discussed, that the central bank focuses primarily on the first day, its optimisation problem is, under this publication scheme, as follows:

$$\begin{cases} m_0 = \arg\min\{(i_1 - i_0^*)^2 \mid \varepsilon_1 = \varepsilon_{1,0}, \varepsilon_2 = \varepsilon_{2,0}, i^* = i_0^*\} \\ i_1 = P(m - a < 0 \mid m = m_0, \varepsilon_1 = \varepsilon_{1,0}, \varepsilon_2 = \varepsilon_{2,0}) \\ = 1 - \Phi\left(\frac{(2m_0 - \varepsilon_{1,0} - \varepsilon_{2,0})}{\sqrt{2 - \sigma_{\varepsilon 1}^2 - \sigma_{\varepsilon 2}^2}}\right) = 1 - \Phi\left(\frac{\gamma}{\sqrt{2 - \sigma_{\varepsilon 1}^2 - \sigma_{\varepsilon 2}^2}}\right) \end{cases} \tag{8}$$

Similarly to the case of the one day maintenance period, it can easily be shown that the allotment strategy $\gamma(i_0^*) + \varepsilon_{1,0} + \varepsilon_{2,0}$ with $\gamma(i_0^*) = \sqrt{2 - \sigma_{\varepsilon 1}^2 - \sigma_{\varepsilon 2}^2}\,\Phi^{-1}(1 - i_0^*)$ allows for a perfect steering of the interest rate on the first day. Similarly to the one day maintenance period case, the better the qualities of liquidity forecasts at the different horizons, the smaller the expected squared recourse to standing facilities. Denoting for instance by $g_{1-\sigma_{\varepsilon 1}^2}$ the density of a normal distribution with expected value zero and variance $1 - \sigma_{\varepsilon 1}^2$, the expected interest rate on day 2 will be:

$$E_1(i_2) = E(P(m - a < 0 \mid m = m_0, a_1 = a_{1,0}, \varepsilon_1 = \varepsilon_{1,0}, \varepsilon_2 = \varepsilon_{2,0})$$
$$= \int_{-\infty}^{+\infty}\left(1 - \Phi\left(\frac{(\gamma + \eta_1)}{\sqrt{1 - \sigma_{\varepsilon 2}^2}}\right)\right) g_{1-\sigma_{\varepsilon 1}^2}(\eta_1)\, d\eta_1) \tag{9}$$

As should have been expected on the basis of the assumptions of the model, the martingale property holds for any values of $\sigma_{\varepsilon 1}^2$ and for any value of γ, i.e. that:[11]

$$\begin{aligned}&\int_{-\infty}^{+\infty}\left(1-\Phi\left(\frac{(\gamma+\eta_1)}{\sqrt{1-\sigma_{\varepsilon 2}^2}}\right)\right)g_{1-\sigma_{\varepsilon 1}^2}(\eta_1)d\eta_1 = 1-\Phi\left(\frac{\gamma}{\sqrt{2-\sigma_{\varepsilon 1}^2-\sigma_{\varepsilon 2}^2}}\right)\\ \Leftrightarrow &\int_{-\infty}^{+\infty}\left(\Phi\left(\frac{(\gamma+\eta_1)}{\sqrt{1-\sigma_{\varepsilon 2}^2}}\right)\right)g_{1-\sigma_{\varepsilon 1}^2}(\eta_1)d\eta_1 = \Phi\left(\frac{\gamma}{\sqrt{2-\sigma_{\varepsilon 1}^2-\sigma_{\varepsilon 2}^2}}\right)\end{aligned} \tag{10}$$

The martingale property shows that two effects exactly compensate for any combinations of quality of liquidity forecasts: The first effect consist in the reduction of residual uncertainty between the two market sessions, i.e. the decrease of the denominator in the cumulative Gauss function implies that interest rates should approach more and more the corridor rate which is closer to the target rate. In other words: assuming that future autonomous factors correspond to their expected value (zero), the steepening of the cumulative Gauss function relating to the vanishing of uncertainty should map any liquidity imbalance closer and closer to a standing facility rate. The second effect, which compensates the first one, is due to the increasing uncertainty relating to past autonomous factors when approaching the end of the reserve maintenance period. The bigger this uncertainty, the more relevant the convexity of the cumulative Gauss-Function becomes when the expectation is built over all possible values of autonomous factors on the first day. Furthermore, one could say that the convexity of the cumulative Gauss function increases when the remaining uncertainty vanishes and it steepens correspondingly.

As suggested by Bartolini, Bertola, Prati [2000], the variance of interest rates will however increase when approaching the end of the reserve maintenance period. Here, two effects go in the same direction: the autonomous factor shocks on day 1 impact on the interest rate, and the related vanishing of uncertainty steepens the cumulative density function such that liquidity imbalances are mapped more strongly into deviations of rates from the mid point of the corridor set by standing facilities rates. The variance of interest rates on day 2 is defined as:

$$\operatorname{var}(i_2-i_0^*) = \int_{-\infty}^{+\infty}\left(1-\Phi\left(\frac{(\gamma(i_0^*)+\eta_1)}{\sqrt{1-\sigma_{\varepsilon 2}^2}}\right)-i_0^*\right)^2 g_{1-\sigma_{\varepsilon 1}^2}(\eta_1)d\eta_1 \tag{11}$$

Chart 1 below draws for $i_0^* = 0.7$ the variance of the deviation of interest rates on day 2 from targets, $\operatorname{var}(i_2 - i_0^*)$ for $\sigma_{\varepsilon 1}^2 = 0.8$ for different values of $\sigma_{\varepsilon 2}^2 \le 0.8$. The function increases monotonously in the quality of the liquidity forecasts for the second day. This is intuitive in so far as a good quality of

[11] The property has been verified through calculations of various examples in *Mathematica*. The *Mathematica* code underlying this and other calculus in the paper can be obtained from the author.

forecasts for the second day implies little residual uncertainty regarding autonomous factors, and hence a strong impact of news regarding the expected liquidity situation on interest rates.

Chart 2 draws, again for $i_0^* = 0.7$ the function $\text{var}(i_2 - i_0^*)$ of $\sigma_{\varepsilon 1}^2 \geq 0.8$ for $\sigma_{\varepsilon 2}^2 = 0.1$. The function decreases monotonously in the quality of the liquidity forecasts for the first day. This is again intuitive in so far as a good quality of forecasts for the first day implies that only little news are likely to have emerged between the first and the second day's money market session, and there are hence little reasons for a change in the market interest rate. One may conclude that in the two maintenance period model with a separate publication of autonomous factors forecasts for each day, the following observation holds:

> ***Observation 3****: To achieve the best control of overnight rates also on the second day of the reserve maintenance period, the central bank, which publishes separately its autonomous factor forecasts for the two days, should invest all resources devoted to autonomous factor forecasting into the forecasting of autonomous factors on the first day of the maintenance period, and none into the forecasting on the second day.*

The proof of the proposition is provided in the annex 2. Observation 3 contradicts a possible first intuition that the central bank should invest its resources equally into the forecasting of autonomous factors on the different days of the maintenance period. But of course, a central bank may also wish to minimise the expected squared recourse to standing facilities at the end of the reserve maintenance period, which would argue in favour of a more equal allocation of resources to forecasting autonomous liquidity factors on the two days.

4.2.2 The central bank publishes only an average autonomous factor forecast for both days

The ECB decided on 8 June 2000 to publish a forecast of the average of autonomous factors in the relevant period. In our model of a two days reserve maintenance period, this means that the central bank publishes only one figure, namely $\varepsilon = (\varepsilon_1 + \varepsilon_2)/2$, instead of the two separate autonomous factors. For the central bank's allotment decision on the first day, nothing changes, since counterparties can still extract precisely the liquidity target from the allotment volume and the forecast of autonomous factors. However, things change on the second day, when the banks wonder in how far the central bank anticipated the autonomous factor shock on day 1.

The central bank's optimisation problem, again taking the approach to first focus on the interest rate on the first day of the maintenance period, becomes:

$$
\begin{cases}
m_0 = \arg\min\{(i_1 - i_0^*)^2 \mid \varepsilon_1 = \varepsilon_{1,0}, \varepsilon_2 = \varepsilon_{2,0}, i^* = i_0^*\} \\
i_1 = P(m - a < 0 \mid m = m_0, \varepsilon_1 + \varepsilon_2 = \varepsilon_{1,0} + \varepsilon_{2,0}) \\
= 1 - \Phi\left(\dfrac{(2m_0 - \varepsilon_{1,0} - \varepsilon_{2,0})}{\sqrt{2 - \sigma_{\varepsilon 1}^2 - \sigma_{\varepsilon 2}^2}}\right) = 1 - \Phi\left(\dfrac{\gamma}{\sqrt{2 - \sigma_{\varepsilon 1}^2 - \sigma_{\varepsilon 2}^2}}\right)
\end{cases}
\tag{12}
$$

Obviously, the same result regarding the optimal allotment strategy as in the case of publication of separate autonomous factor figures is obtained. However, things are different on the second day of the reserve maintenance period. The interest rate on day 2 equals $P(m - a < 0 \mid m = m_0, a_1 = a_{1,0}, \varepsilon_1 + \varepsilon_2 = \varepsilon_{1,0} + \varepsilon_{2,0})$, with $m - a = \gamma(i_0^*) + \varepsilon_1 + \varepsilon_2 - (\varepsilon_1 + \eta_1 + \varepsilon_2 + \eta_2) = \gamma(i_0^*) - \eta_1 - \eta_2$. While the liquidity target $\gamma(i_0^*)$ can be calculated by market participants precisely on the first day, the non-anticipated autonomous factor component on the first day is, in contrast to the case of a separate publication of autonomous factor forecasts not exactly known on day 2. Nevertheless, counterparties can extract some information regarding the non-anticipated autonomous factor development on day 1 from the observed variables. This signal extraction problem can be represented in its linear matrix form as $z = \Lambda x$, where z is the vector of observed and x the vector of unobserved variables with:

$$
z = \begin{pmatrix} \varepsilon \\ a_1 \end{pmatrix} \quad \Lambda = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \quad x = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \eta_1 \end{pmatrix} \quad E(xx') = \begin{pmatrix} \sigma_{\varepsilon 1}^2 & 0 & 0 \\ 0 & \sigma_{\varepsilon 2}^2 & 0 \\ 0 & 0 & 1 - \sigma_{\varepsilon 1}^2 \end{pmatrix}
\tag{13}
$$

We are looking for the matrix of signal extraction coefficients such that $\hat{x} = Bz$. As is shown at the end of annex 2, $B = (\Lambda E(xx')\Lambda')^{-1}\Lambda E(xx')$. We thus obtain:

$$
(\Lambda E(xx')\Lambda')^{-1} = \frac{1}{\sigma_{\varepsilon 1}^2 + \sigma_{\varepsilon 2}^2 - \sigma_{\varepsilon 1}^4} \begin{pmatrix} 1 & -\sigma_{\varepsilon 1}^2 \\ -\sigma_{\varepsilon 1}^2 & \sigma_{\varepsilon 1}^2 + \sigma_{\varepsilon 2}^2 \end{pmatrix}
\tag{14}
$$

The matrix of signal extraction coefficients is:

$$
B = \frac{1}{\sigma_{\varepsilon 1}^2 + \sigma_{\varepsilon 2}^2 - \sigma_{\varepsilon 1}^4} \begin{pmatrix} \sigma_{\varepsilon 1}^2 - \sigma_{\varepsilon 1}^4 & \sigma_{\varepsilon 2}^2 & -\sigma_{\varepsilon 1}^2(1 - \sigma_{\varepsilon 1}^2) \\ \sigma_{\varepsilon 1}^2 \sigma_{\varepsilon 2}^2 & -\sigma_{\varepsilon 1}^2 \sigma_{\varepsilon 2}^2 & (\sigma_{\varepsilon 1}^2 + \sigma_{\varepsilon 2}^2)(1 - \sigma_{\varepsilon 1}^2) \end{pmatrix}
\tag{15}
$$

Therefore, we obtain as best estimator for η_1:

$$
E(\eta_1 \mid m = m_0, a_1 = a_{1,0}, \varepsilon = \varepsilon_0) = \frac{-\sigma_{\varepsilon 1}^2(1 - \sigma_{\varepsilon 1}^2)}{\sigma_{\varepsilon 1}^2 + \sigma_{\varepsilon 2}^2 - \sigma_{\varepsilon 1}^4}\varepsilon + \frac{(\sigma_{\varepsilon 1}^2 + \sigma_{\varepsilon 2}^2)(1 - \sigma_{\varepsilon 1}^2)}{\sigma_{\varepsilon 1}^2 + \sigma_{\varepsilon 2}^2 - \sigma_{\varepsilon 1}^4} a_1
\tag{16}
$$

The uncertainty in this estimate, $\operatorname{var}(\eta_1 \mid m = m_0, a_1 = a_{1,0}, \varepsilon_1 + \varepsilon_2 = \varepsilon_{1,0} + \varepsilon_{2,0})$, corresponds to the relevant element in the variance covariance matrix of the estimated unobserved variables, which is: $E((\hat{x} - x)(\hat{x} - x)') = E((B'\Lambda x - x)(B'\Lambda x - x)') = B'\Lambda E(xx')\Lambda' B - sB'\Lambda E(xx') + E(xx')$. Since the expression is rather lengthy, it is not displayed here. The interest rate on day 2 of the maintenance period will amount to:

$$i_2 = 1 - \Phi\left(\frac{\gamma + E(\eta_1 \mid m = m_0, a_1 = a_{1,0}, \varepsilon_1 + \varepsilon_2 = \varepsilon_{1,0} + \varepsilon_{2,0})}{\sqrt{1 - \sigma_{\varepsilon 2}^2 + \operatorname{var}(\eta_1 \mid m = m_0, a_1 = a_{1,0}, \varepsilon_1 + \varepsilon_2 = \varepsilon_{1,0} + \varepsilon_{2,0})}} \right) \tag{17}$$

The expected value of the interest rate on day 2 will be:

$$E(i_2) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left(1 - \Phi\left(\frac{(\gamma(i_0^*) + E(\eta_1 \mid m = m_0, a_1 = a_{1,0}, \varepsilon_1 + \varepsilon_2 = \varepsilon_{1,0} + \varepsilon_{2,0})}{\sqrt{1 - \sigma_{\varepsilon 2}^2 + \operatorname{var}(\eta_1 \mid m = m_0, a_1 = a_{1,0}, \varepsilon_1 + \varepsilon_2 = \varepsilon_{1,0} + \varepsilon_{2,0})}} \right) \right) g_{1-\sigma_{\varepsilon 1}^2}(\eta_1) g_{\sigma_{\varepsilon 1}^2}(\varepsilon_1) g_{\sigma_{\varepsilon 2}^2}(\varepsilon_2) d\eta_1 d\varepsilon_1 d\varepsilon_2 \tag{18}$$

Again, calculations verify that the martingale property holds, namely that $\forall i_0^*, \forall \sigma_{\varepsilon 1}^2, \forall \sigma_{\varepsilon 2}^2$: $E_1(i_2) = E_1(i_1) = i_0^*$. The variance of the interest rate on day 2 in this regime will be:

$$\operatorname{var}(i_2 - i_0^*) = \int_{-\infty}^{\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left(1 - \Phi\left(\frac{(\gamma(i_0^*) + E(\eta_1 \mid m = m_0, a_1 = a_{1,0}, \varepsilon_1 + \varepsilon_2 = \varepsilon_{1,0} + \varepsilon_{2,0})}{\sqrt{1 - \sigma_{\varepsilon 2}^2 + \operatorname{var}(\eta_1 \mid m = m_0, a_1 = a_{1,0}, \varepsilon_1 + \varepsilon_2 = \varepsilon_{1,0} + \varepsilon_{2,0})}} \right) - i_0^* \right)^2 g_{1-\sigma_{\varepsilon 1}^2}(\eta_1) g_{\sigma_{\varepsilon 1}^2}(\varepsilon_1) g_{\sigma_{\varepsilon 2}^2}(\varepsilon_2) d\eta_1 d\varepsilon_1 d\varepsilon_2 \tag{19}$$

The following charts (chart 1 and 2) also display the variance of the difference between the interest rate on day 2 and the central bank's target of 0.7 for the case of a publication of a forecast of average autonomous factors over the forecasting horizon. The charts allow comparing directly this case with the previously treated one of a separate publication of autonomous factor forecasts for every single day. As could be expected, publishing a forecast of an average of individual daily figures does not make a big difference if one of the forecast qualities is much higher than the other, since the average then contains nearly the same information as the forecast for the day for which the forecast is much better. In contrast, the variance of day 2 interest rates is systematically lower if both qualities of autonomous factor forecasts are relevant, such that averaging dissipates information. Furthermore, the U-shaped form of the variance function displayed in chart 1 is noteworthy. In contrast to the case of published individual autonomous factors, one cannot conclude in the case of a published average that investing no resources into forecasting of autonomous factors on day 2 is necessarily best (if the central bank does not care about the expected squared recourse to standing facilities).

Chart 1: The function $\mathrm{var}(i_2 - i_0^*)$ *for* $i_0^* = 0.7$, $\sigma_{\varepsilon 1}^2 = 0.8$ *for different values of* $\sigma_{\varepsilon 2}^2 \leq 0.8$.

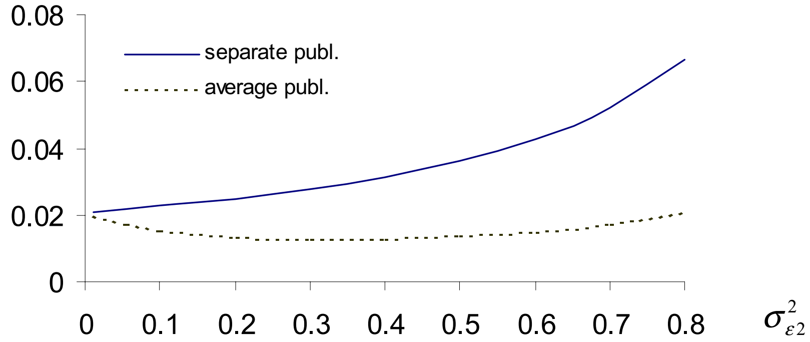
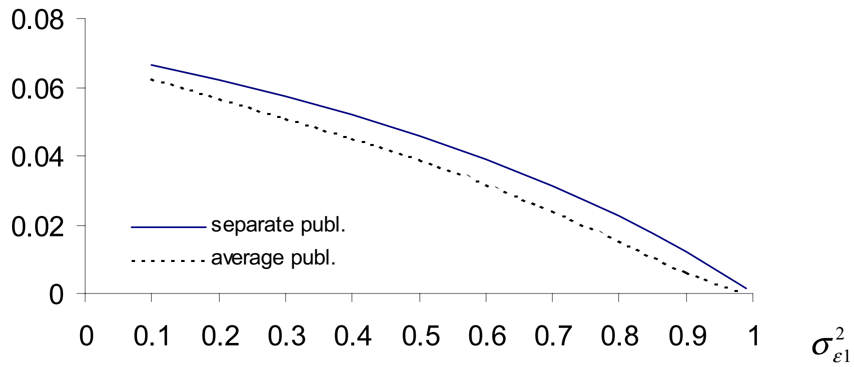


Chart 2: The function $\mathrm{var}(i_2 - i_0^*)$ *for* $\sigma_{\varepsilon 2}^2 = 0.1$ *for different values of* $\sigma_{\varepsilon 1}^2 \geq 0.1$.



The results suggested by the charts are summarised in the following observation.

> ***Observation 4:*** *The publication of an average of forecast autonomous factors unambiguously improves the steering of interest rates on the second day of the maintenance period, relative to the separate publication of autonomous factor forecasts for the two days.*

No formal proof is provided. However, beyond the numerical evidence presented above, the observation is rather intuitive as the uncertainty regarding the end of maintenance period liquidity situation is identical under both approaches on day 1 of the maintenance period, while the reduction of uncertainty implied by the publication of the value of autonomous factors on the first day is stronger under the separate publication (under a separate publication, at the start of the second market session, the banks know perfectly the forecast error of the central bank for the autonomous factors on day 1, while they only have a noisy estimate if an average forecast has been published). Hence, the innovation induced by news is stronger under a separate publication, and since the steering of rates was perfect on day 1, the steering on day 2 is better under an average publication. Since the publication of an average appears in any case to be simpler, one may conclude from this analysis that there are good reasons for a central bank to publish an average of forecast autonomous factor instead of separate forecasts for single days, as also decided by the ECB. One should note here the relation of this result with a result of a former literature represented for instance by Dotsey [1987]. This literature also modelled lack of information as an additional random variable which makes the signal extraction of market players less precise and hence tends to reduce the reaction of asset prices to the arrival of new information. Through this channel, additional uncertainty reduces also in these models the unconditional variance of asset prices (see also Tabellini, [1987, 426]).

Finally, chart 2 also suggests the following proposition, which is straightforward to prove in both cases analysed above (separate and average publication):

> ***Observation 5:*** *A perfect steering of overnight rates on both days of the maintenance period through the open market operation in the morning of day 1 is possible if and only if the forecast of autonomous factors for day 1 is perfect, i.e. if* $\sigma_{\varepsilon 1}^2 = 1$.

The intuition of this proposition is also evident: if forecasts of autonomous factors are perfect for day 1, then no news can emerge between the 2 market sessions, and the uncertainty regarding the end of the reserve maintenance period also remains unchanged throughout the maintenance period.[12]

4.3 Outline of a more general normative analysis

Observations 3 and 4 consisted in first normative propositions derived from the presented model. This section more generally outlines elements of a normative analysis. For that purpose, several costs have to be specified. First, to explain that the central banks do not opt systematically for the publication of autonomous factor forecasts, we have to assume that the publication of autonomous factors implies for the central bank a certain fixed cost which may be related to the required set up and security procedures, the risks of loosing reputation if the quality of the forecasts is regarded as poor by market participants, etc. Call these costs q. Secondly, we have to assume that the central bank assigns a

---

[12] The proof of the proposition is immediate by inserting $\sigma_{\varepsilon 1}^2 = 1$ into the formulas for i_1 and i_2.

certain welfare loss to the non-controllability of overnight rates. For this loss function, one may assume the very basic specification: $d = d_0(\mathrm{var}(i^* - i_1) + \mathrm{var}(i^* - i_2))$ [13]. Thirdly, improving the quality of liquidity forecasts is not free of costs either. We can assume functions $c_1(\sigma_{\varepsilon 1}^2)$, $c_2(\sigma_{\varepsilon 2}^2)$ such as for instance, for i=1,2, $c_i = c_{i,0} / (1 - \sigma_{\varepsilon i}^2) - c_{i,0}$ with, normally, $c_{1,0} < c_{2,0}$. Finally, the quality of liquidity forecasts may be regarded as a substitute for *fine-tuning* in achieving a certain quality of the steering of interest rates. Through fine-tuning on the second day, the central bank can always achieve i^* on that day, similarly to the first day where a perfect steering of rates could be achieved through the regular operation (independently of the quality of liquidity forecasts). One may assume that a fine tuning operation is regarded as creating a cost of w. One can then derive an optimal frequency of fine-tuning operations such as to equalise the marginal cost of fine tuning with the marginal benefit of it in terms of reducing the difference between actual and target overnight rates. Improving the quality of liquidity forecasts and conducting fine tuning operations are then substitutes for reducing the costs associated to imperfect steering of overnight rates.

The decisions of the central bank regarding its liquidity management strategy will depend on the parameters $q, w, c_{1,0}, c_{2,0}, d_0$. We do not further analyse here the mapping of these parameters into an optimal liquidity management strategy. Just note that the liquidity management strategy in our model may be characterised by the array $\{\rho, \sigma_{\varepsilon 1}^2 *, \sigma_{\varepsilon 1}^2 *, \eta_1^{\#}\}$ defining the parameters $\rho \in \{0,1\}$ as taking the value 1 if the central bank publishes its autonomous factor forecasts, otherwise taking the value zero; the parameter $\sigma_{\varepsilon i}^2 * \in [0,1]$ constitutes the optimal quality of liquidity forecasts for day i. Finally, the parameter $\eta_1^{\#} \in \Re_+$ constitutes the critical level of the absolute value of the non-anticipated autonomous factor change on day 1, beyond which a fine tuning operation is carried out in the morning of day 2. A normative theory of liquidity management in the basic framework presented here then consists in the specification of the mapping between the space of the environmental and preference parameters into the space of optimal liquidity management strategies.

5. A CENTRAL BANK WITH A STOCHASTIC END OF THE MAINTENANCE PERIOD LIQUIDITY TARGET

The previous section led for the case of a private autonomous factor forecast to the somewhat non-satisfying result that we cannot easily characterise the resulting equilibrium. To get nevertheless some

[13] Obviously, this loss function differs from the previously assumed one since it directly incorporates both days' interest rates. The previously adopted "sequential" approach allowed to derive the relevant results in a much simpler way.

feeling about the way the signal extraction by counterparties shapes in that case overnight rates, we take in this section a purely "quantitative" approach by assuming that the central bank does not have interest rate targets, but only liquidity targets. This allows solving the signal extraction problem in the non-trivial case of non-disclosed, but used autonomous factor forecasts.

Specifically, it is assumed that in deciding on the allotment volume in its open market operation, the central bank takes into account its autonomous factor forecasts and the liquidity surplus or deficit it would like to see at the end of the maintenance period. This liquidity target is denoted in the following by γ. The targeted γ may change from one maintenance period to the other. The liquidity target may for instance be derived from some pedagogical aim (e.g. provide more or less incentives to the market to participate in open market operations) or even to give a signal about e.g. possible future changes of standing facility rates. Assume that from the point of view of the market, γ also follows a white noise process, i.e. that it has an expected value of zero and a variance of $\sigma_\gamma^2 \in \Re_+$. Formally, the open market operation volume m is assumed to be chosen by the central bank as $m = \arg\min\{E(\gamma - (m-a))^2 \mid \varepsilon = \varepsilon_0\}$, i.e. m is chosen by the central bank, which knows ε, such that the expected squared difference between the end of period liquidity situation and the liquidity target is minimised. It is straightforward to show that this implies $m = \varepsilon + \gamma$. The following subsection will look at the case that the central bank does not publish its autonomous factor forecasts. The subsequent subsection will treat the case of published autonomous factors.

5.1 If the central bank makes use of autonomous factor forecasts, but does not publish them

The overnight inter-bank interest rate is determined in this case by the following equation:

$$i = P((m - a \mid m = m_0) < 0), \tag{20}$$

which is a variant (for the chosen values of the standing facility rates) of the well-known basic relationship between quantities and rates on the money market. In words: the interest rate equals the probability that there is an end of reserve maintenance period shortage of liquidity, knowing that $m = m_0$. Counterparties observe the allotment amount m and know the linear structure $m = \varepsilon + \gamma$. Applying the standard signal extraction formula, one obtains the following estimators for the unobserved variables, after observing $m = m_0$:

$$E(\gamma \mid m = m_0) = \frac{\sigma_\gamma^2}{\sigma_\gamma^2 + \sigma_\varepsilon^2} m_0, \quad E(\varepsilon \mid m = m_0) = \frac{\sigma_\varepsilon^2}{\sigma_\gamma^2 + \sigma_\varepsilon^2} m_0 \tag{21}$$

The variances of the errors of the estimates will be:

$$E(\gamma - E(\gamma \mid m = m_0))^2 = \sigma_\gamma^2 - \frac{\sigma_\gamma^4}{\sigma_\gamma^2 + \sigma_\varepsilon^2}, \quad E(\varepsilon - E(\varepsilon \mid m = m_0))^2 = \sigma_\varepsilon^2 - \frac{\sigma_\varepsilon^4}{\sigma_\gamma^2 + \sigma_\varepsilon^2} \tag{22}$$

The overnight rate in the inter-bank market will amount to:

$$i = P(m-a<0 \mid m=m_0) = \int_{-\infty}^{0} f_{(m-a|m=m_0)}(x)dx = F_{(m-a|m=m_0)}(0) \tag{23}$$

To specify $f_{(m-a|m=m_0)}()$, we note that the expected value of the underlying random variable is: $E(m-a \mid m=m_0) = E(\varepsilon+\gamma-\varepsilon-\eta \mid m=m_0) = E(\gamma \mid m=m_0)$. Its variance is $Var(m-a \mid m=m_0) = Var(\gamma-\eta \mid m=m_0) = Var(\gamma \mid m=m_0) + Var(\eta \mid m=m_0)$. Hence, $(m-a \mid m=m_0)$ is normally distributed with an expected value of $\frac{\sigma_\gamma^2}{\sigma_\gamma^2+\sigma_\varepsilon^2}m_0$ and a variance of

$\sigma_\gamma^2 - \frac{\sigma_\gamma^4}{\sigma_\gamma^2+\sigma_\varepsilon^2} + 1 - \sigma_\varepsilon^2 = \frac{\sigma_\varepsilon^2\sigma_\gamma^2}{\sigma_\gamma^2+\sigma_\varepsilon^2} + 1 - \sigma_\varepsilon^2 = 1 - \frac{\sigma_\varepsilon^4}{\sigma_\gamma^2+\sigma_\varepsilon^2}$.

Therefore, using the Gaussian (standard normal) cumulative density function, we can write:

$$i = \Phi\left(\frac{-E(m-a \mid m=m_0)}{\sqrt{\operatorname{var}(m-a \mid m=m_0)}}\right) = 1-\Phi\left(\frac{\frac{\sigma_\gamma^2}{\sigma_\gamma^2+\sigma_\varepsilon^2}m_0}{\sqrt{1-\frac{\sigma_\varepsilon^4}{\sigma_\gamma^2+\sigma_\varepsilon^2}}}\right) = 1-\Phi(Z) \tag{24}$$

The random variable Z has an expected value of zero and a variance of $\operatorname{var}(Z) = \left(\frac{\sigma_\gamma^2}{\sigma_\gamma^2+\sigma_\varepsilon^2} \Big/ \sqrt{1-\frac{\sigma_\varepsilon^4}{\sigma_\gamma^2+\sigma_\varepsilon^2}}\right)^2 \left(\sigma_\gamma^2+\sigma_\varepsilon^2\right) = \frac{\sigma_\gamma^4}{\sigma_\gamma^2+\sigma_\varepsilon^2-\sigma_\varepsilon^4}$. Denote by $\operatorname{var}(i)_{\sigma_\varepsilon^2,\sigma_\gamma^2}^{non-publicAF}$ the variance of the overnight rate if autonomous factors are not published for the given variance of the underlying random variables ε and γ.

As indicated by the mean squared errors of the estimated values of the liquidity target and the autonomous factor forecast, the signal extraction will not be perfect and the relationship between the liquidity target and the resulting interest rate will hence be noisy. Assume that the central bank would like to achieve a certain maximum variance of the interest rate, which may be motivated as follows: ranking behind its liquidity target, the central bank may have a secondary overnight interest rate target being always in the middle of the corridor set by standing facility rates.[14] Then, the central bank may calculate how it has to adjust σ_γ^2 in order to keep the variance of overnight interest rates below a

[14] For instance, the central bank may argue that moderate and transitory fluctuations of the overnight rate within the corridor are not relevant for the longer term rates, which are deemed to play a role in the transmission of monetary policy.

certain maximum if other parameters, such as the quality of liquidity forecasts, or the publication policy, change.

For the related following analysis, we will need the following basic Lemma.

Lemma 1*: If* $x \approx N(0,\sigma_x^2)$*,* $y \approx N(0,\sigma_y^2)$ *with* $\sigma_y^2 > \sigma_x^2$ *and g( ) a monotonous decreasing function which is point-symmetric in* $(0, g(0))$ *in the sense that* $\forall x_1 \in \Re : g(0) - g(x_1) = -g(0) + g(-x_1)$ *, then* $\operatorname{var}(g(y)) > \operatorname{var}(g(x))$.

The proof of Lemma 1 is provided in annex 2. Now, a series of interesting propositions can be derived. The following observation 6 shows that the appearing basic intuition that good autonomous factor forecasts should, ceteris paribus, always lead to a more precise steering of interest rates, is mistaken.

Observation 6*: In the case of private autonomous factor forecasts, For given* σ_γ^2*, the variance of overnight rates,* $\operatorname{var}(i)_{\sigma_\varepsilon^2,\sigma_\gamma^2}^{non-publicAF}$ *, is U-shaped in the quality of autonomous factor forecasts. When the quality of autonomous factor forecasts,* σ_ε^2*, increases, starting from zero, the volatility of the overnight rate decreases until* $\sigma_\varepsilon^2 = 0.5$ *and then increases again:*

$$\forall \sigma_\varepsilon^2 \in \left[0,\frac{1}{2}\right] : \frac{\partial \operatorname{var}(i)_{\sigma_\varepsilon^2,\sigma_\gamma^2}^{non-publicAF}}{\partial \sigma_\varepsilon^2} \leq 0, \forall \sigma_\varepsilon^2 \in \left]\frac{1}{2},1\right] : \frac{\partial \operatorname{var}(i)_{\sigma_\varepsilon^2,\sigma_\gamma^2}^{non-publicAF}}{\partial \sigma_\varepsilon^2} > 0 \tag{25}$$

The proof of observation 6 is provided in the annex 2.

Observation 6 however does not necessarily imply that the central bank should not try to have the best possible autonomous factor forecasts, since indeed, the ceteris paribus condition does not have to be applicable. Specifically, we now look at how σ_γ^2 has to be adjusted if the quality of liquidity forecasts changes. We assume that it is the intention to keep the variance of i constant at $\operatorname{var}^*(i)$, which implies the same for Z, i.e. $\frac{\sigma_\gamma^4}{\sigma_\gamma^2 + \sigma_\varepsilon^2 - \sigma_\varepsilon^4} = \operatorname{var}^*(i)$. The positive root of the obtained quadratic equation is: $\sigma_{\gamma,0}^2(\operatorname{var}^*(i),\sigma_\varepsilon^2) = \frac{-\operatorname{var}^*(i) + \sqrt{(\operatorname{var}^*(i))^2 + 4\operatorname{var}^*(i)(\sigma_\varepsilon^2 - \sigma_\varepsilon^4)}}{2}$. This result is summarised in the following observation:

Observation 7: *In the case of a one day maintenance period and non-published autonomous factors, the central bank has to adjust its variance of liquidity targets in a tent-shaped way to changes of the quality of liquidity forecasts, in order to maintain a certain variance of interest rates*.

The proof is obvious from the functional form of the positive root of the obtained quadratic equation.

5.2 If the central bank does publish its autonomous factor forecasts

Publishing forecasts of autonomous factors is equivalent to reducing the uncertainty with regard to γ to zero and the residual uncertainty with regard to the end of maintenance period liquidity position to $\sigma_\eta^2 = 1 - \sigma_\varepsilon^2$. The expected end of maintenance period liquidity position is γ, which can be extracted perfectly. Hence, interest rates will be determined by the following relationship:

$$i = P(m - a < 0 \mid m = m_0, \varepsilon = \varepsilon_0) = \int_{-\infty}^{0} f_{(m-a|m=m_0,\varepsilon=\varepsilon_0)}(x)dx = F_{(m-a|m=m_0,\varepsilon=\varepsilon_0)}(0) \qquad (26)$$

Where $f_{(m-a|m=m_0,\varepsilon=\varepsilon_0)}()$ is the density function of a normally distributed random variable with expected value γ and variance $1-\sigma_\varepsilon^2$. Hence:

$$i = \Phi\left(\frac{-\gamma}{\sqrt{1-\sigma_\varepsilon^2}}\right) = 1 - \Phi\left(\frac{\gamma}{\sqrt{1-\sigma_\varepsilon^2}}\right) = 1 - \Phi(\widetilde{Z}) \qquad (27)$$

with $\mathrm{var}(\widetilde{Z}) = \dfrac{\sigma_\gamma^2}{(1-\sigma_\varepsilon^2)}$. Denote the variance of this interest rate for given variances of the underlying variables as $\mathrm{var}(i)_{\sigma_\varepsilon^2,\sigma_\gamma^2}^{publicAF}$. Now, a series of 4 observations can be derived which characterise the relation between the variance of interest rates and the variances of the underlying random variables in the case of published autonomous factors, as well as the relationship to the case of private autonomous factors. Observation 8, which is somewhat related to observation 6, contradicts the possible intuition that publishing autonomous factor forecasts ceteris paribus necessarily reduces the volatility of overnight rates.

Observation 8: *For a given variance of liquidity targets,* σ_γ^2*, the volatility of overnight rates is at least as high under public autonomous factor as under non-public autonomous factor forecasts:*

$$\forall \sigma_\varepsilon^2 \in [0,1], \forall \sigma_\gamma^2 \in \Re_+ : \quad \mathrm{var}(i)_{\sigma_\varepsilon^2,\sigma_\gamma^2}^{publicAF} \geq \mathrm{var}(i)_{\sigma_\varepsilon^2,\sigma_\gamma^2}^{non-publicAF}$$

The proof is provided in annex 2. The following observation continues in the same line by suggesting that ceteris paribus, better autonomous factor forecasts also mean more volatile overnight rates in the case of public autonomous factor forecasts.

Observation 9: *With public autonomous factor forecasts, for a given variance of end of maintenance period targets,* σ_γ^2*, the volatility of interest rates increases monotonously with an increasing quality of liquidity forecasts:*

$$\forall \sigma_\varepsilon^2 \in [0,1], \forall \sigma_\gamma^2 \in \Re_+ : \quad \frac{\partial \operatorname{var}(i)_{\sigma_\varepsilon^2,\sigma_\gamma^2}^{publicAF}}{\partial \sigma_\varepsilon^2} > 0 \tag{28}$$

The proof is obvious: the variance of $\widetilde{Z}$ increases monotonously in σ_ε^2 . Therefore, we know with the help of lemma 1 that the volatility of interest rates $\operatorname{var}(i)_{\sigma_\varepsilon^2,\sigma_\gamma^2}^{publicAF}$ also increases monotonously in σ_ε^2.

Now, we look again at how σ_γ^2 has to be adjusted if the quality of liquidity forecasts changes.

Observation 10: *In the case of a one day maintenance period and non-published autonomous factors, the central bank has to adjust its variance of liquidity targets to changes of the quality of liquidity forecasts as follows:*

$$\sigma_\gamma^2 = \operatorname{var}^*(i)(1-\sigma_\varepsilon^2) \qquad (24)$$

in order to maintain the variance of interest rates at a given level var(i).*

Above, it has been suggested that the switch to a publication of forecasts of autonomous factors increases ceteris paribus the volatility of interest rates. However, the central bank may complement the switch to a publication of autonomous factor forecasts by a lowering of the volatility of autonomous factor forecast. The following proposition reflects this option:

Observation 11. *The effect of the decision to publish autonomous factors on the volatility of overnight rates can always be neutralised by an adequate reduction of the volatility of targets with regard to the end of maintenance period liquidity imbalance.*

$$\forall \sigma_\varepsilon^2, \in [0,1], \forall \sigma_\gamma^2 \in \Re_+, \quad \exists \widetilde{\sigma}_\gamma^2 \in \Re_+ : \operatorname{var}(i)_{\sigma_\varepsilon^2,\widetilde{\sigma}_\gamma^2}^{publicAF} = \operatorname{var}(i)_{\sigma_\varepsilon^2,\sigma_\gamma^2}^{non-publicAF} \tag{29}$$

The proof is provided in annex 2. Of course, the effect of a publication is the stronger, the better the quality of liquidity forecasts. If the quality is zero, the variance of targets does not have to be changed (it stays at 0.5). In the case of quasi-perfect forecasts, the variance of targets should approach zero.

The conclusions from this section may be summarised as follows: the central bank should reduce the variance of its liquidity targets σ_γ^2 if it improves the quality of its liquidity forecasts. It should also do so if it switches from private to public autonomous factors. Since reducing σ_γ^2 is equivalent to reducing $\mathrm{var}(m-a)$, the conclusions reached in the previous section in a framework of targeting interest rates, namely that publishing autonomous factors and improving the quality of liquidity forecasts is useful, are confirmed. In particular, this section allowed representing the relationships between the variables in question also for the case of non-public and used autonomous factor forecasts.

6. CONCLUSIONS

A simple model of the interaction between central bank liquidity management and the setting of the overnight rate in the money market was presented which allowed addressing analytically the recently decided publication of estimates of autonomous liquidity factors by the European Central Bank. It was shown in the context of the model, that the main practical advantage of the publication of forecasts of autonomous factors is that it makes the signal extraction problem with regard to the central bank's intentions trivial and hence allows establishing a transparent behavioural equilibrium between the central bank and the money market. In this equilibrium, the central bank can perfectly steer overnight rates at least on the day of the open market operation, with the smallest possible variance of imbalances of liquidity at the end of the maintenance period. The paper distinguished between three variants of the main model. Sections 3 and 4 considered the case of interest rate targeting, for the one and two days reserve maintenance period case, respectively. Section 5 restated the one day case in pure quantitative terms allowing to solve analytically the case of private and used autonomous factor forecasts. The analysis of each of the three cases (interest rate target for one and two days maintenance periods, quantitative target) allowed representing in the simplest way different aspects of the signal extraction problem banks face in the money market. The two days model also allowed to analyse whether a central bank should publish separately autonomous factor forecasts for single days of the reserve maintenance period, or whether it should publish only a forecast average, such as done by the ECB since July 2000. The model suggests unambiguously that the approach adopted by the ECB allows for a better steering of interest rates on the second day.

Of course, all assumptions of the models constitute strong simplifications of reality. The maintenance period is much longer in the case of the ECB than one or two days. There are several operations in the maintenance period. Autonomous factors are not revealed suddenly at the end of the day, but more smoothly in the course of the day; furthermore, they may exhibit auto-correlation. Averaging capacities are not unlimited at the level of individual banks, such that a preference for back-loading of reserve requirements may emerge (as described by Peres Quiros and Mendizabal [2000]). Expectations of banks are not always rational and their behaviour may be more accurately modelled through some "bounded rationality" model. Finally, the inter-bank market is not a perfect market and friction may imply that the volatility of rates is determined by other, temporarily even dominant factors. Nevertheless, the models allow representing the main elements, which determine the relationship between liquidity management, information policy, and overnight rates in a full rationality, zero transaction cost setting, which should be the starting point for further research. Finally, the analysis contributed a first step to base two inevitable policy decisions of any central bank, namely whether or not to publish liquidity forecasts, and how much resources to invest into the quality of liquidity forecasts, on analytical grounds.

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Annex 1:Excerpt of the press release of the ECB of 16 June 2000, "Switch to variable rate tenders in main refinancing operations: some technical details"

At its meeting on 8 June 2000 the Governing Council of the ECB decided that, starting from the operation to be settled on 28 June 2000, the main refinancing operations of the Eurosystem will be conducted as variable rate tenders, using the multiple rate auction procedure. […]

The tender announcement will include, in addition to the standard information, […] an indication of the expected liquidity needs of the banking system.

This indication refers to an average for the period from the day of announcement until (and including) the day before the settlement of the following main refinancing operation. If this time interval goes beyond the end of a reserve maintenance period, an estimate of the average liquidity needs until the end of the reserve maintenance period will also be provided. An estimate of the liquidity needs of the banking system is necessarily surrounded by a significant degree of uncertainty. The ECB provides its estimates to the best of its knowledge at the time of publication, drawing from the information provided by national central banks. It should also be stressed that the ECB bases its allotment decisions on a number of factors, including, but not limited to, the expected liquidity needs of the banking system.

The liquidity needs are defined as the average, over the relevant period, of the daily sum of reserve requirements and of all factors other than monetary policy operations of the Eurosystem which affect the banking system's liquidity (the so-called autonomous factors, e.g. banknotes and government deposits with the Eurosystem; see the box in each issue of the ECB Monthly Bulletin entitled "Monetary policy operations and liquidity conditions in the reserve maintenance period ending on ...", for instance pages 18 to 19 in the June 2000 issue). The ECB pages providing daily information on liquidity conditions will display ex post data on liquidity factors other than monetary policy operations, to allow counterparties easily to assess the deviation of actual figures from the published estimates. […]

Annex 2: Proofs of observations

Proof of Observation 1

First, we show that there exists a strategy of the type $m_0 = \varepsilon_0 + \gamma(i^*)$ that allows for a perfect steering of interest rates. Note that $F_{(m-a|m=m_0,\varepsilon=\varepsilon_0)}$ is the cumulative distribution function of an $N(m_0 - \varepsilon_0, 1-\sigma_\varepsilon^2)$ distributed random variable since $E(m-a \mid m=m_0, \varepsilon=\varepsilon_0) = E(m_0 - (\varepsilon_0 + \eta)) = E(m_0 - \varepsilon_0))$ and $Var(m-a \mid m=m_0, \varepsilon=\varepsilon_0) = Var(\eta) = 1-\sigma_\varepsilon^2$. Hence, we can write:

$$i = 1 - \Phi\left(\frac{m_0 - \varepsilon_0}{\sqrt{1-\sigma_\varepsilon^2}}\right) = 1 - \Phi\left(\frac{\gamma}{\sqrt{1-\sigma_\varepsilon^2}}\right) \tag{30}$$

This relationship can be used by the central bank to steer interest rates. The central bank has simply to choose an allotment volume corresponding to the sum of the expected autonomous factor and the i* quantile of a normal distribution with variance $1-\sigma_\varepsilon^2$:

$$\gamma(i^*) = \sqrt{1-\sigma_\varepsilon^2}\,\Phi^{-1}(1-i^*) \tag{31}$$

If the central bank adopts this allotment policy, the allotted amounts do not reveal anything that would not already been known to counterparties, such that the assumptions made above regarding the signal extraction remain valid. Hence, we obtain a behavioural equilibrium between the market and the central bank in which the central bank can perfectly steer overnight rates in this equilibrium.

Secondly, we show that the set of alternative allotment strategies allowing for a perfect steering of interest rates is limited to the one in which the central bank deviates from the one proposed above by reducing the quality of its liquidity forecasts. Define as the set of possible allotment strategies the set of functions $m_0 = m_0(\varepsilon_0, i^*, \omega)$, whereby ω represents all the other variables that the central bank may define as being relevant for its allotment decisions. Obviously, any allotment rule, which would allow other variables to be relevant would by definition not allow for a perfect steering of interest rates. We can hence restrict the set of strategies conducive at a perfect steering of interest rates to the set of strategies defined by $m_0 = m_0(\varepsilon_0, i^*)$. This set of strategies may alternatively be represented by the set $m_0 = m'_0(\varepsilon_0, i^*) + \varepsilon_0$. Substituting this into the interest rate determining formula above, it appears that the autonomous factor forecast ε_0 in $m'_0(\varepsilon_0, i^*)$ will introduce the same noise into the interest rate as any other further random variable, since the actual effect of the autonomous factor forecast has already been included in the linear component of the allotment function outside $m'_0(\varepsilon_0, i^*)$. Therefore, a perfect steering of interest rates will only be possible for functions of the form $m_0 = m'_0(i^*) + \varepsilon_0$, whereby the exact form of this function has been derived above.

Finally, it has to be shown that this allotment strategy minimises the expected squared variance of the recourse to standing facility at the end of the reserve maintenance period if the central bank makes full use of its autonomous factor forecasts. The expected squared recourse to standing facilities expected before the interest rate target becomes known is $Var(m-a) = Var(\varepsilon_0 + \gamma(i^*) - \varepsilon_0 - \eta) = Var(\gamma(i_0^*)) + Var(\eta)$. Both terms increase if the central bank does not make full use of its autonomous factor forecast. ■

Proof of observation 3

The proof proceeds in two parts.

First, it is shown that for any given forecast quality for the autonomous factors on day 1, improving the forecast quality for day 2 is counterproductive as it increases volatility of interest rates on day 2. Consider the formula of $\mathrm{var}(i)$. The quality of autonomous factor forecasts for day 2, $\sigma_{\varepsilon 2}^2$, is only present in the term $\left(1 - \Phi\left(\frac{(\gamma(i_0^*) + \eta_1)}{\sqrt{1-\sigma_{\varepsilon 2}^2}}\right) - i_0^*\right)$, which obviously, $\forall i_0^*, \forall \eta_1$, increases monotonously with $\sigma_{\varepsilon 2}^2$. This property is preserved if the term is squared and if the weighted integral over all values of η_1 are taken, which then yields the formula for the variance.

Secondly, it has to be shown that for any given forecast quality for the autonomous factors on day 2, improving the forecast quality for day 1 reduces the volatility of interest rates on day 2. This is immediately obvious from the formula for the variance of interest rates on day 2. ■

Proof of Lemma 1

Assume $y = \psi x$ with ψ>1 such that E(y)=E(x)=0 and $Var(y) = \psi^2 Var(x)$. We have to show that for a function g( ) with the properties indicated in lemma 1, $\mathrm{var}(g(y)) > \mathrm{var}(g(x))$.

First, we note that the E(g(x))=g(E(x))=g(0). This is the case since the function g( ) maintains by definition the symmetry of the probability distribution around g(E(x))=g(0): $\forall x \in \Re : g(0) - g(x) = -g(0) + g(-x) \Leftrightarrow g(0) = (g(x) + g(-x))/2$. Of course, we also have E(g(y))=g(E(y))=g(0) and hence E(g(x))=E(g(y)). Secondly, we calculate Var(g(y)):

$$= \int_{\Re}((g(y) - E(g(y)))^2 f_y(y)dy = \int_{\Re}((g(\psi x) - E(g(0)))^2 \frac{f_x(x)}{\psi} d(\psi x) = \int_{\Re}((g(\psi x) - E(g(0)))^2 f_x(x)dx$$

(This is the case since: $f_y(y) = \frac{1}{\sigma_y \sqrt{2\pi}} e^{\frac{(y)^2}{2\sigma_y^2}} = \frac{1}{\psi\sigma_x \sqrt{2\pi}} e^{\frac{\psi^2 (x)^2}{2\psi^2\sigma_x^2}} = \frac{f_x(x)}{\psi}$). Since g( ) is monotonously decreasing, we know that $\forall y \in \Re: \quad |y| = |\psi x| > |x| \Rightarrow |g(y) - g(0)| > |g(x) - g(0)|$. This implies that: $\forall x : ((g(\psi x) - E(g(0)))^2 > ((g(x) - E(g(0)))^2$. Hence:

$$\int_{\Re}((g(\psi x) - E(g(0)))^2 f_x(x)dx > \int_{\Re}((g(x) - E(g(0)))^2 f_x(x)dx \Leftrightarrow \mathrm{var}(g(y)) > \mathrm{var}(g(x)). \blacksquare$$

Proof of observation 6

Since $1 - \Phi()$ is a monotonously decreasing function with the properties required in lemma 1 and since Z is normally distributed with expected value of zero, to prove now observation 6, it is sufficient to show that the variance of the expression in the Gaussian cumulative distribution, i..e $\mathrm{var}(Z) = \frac{\sigma_\gamma^4}{\sigma_\gamma^2 + \sigma_\varepsilon^2 - \sigma_\varepsilon^4}$ has the property to decrease with σ_ε^2 until $\sigma_\varepsilon^2 = 0.5$ and to increase afterwards. This is straightforward by calculating the first derivative of $\mathrm{var}(Z)$: $\frac{\partial\, \mathrm{var}(Z)}{\partial \sigma_\varepsilon^2} = -\sigma_\gamma^4 \frac{1 - 2\sigma_\varepsilon^2}{(\sigma_\gamma^2 + \sigma_\varepsilon^2 - \sigma_\varepsilon^4)^2}$. This expression is negative for $\sigma_\varepsilon^2 < 0.5$, zero for $\sigma_\varepsilon^2 = 0.5$ and positive for $\sigma_\varepsilon^2 > 0.5$. $\blacksquare$

Proof of observation 8

It has to be shown that the variance of $\widetilde{Z}$ is at least as high as the one of Z, for every possible pair of values ($\sigma_\varepsilon^2, \sigma_\gamma^2$):

$$\begin{gathered} \mathrm{var}(Z) > \mathrm{var}(\widetilde{Z}) \Leftrightarrow \frac{\sigma_\gamma^2}{(1 - \sigma_\varepsilon^2)} > \frac{\sigma_\gamma^4}{\sigma_\gamma^2 + \sigma_\varepsilon^2 - \sigma_\varepsilon^4} \\ \Leftrightarrow (\sigma_\gamma^2 + \sigma_\varepsilon^2 - \sigma_\varepsilon^4) > \sigma_\gamma^2 (1 - \sigma_\varepsilon^2) \Leftrightarrow 1 - \sigma_\varepsilon^2 > -\sigma_\gamma^2 \end{gathered} \tag{32}$$

This is indeed always verified. Taking lemma 1, the observation is proven. $\blacksquare$

Proof of observation 11

The equation $\mathrm{var}(i)_{\tilde{\sigma}_\gamma^2}^{publicAF} = \mathrm{var}(i)_{\sigma_\gamma^2}^{non-publicAF}$ is equivalent to:

$$\frac{\tilde{\sigma}_\gamma^2}{(1-\sigma_\varepsilon^2)} = \frac{\sigma_\gamma^4}{\sigma_\gamma^2 + \sigma_\varepsilon^2 - \sigma_\varepsilon^4} \Leftrightarrow \tilde{\sigma}_\gamma^2(\sigma_\gamma^2, \sigma_\varepsilon^2) = \frac{\sigma_\gamma^4(1-\sigma_\varepsilon^2)}{\sigma_\gamma^2 + \sigma_\varepsilon^2 - \sigma_\varepsilon^4} \tag{33}$$

This can indeed always be achieved through choosing an appropriate $\tilde{\sigma}_\gamma^2 \in \Re_+$.■

The solution to the linear signal extraction problem with m observed and n unobserved variables[15]

The subjects in question are aware of all coefficients (the matrix Λ) of the structure:

$$\begin{pmatrix} z_1 \\ z_2 \\ \dots \\ z_m \end{pmatrix} = \begin{pmatrix} \lambda_{1,1} & \lambda_{1,2} & \dots & \lambda_{1,n} \\ \lambda_{2,1} & \lambda_{1,1} & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \lambda_{m,1} & \dots & \dots & \lambda_{m,n} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{pmatrix} \equiv \quad z = \Lambda.x \tag{34}$$

as well as of the variance-covariance matrix of *x, E(xx')*, whereby *z* is the vector of observed variables, *x* is the vector of unobserved variables (to be extracted), and *A* is the matrix of the linear coefficients describing the relationship between the unobserved and the observed variables. The signal extraction problem consists in finding the vector $\beta_k \in R^m$ for which $\beta_k = \arg\min(E(\hat{x}_k - x_k)^2)$ with $\hat{x}_k = \beta_k' z$. By substituting, we obtain:

$$\begin{aligned} &E(\hat{x}_k - x_k)^2 = E(\beta_k'\Lambda x - x_k)^2 = E((\beta_k'\Lambda x)^2 - 2\beta_k'\Lambda x x_k + x_k^2) \\ &= E(\beta_k'\Lambda x(\Lambda x)'\beta_k - 2\beta_k'\Lambda x x_k + x_k^2) \end{aligned} \tag{35}$$

The first derivative of this term has to be set to zero to obtain the necessary (and in this case sufficient) condition for a minimum. Following Luetkepohl [1991, 470], proposition 2, the derivative of the first term in the brackets is:

$$\frac{\partial \beta_k'\Lambda x(\Lambda x)'\beta_k}{\partial \beta_k} = (\Lambda x(Ax)' + (\Lambda x(Ax)')')\beta_k = 2\Lambda x x'\Lambda\beta_k \tag{36}$$

Following Luetkepohl [1991, 470], proposition 1, the derivative of the second term in (A.2) is:

$$\frac{\partial 2\beta_k'\Lambda x x_k}{\partial \beta_k} = 2(\Lambda x x_k) \tag{37}$$

The first derivative of the third term in (A.2) is zero. The entire derivative of expression ( ) set equal to zero hence yields the following equation:

$$2AE(xx')\Lambda\beta_k - 2\Lambda E(x x_k) = 0 \tag{38}$$

If multiplied by the left hand side by $(2\Lambda E(xx')\Lambda)^{-1}$, we obtain the signal extraction vector minimising the mean squared error to estimate x_k:

$$\beta_k = (\Lambda E(xx')\Lambda')^{-1}\Lambda E(x x_k) \tag{39}$$

[15] As in Bindseil [2000]

Defining $B = (\beta_1 \quad \beta_2 \ldots \beta_k \ldots \beta_n) \in R^{m \times n}$, one can summarise the solution to the signal extraction problem for all unobserved variables as:

$$B = (\Lambda E(xx')\Lambda')^{-1} \Lambda E(xx) \tag{40}$$

The expected squared error of the estimates of unobserved variables can be obtained by calculating:

$$E(\hat{x} - x)(\hat{x} - x)' = E(B'\Lambda x - x\ )^2 = E(B'\Lambda xx'A'B - 2B'\Lambda xx' + xx') \tag{41}$$

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