

# **Working Paper Series**

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olt Rational inattention during an RCT



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#### Abstract

We introduce an information provision experiment into a standard dynamic rational inattention model. We derive analytical results about how the treatment effect varies with characteristics of the environment and the individual. We use these results to discuss findings in the empirical literature on information provision experiments that can be explained by rational inattention of survey respondents and what this interpretation implies about behavior outside the survey.

**Keywords:** rational inattention, information provision experiment, randomized control trial (JEL: D8, D9, E7).

### Non-technical summary

In surveys with information provision experiments, one can observe how people change their beliefs, and sometimes also actions, after having been confronted with information. How can we explain findings from information provision experiments? What do these findings tell us about economic behavior outside the survey? What are the implications for communication policy?

As a step toward answering these questions, this paper studies what the theory of rational inattention predicts about information provision experiments. "Rational inattention" is the idea that information is in principle available, but absorbing information requires paying attention which is costly. We introduce an information provision experiment into a standard dynamic rational inattention model. We derive analytical results about how the treatment effect – how much people respond to information presented during a survey – depends on the characteristics of the environment and the individual.

We prove that the treatment effect during an information provision experiment is strictly *decreasing* in the importance of being informed, so long as agents pay positive attention outside and during the survey. When being informed becomes more important, two counteracting effects arise: people want to pay more attention in daily life, which lowers the treatment effect; and people want to pay more attention during the survey, which raises the treatment effect. The attention-in-daily-life effect dominates.

Next, we consider the case when agents pay attention during the survey but not in daily life (because the cost of paying attention in daily life is much higher than during a survey, or the relevant information is simply unavailable outside the survey). In this case, the treatment effect is strictly *increasing* in the importance of being informed. The reason is that the first of the two aforementioned effects, the attention-in-daily-life effect, is absent. Hence, the nature of the information – whether the information was available to respondents before the survey or not – affects the interpretation of the findings.

We use our results to interpret findings from information provision experiments in the literature. As an example, the literature finds that the treatment effect is smaller when inflation is high. Our interpretation of this finding is that the importance of being informed about inflation and/or the size of inflationary shocks increase with inflation, and such increases reduce the treatment effect in the model. The literature also finds that individuals who are less informed about monetary policy revise beliefs by more after an information treatment. Our model-based interpretation is that these are individuals for whom monetary policy is relatively unimportant in daily life.

Our analysis suggests that communication policy may have less effect among individuals who care more about being informed and at times when being informed is more important, so long as communication involves merely restating information that has already been publicly available. If communication involves expanding the set of publicly available information, then it may be more effective among individuals who care more about being informed and at times when being informed is more important.

### 1 Introduction

Surveys with information provision experiments, a form of randomized control trials (RCTs), have become popular in economics. Naturally, economists are interested in comparing the information treatment effects, on beliefs and on actions, measured in this literature with theories of behavior under incomplete information or bounded rationality, including rational inattention (RI) introduced by Sims (2003). This comparison is complicated because RI implies optimal attention choice *before* and *during* a survey. The goal of this paper is to make this comparison easier.<sup>1</sup>

As a motivation, consider a basic research question in information provision experiments: Does one expect a larger or a smaller treatment effect when respondents care more about being informed? It is intuitive that when being informed is more important, people pay more attention in their daily life, and consequently they enter a survey with a sharper prior, which reduces the treatment effect. But it is likewise intuitive that when being informed is more important, respondents have more incentive to absorb information presented to them during the survey, which tends to increase the treatment effect.

This paper addresses three questions: What does the standard RI model predict about the treatment effect in an information provision experiment? Are these predictions consistent with findings in the empirical literature on information provision experiments? Given the theory, what can one infer from behavior during the survey about behavior outside the survey?

To address these questions, we introduce an information provision experiment into an otherwise standard dynamic RI model. Individuals track a state variable that follows a first-order autoregressive process. They decide optimally how much attention to devote to the state in every period. At some point, a subset of the population is unexpectedly called into a survey and is provided with information about the current value of the state. The marginal cost of attention is assumed to be lower during the survey than outside the survey. We derive analytical results about the treatment effect as a function of the parameters of the model: the importance of being informed, the marginal cost of attention outside the survey, the marginal cost of attention during the survey, and the variance of the innovation in the state.

We prove that the treatment effect during the information provision experiment is strictly  $$^{1}$See Fuster and Zafar (2023) and Haaland et al. (2023) for reviews of the literature on information provision experiments and Maćkowiak et al. (2023) for a review of the literature on rational inattention.$ 

*decreasing* in the importance of being informed about the state, so long as the state is correlated over time and agents pay positive attention to the state outside and during the survey. Thus, a strong response to information during an information provision experiment is a sign of a low importance of being informed about the state. The first of the two counteracting forces from the motivating example (attention outside the survey) turns out to dominate the second (attention during the survey).

This comparative static result extends to other parameters. Under the same conditions, the treatment effect is strictly decreasing in the variance of the innovation in the state. Again, the attention-outside-the-survey effect dominates the attention-during-the-survey effect.

We also characterize how the treatment effect depends on the parameters in the case of corner solutions of no attention. When agents do not pay attention to the state outside the survey but pay attention to the state during the survey (because the cost of paying attention to the state in daily life is much higher than during a survey, or the relevant information is simply unavailable outside the survey), then the treatment effect is strictly *increasing* in the importance of being informed. The reason is that the attention-outside-the-survey effect is absent. Hence, the nature of the provided information—whether the information was already available to respondents before the experiment or not—affects whether the treatment effect is decreasing or increasing in the importance of being informed.

The paper's main results are stated as three propositions. Given these propositions, one can begin using the standard RI model to interpret information provision experiments. We give a few examples in the paper. Let us focus here on the benchmark case with an interior solution. According to the model, two individuals who pay attention to the state outside and during the survey but show different treatment effects have a different importance of being informed or a different marginal cost of attention. The individual with the *lower* treatment effect is predicted to pay more attention to the state in daily life but is also predicted to react less strongly to an unexpected, temporary reduction in the marginal cost of attention. By contrast, if the information was unavailable before the survey, then the individual with the *higher* treatment effect is predicted to pay more attention to the state outside the survey if information about the state were to become available outside the survey.

Section 2 presents the model. The main results are in Section 3. Section 4 uses the model to

interpret information provision experiments from the literature (therefore we postpone a discussion of the literature until that section). Section 5 concludes. We include an Appendix with proofs and an Online Appendix with additional results.

## 2 Model: RI before, during, and after the survey

We introduce an information provision experiment into an otherwise standard dynamic rational inattention model.<sup>2</sup>

The state follows a Gaussian Markov process:

$$\pi_t = \rho \pi_{t-1} + \varepsilon_t \tag{1}$$

where  $\rho \in [0,1)$  and  $\varepsilon_t \sim i.i.d.N(0, \sigma_{\varepsilon}^2)$ . We denote the state by  $\pi_t$  because in some information provision experiments respondents are asked about their beliefs about inflation, but of course one could have used any other letter to denote the state.

There are three stages:

- stage 1: before the survey (periods t = 0, -1, -2, ...);
- stage 2: during the survey (end of period t = 0);
- stage 3: after the survey (periods t = 1, 2, ...).

One can think of a period as a quarter, a month, a week, or a day. The survey with an information provision experiment occurs at the end of period zero. At this point, some agents ("the treatment group") are unexpectedly called into the survey. During the survey, the agents undergo an information treatment and report their conditional expectation of the current value of the state, before and after the treatment. The treatment consists of showing the agents some information relevant for updating their belief. Agents who are not called into the survey ("the control group") move directly from stage 1 to stage  $3.^3$ 

Our model of attention is rational inattention, as in Sims (2003), in all stages. "Rational inattention" means that information is in principle available, but absorbing information requires

<sup>&</sup>lt;sup>2</sup>Sims (2003), Maćkowiak et al. (2018), Afrouzi and Yang (2021), Miao et al. (2022), Jurado (2023).

<sup>&</sup>lt;sup>3</sup>In a real-world survey, the control group are typically agents who are called into the survey but do not receive the information treatment during the survey. In the model, such agents move directly from stage 1 to stage 3.

paying attention which is costly; and the cost of attention is linear in entropy reduction.<sup>4</sup> We now state the decision problem that agents solve in each of the three stages.

**Stage 3:** Agents solve the following RI problem at the beginning of period t = 1:

$$V_1\left(\Sigma_{1|0}\right) = \min_{\left\{\sum_{t|t}\right\}_{t=1}^{\infty}} \left\{ \sum_{t=1}^{\infty} \beta^{t-1} \left( \gamma \Sigma_{t|t} + \mu ln \left( \frac{\Sigma_{t|t-1}}{\Sigma_{t|t}} \right) \right) \right\}$$
(2)

subject to

$$\Sigma_{t|t} \le \Sigma_{t|t-1} \tag{3}$$

$$\Sigma_{t+1|t} = \rho^2 \Sigma_{t|t} + \sigma_{\varepsilon}^2 \tag{4}$$

$$\Sigma_{1|0}$$
 given. (5)

In an RI model, agents decide how informed they want to be about the state. Agents enter period one (the period after the survey) with some prior uncertainty about the state. We denote by  $\Sigma_{1|0}$ the prior variance of the state in period one given information in period zero. Agents choose the posterior variance of the state (the conditional variance of the state after paying attention to the state) in period one,  $\Sigma_{1|1}$ , and they make a plan for the future  $\{\Sigma_{t|t}\}_{t=2}^{\infty}$ . Agents face the constraint that, in any period, the posterior variance of the state cannot exceed the prior variance, inequality (3). If this constraint holds with equality for some  $t \ge 1$ , the agent chooses to pay no attention in that period. The constraint states that the agent cannot pay "negative" attention. Equation (4) states that the prior variance in any period is determined by the process for the state, equation (1). The first term in the per-period loss function in (2) is the loss from being uninformed about the current value of the state and the second term in the per-period loss function in (2) is the cost of paying attention, which equals the parameter  $\mu$  times the entropy reduction in that period.  $V_1(\Sigma_{1|0})$  denotes the value function in period one and  $\beta \in (0, 1)$  is a discount factor.

One could add a subscript *i* for an individual agent *i* in the parameters  $\gamma, \mu, \beta$  as well as in the prior and posterior variances. We suppress the subscript for ease of exposition. Likewise for ease of exposition, we express the per-period loss directly in terms of the conditional variance of

<sup>&</sup>lt;sup>4</sup>Regarding inattention during the survey, Haaland et al. (2023), p.25, write: "One concern in online surveys is that respondents are inattentive and speed through the surveys (...)." Knotek et al. (2024), p.3, write: "(...) while our RCTs explicitly communicate information to the treatment groups, it is hard to know how much of this information is really read and/or absorbed by respondents." Knotek et al. measure attention during the survey, which they refer to as compliance, through the time that respondents spend reading the information treatment.

the state. One can think of the agent as ultimately choosing an action where the period-t optimal action depends on the period-t state. The parameter  $\gamma$  reflects both the cost of a given mistake in the action and the sensitivity of the optimal action to the state. With this understanding, we refer to  $\gamma$  simply as the importance of being informed about the state.

Stage 1: Agents solve the same problem as in stage 3, because they are in the same environment as in stage 3 (the same  $\rho$ ,  $\sigma_{\varepsilon}^2$ ,  $\gamma$ ,  $\mu$ , and  $\beta$  as in stage 3) and they do not expect to be called into the survey. In this type of problem, the conditional variances converge to steady-state values, denoted  $\bar{\Sigma}_{t+1|t}$  and  $\bar{\Sigma}_{t|t}$ . We assume that in stage 1 the convergence has taken place by period t = 0,  $\Sigma_{0|0} = \bar{\Sigma}_{t|t}$ .<sup>5</sup>

**Stage 2:** Agents who are called into the survey and receive the information treatment solve the following RI problem at the end of period t = 0 (during the survey):

$$\min_{\Sigma_{0|0,s}} \left\{ \left( \gamma \Sigma_{0|0,s} + \lambda ln \left( \frac{\Sigma_{0|0}}{\Sigma_{0|0,s}} \right) \right) + \beta V_1 \left( \Sigma_{1|0} \right) \right\}$$
(6)

subject to

$$\Sigma_{1|0} = \rho^2 \Sigma_{0|0,s} + \sigma_{\varepsilon}^2 \tag{7}$$

$$\Sigma_{0|0}$$
 given. (8)

Agents choose how much attention to devote to the information provided during the survey. Agents enter the survey with some before-the-survey posterior variance of the state,  $\Sigma_{0|0}$ , determined in stage 1. Agents exit the survey with some after-the-survey posterior variance of the state,  $\Sigma_{0|0,s}$ , determined by the decision problem (6)-(8). Agents can choose zero attention, implying  $\Sigma_{0|0,s} = \Sigma_{0|0}$ , or any positive amount of attention, implying  $\Sigma_{0|0,s} < \Sigma_{0|0}$ , during the survey. In the loss function in (6), the first term is the loss from being uninformed about the current value of the state, the second term is the cost of paying attention during the survey, which equals the parameter  $\lambda$  times the entropy reduction during the survey, and the third term captures the value of entering better informed into the first period after the survey. We assume that the marginal cost

<sup>&</sup>lt;sup>5</sup>It turns out that if the inequality constraint (3) is non-binding, the convergence in this model is immediate (the posterior variance reaches  $\bar{\Sigma}_{t|t}$  within a period for any given prior variance). If the inequality constraint (3) is binding, the convergence is slower (the posterior variance  $\Sigma_{t|t}$  gradually converges to  $\bar{\Sigma}_{t|t}$  from below). The assumption that also in this case the convergence has taken place by period t = 0,  $\Sigma_{0|0} = \bar{\Sigma}_{t|t}$ , is only used in the proof of Case 2 of Proposition 2.

of attention during the survey is strictly smaller than outside the survey,  $\lambda < \mu$  (the opportunity cost of one's attention to the variable of interest declines once one participates in a survey about this variable).

**Discussion.** We introduced an information provision experiment into an otherwise standard dynamic RI model. The survey experiment (stage 2) is modeled as follows: (i) agents are called unexpectedly into the survey, and (ii) the marginal cost of attention is lower during the survey than outside the survey ( $\lambda < \mu$ ). There may be other instances where agents face an unexpected, temporary reduction in the marginal cost of attention. The results of Section 3 also apply to those instances, because the unexpectedness and the lower marginal cost of attention are the only features of the information provision experiment in our model. In some experiments, agents receive additional monetary incentives to pay attention during the experiment. We study this extension of the model in the Online Appendix, and we summarize the findings at the end of Section 3.

The treatment effect. For an agent who receives the information treatment during the survey, the posterior (after-the-survey) mean of the period-0 state,  $\pi_{0|0,s}$ , is a convex combination of the prior (before-the-survey) mean,  $\pi_{0|0}$ , and the signal absorbed from the treatment, S:

$$\pi_{0|0,s} = \pi_{0|0} + G\left(S - \pi_{0|0}\right). \tag{9}$$

During the survey, an agent updates their belief as if they observed a realization of  $S = \pi_0 + \eta$ , where  $\eta \sim N(0, \sigma_{\eta}^2)$  with  $\sigma_{\eta}^2$  chosen optimally (stage-2 problem). This is a simple way to model attention choice during the survey; furthermore, a signal of this form is optimal under RI.<sup>6</sup> Think of the treatment as showing the agent the current value of the state. The agent is "speeding through the survey" (footnote 4) with the speed that reflects their environment and preferences, and therefore the agent absorbs a noisy signal where the variance of the noise is endogenous.<sup>7</sup>

From Bayesian updating,  $G = \Sigma_{0|0} / (\Sigma_{0|0} + \sigma_{\eta}^2) = 1 - \Sigma_{0|0,s} / \Sigma_{0|0}$ . We refer to the updating coefficient  $G \in [0, 1]$  as the treatment effect. The treatment effect G is increasing in *the ratio* of the before-the-survey posterior variance,  $\Sigma_{0|0}$ , to the after-the-survey posterior variance,  $\Sigma_{0|0,s}$ , both

<sup>&</sup>lt;sup>6</sup>In the Gaussian AR(1) case with a quadratic objective, the optimal signal under RI has the form "current value of the state plus Gaussian noise" (see the papers in footnote 2). It follows that outside the survey an agent updates their belief as if in each period t they observed a realization of  $\pi_t + \psi_t$ , where  $\psi_t$  is distributed normally and independently over time with mean zero and variance  $\sigma_{\psi_t}^2$  which is chosen optimally (stage-1 and stage-3 problems).

<sup>&</sup>lt;sup>7</sup>One can also think of the treatment as showing the agent some non-quantitative information (e.g., a news report) from which the agent extracts the signal S.

of which are endogenous in the RI model. Think of the incentives to pay attention outside the survey and during the survey, respectively.

Link to the empirical literature. The treatment effect is a key object of interest in the empirical literature on information provision experiments. As a recent example, Weber et al. (2025) use the following regression specification to estimate the treatment effect (the subscript i is for individuals):

$$Posterior_i = \beta_0 + \beta_1 Prior_i + \beta_2 I_i + \beta_3 I_i Prior_i + \epsilon_i$$
(10)

where  $Posterior_i$  and  $Prior_i$  are the respondent's posterior and prior about the variable of interest, and  $I_i$  is the indicator variable for being in the treatment group as opposed to being in the control group. Comparing equations (9) and (10), the model predicts  $\beta_3 = -G$  (also,  $\beta_1 = 1$  and  $\beta_2 = GS$ ). Weber et al. estimate the treatment effect as  $\hat{\beta}_3/\hat{\beta}_1$ , allowing for  $\beta_1 \neq 1$  since priors and posteriors are measured using different questions in their data (they also report  $\hat{\beta}_3$  separately). Hence, by studying G we focus on the key coefficient of interest in the empirical literature.<sup>8</sup>

### 3 Main results

**Proposition 1 (Benchmark)**: Suppose the discount factor is strictly positive,  $\beta > 0$ , and the state is correlated over time,  $\rho > 0$ . If the inequality constraint (3) is non-binding in period t = 0 before the survey, in period t = 0 during the survey, and in period t = 1 after the survey, then the treatment effect is strictly decreasing in the importance of being informed:

$$\frac{\partial G}{\partial \gamma} < 0.$$

The intuition for this result can be illustrated with a few equations. Let  $x \equiv \Sigma_{0|0}$  denote the posterior variance of the state in the period of the survey *before* participating in the survey, and let  $y \equiv \Sigma_{0|0,s}$  denote the posterior variance of the state in the same period *after* participating in the survey. The chain rule implies that the derivative of the treatment effect G = 1 - (y/x) with

<sup>&</sup>lt;sup>8</sup>Haaland et al. (2023) include in their survey article a table with the estimated G's from the literature (their Table 1). In some experiments, priors are unobserved and one cannot estimate the treatment effect G (one can see how ex-post beliefs differ between the control group and the treatment group).

respect to the importance of being informed  $\gamma$  equals

$$\frac{\partial G}{\partial \gamma} = \frac{y}{x} \left[ -\frac{\frac{\partial y}{\partial \gamma}}{y} - \left( -\frac{\frac{\partial x}{\partial \gamma}}{x} \right) \right]. \tag{11}$$

The first-order condition for the optimal posterior uncertainty *before* the survey, if the inequality constraint (3) is non-binding in period t = 0 before the survey, reads

$$\gamma - \mu \frac{1}{x} + \beta \mu \frac{1}{x + \frac{\sigma_{\varepsilon}^2}{\rho^2}} = 0.$$

$$\tag{12}$$

Choosing a marginally lower posterior variance has three effects: It reduces the current loss from inattention (the first term), it increases the current attention cost (the second term), and it reduces the future attention cost (the third term). The third term captures that, in a dynamic environment, current attention reduces future prior uncertainty thereby saving future attention. The first-order condition for the optimal posterior uncertainty during the survey, if the inequality constraint (3) is non-binding in period zero during the survey and in period one after the survey, reads

$$\gamma - \lambda \frac{1}{y} + \beta \mu \frac{1}{y + \frac{\sigma_z^2}{\rho^2}} = 0.$$
<sup>(13)</sup>

The three terms have the same interpretation as before. The only difference between equations (12) and (13) is the lower marginal cost of attention during the survey ( $\lambda < \mu$ ). It is this lower marginal cost of attention during the survey ( $\lambda < \mu$ ). It is this lower marginal cost of attention during the survey which implies further updating and a positive treatment effect ( $\lambda < \mu \Leftrightarrow y < x \Leftrightarrow G > 0$ ). Next, one can use the implicit function theorem to compute the two terms  $\left(\frac{\partial x}{\partial \gamma}/x\right)$  and  $\left(\frac{\partial y}{\partial \gamma}/y\right)$  in equation (11) from equations (12) and (13). One obtains two results. First, when being informed becomes more important ( $\gamma$  rises), agents choose a lower posterior variance before the survey (first effect) and agents choose a lower posterior variance during the survey (second effect);  $\left(\frac{\partial x}{\partial \gamma}/x\right) < 0$  and  $\left(\frac{\partial y}{\partial \gamma}/y\right) < 0$ . Second, the percentage reduction in the optimal posterior variance during the survey;  $\left(\frac{\partial x}{\partial \gamma}/x\right)$  is strictly larger than  $\left(\frac{\partial y}{\partial \gamma}/y\right)$  in absolute value. Hence, the treatment effect *G* is strictly decreasing in the importance of being informed  $\gamma$ .

The variation in  $\gamma$  can be a time-series variation or a cross-sectional variation. In the first case, the proposition states that, in times when it is more important to be informed about the state, the treatment effect will be smaller. In the second case, the proposition states that, for individuals who care more about being informed about the state, the treatment effect will be smaller.<sup>9</sup>

The first condition of Proposition 1 is that the state is correlated over time, which implies that information processed in the period of the survey also has value in the period after the survey (see the third term in equations (12)-(13)). The other condition of Proposition 1 is that agents find it optimal to pay attention to the state before, during, and after the survey. Proposition 2 covers the case when this other condition is not satisfied.

**Proposition 2 (Corner Solutions)**: Suppose the discount factor is strictly positive,  $\beta > 0$ , and the state is correlated over time,  $\rho > 0$ . Suppose the other condition of Proposition 1 is not satisfied. Case 1: If the inequality constraint (3) is binding in period t = 0 before and during the survey,

$$\frac{\partial G}{\partial \gamma} = 0.$$

Case 2: If the inequality constraint (3) is binding in period t = 0 before the survey but not during the survey,

$$\frac{\partial G}{\partial \gamma} > 0.$$

Case 3: If the inequality constraint (3) is non-binding in period t = 0 and binding in period t = 1, then G is increasing, decreasing, or non-monotonic in  $\gamma$ , depending on parameter values.

Case 1 arises when  $\gamma$  is so low or  $\mu, \lambda$  are so high that the agent pays no attention to the state, either before or during the survey. Since the agent pays no attention during the survey, the treatment effect G equals zero and the derivative of the treatment effect G with respect to  $\gamma$  equals zero. Small variation in  $\gamma$  does not change the fact that the agent chooses to process no information.

Case 2 arises when the agent pays no attention to the state outside the survey but  $\lambda$  is sufficiently low that the agent pays attention during the survey. In this case, the first effect described below Proposition 1 is absent, since the agent pays no attention outside the survey, but the second effect described below Proposition 1 is present, since the agent pays attention during the survey. Hence,

<sup>&</sup>lt;sup>9</sup>As a time-series comparative static experiment in the model, one can think of an unanticipated, permanent change in  $\gamma$  during stage 1. Note that: such a parameter change affects the steady-state conditional variances; stage 1 assumes that agents have converged to the steady-state conditional variances before the survey; the convergence is immediate (occurs in the same period as the parameter change) if the inequality constraint (3) is non-binding (footnote 5).

the treatment effect G is strictly increasing in the importance of being informed.

Case 3 arises when the cost of attention during the survey is so low relative to the cost of attention outside the survey ( $\lambda \ll \mu$ ) that the agent chooses to use this opportunity to update beliefs a lot and not pay attention to the state for a while afterwards. Since the agent pays attention before the survey, x is given by equation (12), and since the agent pays attention during the survey, y is given by an equation that is similar to equation (13). The only difference from equation (13) is the third term since the continuation value of information is now different, which creates the ambiguous and potentially positive sign of the derivative of the treatment effect G with respect to the importance of being informed  $\gamma$ .

Figure 1 illustrates all four cases (the three cases of Proposition 2 and the one case of Proposition 1) in a single figure. The figure shows the endogenous treatment effect, G, as a function of the exogenous importance of being informed,  $\gamma$ .<sup>10</sup> In three out of four regions for the parameter  $\gamma$ , the model makes an unambiguous prediction about how the treatment effect varies with  $\gamma$ . In a survey, one could potentially elicit whether an individual is in region 1, 2, 3, or 4 (benchmark) by measuring attention during the survey, by asking individuals whether they regularly paid attention to the state before the survey, and by asking individuals whether they plan to pay attention to the state in the near future. Conditional on being in region 1, 2, or 4, the model makes an unambiguous prediction about how the treatment of being informed.

These comparative static results extend to other parameters: the volatility of the innovation in the state,  $\sigma_{\varepsilon}^2$ , and the marginal cost of processing information,  $\mu$ , for given  $\lambda/\mu$ . In the comparative static exercise varying  $\mu$ , we are holding constant  $\lambda/\mu$ , because the idea is that some individuals may have a higher cost of attention (hence higher  $\mu$ ) but they may still have a relatively lower cost of attention during the survey than outside the survey (hence constant  $\lambda/\mu$ ).

**Proposition 3 (Other Parameters):** If the conditions of Proposition 1 are satisfied, then  $1^{10}$ We set  $\rho = 0.9$ ,  $\sigma_{\varepsilon}^2 = 1$ ,  $\beta = 0.99$ ,  $\mu = 2/ln(2)$ ,  $\lambda = 0.75\mu$ , and we varied  $\gamma$  from 0.001 to 3. One can show that, for all sets of parameter values with  $\rho > 0$ , there exist three thresholds  $\gamma_1 < \gamma_2 < \gamma_3$  such that: for  $\gamma \leq \gamma_1$  the agent pays no attention outside and during the survey (region 1), for  $\gamma \in (\gamma_1, \gamma_2]$  the agent pays attention only during the survey (region 2), for  $\gamma \in (\gamma_2, \gamma_3]$  the agent pays attention before the survey, during the survey and with some delay after the survey (region 3), and for  $\gamma > \gamma_3$  the agent pays attention throughout (region 4). Cases 1-3 of Proposition 2 and Proposition 1 characterize how G depends on  $\gamma$  in the interior of those regions.

Figure 1: Treatment effect, G, as a function of the parameter  $\gamma$ 



The four gray shaded areas show the four regions, from the left: (i) no attention outside and during survey, (ii) attention only during survey, (iii) attention before, during and with some delay after survey, (iv) attention throughout.

the treatment effect is strictly decreasing in the variance of the innovation:

$$\frac{\partial G}{\partial \sigma_{\varepsilon}^2} < 0,$$

and the treatment effect is strictly increasing in the marginal cost of attention  $\mu$  for given  $\lambda/\mu$ :

$$\frac{\partial G}{\partial \mu} > 0.$$

Nature of provided information. In some information provision experiments, it is straightforward to place participants in the different regions of Figure 1. If agents could not have attended to the state before the survey ( $\mu \rightarrow \infty$ ), there are no regions 3 and 4 and the individual is necessarily in region 1 or 2. In this case, the treatment effect is weakly increasing in the importance of being informed (the treatment effect is zero in region 1 and strictly increasing in region 2).

Similarly, in lab experiments on rational inattention (Khaw, Stevens, and Woodford, 2017, Dean and Neligh, 2023), the state, such as the number of balls of a certain color, could not have been attended to before the experiment, and the prior uncertainty is fixed at a value that is independent of how much the individual cares about being informed. One can think of this setting as  $\mu \to \infty$ , where the uncertainty at the beginning of the survey equals the unconditional variance,  $\Sigma_{0|0} = \sigma_{\varepsilon}^2 / (1 - \rho^2)$ , which is independent of  $\gamma$ . In this case, again, the individual is necessarily in region 1 or 2.

Alternative models. The result "the treatment effect declines with  $\gamma$  and  $\sigma_{\varepsilon}^{2^n}$  (Propositions 1 and 3) does not naturally arise in other models of costly information acquisition or in models with exogenous imperfect information. As an example, the same result does not even arise in the model "RI only outside the survey" where the survey itself is modeled without attention choice. Suppose agents solve the same problem in stage 1 and they pay strictly positive attention in the steady state. During the survey each agent updates their prior based on a realization of the signal "current state plus exogenous noise," with noise  $\eta$  drawn from a normal distribution with mean zero and variance  $\sigma_{\eta}^2$ . In this model, the treatment effect is equal to  $G = \Sigma_{0|0} / (\Sigma_{0|0} + \sigma_{\eta}^2)$  with an exogenous  $\sigma_{\eta}^2$ .  $\Sigma_{0|0}$  is decreasing in  $\gamma$  and increasing in  $\sigma_{\varepsilon}^2$ , and hence in this model G is decreasing in  $\gamma$  and agents reduce their attention during the survey when  $\gamma$  or  $\sigma_{\varepsilon}^2$  increases.

As another example, consider an exogenous-imperfect-information model in which: (i) outside the survey, in every period each agent updates their prior based on a realization of the signal "current state plus exogenous noise," with noise  $\psi$  drawn from a normal distribution with mean zero and variance  $\sigma_{\psi}^2$ , (ii) the survey is modeled as in the previous paragraph. Here the noise outside the survey and the noise during the survey are exogenous. In this model, the treatment effect is invariant to  $\gamma$  and increasing in  $\sigma_{\varepsilon}^2$ .

Additional results. The Online Appendix contains additional results. We give an analytical solution for the posterior variances  $\Sigma_{0|0}$  and  $\Sigma_{0|0,s}$ . We study the persistence of the treatment effect. Rationally inattentive agents optimally decrease attention for some time after an information treatment, which reduces the persistence of the treatment effect compared with a model with exogenous imperfect information. We also study an extension of the model where respondents are incentivized to pay attention during the survey. We find that an additional monetary incentive that is only present during the experiment makes agents increase attention during the experiment but it does not affect the result in Proposition 1: the treatment effect *G* is strictly decreasing in  $\gamma$ . Finally, we study two other extensions of the model: a version where the treatment consists of showing respondents a noisy measure of the state; and a version where the treatment consist of the treatment consist of the state (with noise that

is beyond the respondents' control).

### 4 Interpreting information provision experiments

In this section, we use the model to interpret information provision experiments from the literature. We give examples of findings, from macro- and microeconomics, that can be explained by rational inattention of survey participants.

In many experiments in macroeconomics, the state variable is inflation. Cavallo et al. (2017) and Weber et al. (2025) find that the treatment effect is smaller when inflation is high. Our interpretation of this finding is that the importance of being informed about inflation and/or the volatility of inflation increase with inflation, and increases in  $\gamma$  and  $\sigma_{\varepsilon}^2$  reduce the treatment effect (Propositions 1 and 3). A joint increase in  $\gamma$  and  $\sigma_{\varepsilon}^2$  can also match the other finding in Weber et al. that agents' uncertainty about inflation did not decline, as  $\gamma$  and  $\sigma_{\varepsilon}^2$  have opposing effects on the posterior uncertainty about inflation.<sup>11</sup>

Weber et al. (2025) also provide a model that can explain their findings. It is a two-period model with another assumption beyond RI: in addition to processing information about inflation subject to a linear entropy-reduction cost, agents can pay a fixed cost to learn exactly the official inflation rate. The authors combine the fixed cost with a parametric assumption such that, as a result, an agent's posterior uncertainty about inflation is independent of whether the agent paid the fixed cost and knows the official inflation rate, or did not pay the fixed cost and does not know the official inflation rate. In this setting, a decrease in the fixed cost—or an increase in the informativeness of the official inflation rate—weakens the treatment effect without affecting the posterior uncertainty.<sup>12</sup> Their model is different from ours. Our model has more than two periods,

<sup>12</sup>On the distinction between "official" and "relevant" inflation: In the Online Appendix, we study numerically a version of our model where the treatment consists of showing respondents a noisy measure of the current value of the state (instead of the current value of the state). We find qualitatively the same comparative statics as in the baseline

<sup>&</sup>lt;sup>11</sup>Recall that  $\gamma$  reflects both the sensitivity of the optimal action (e.g., the optimal nominal savings of a household) to inflation and the cost of a mistake in the action, which depends on the marginal utility of consumption (see Maćkowiak and Wiederholt, 2015, for a model with rationally inattentive households who hold nominal bonds). It is plausible that the sensitivity of the optimal action and the marginal utility rise with high realizations of inflation, especially extreme ones. Likewise, it is plausible that high realizations of inflation such as during the COVID pandemic are driven, to some extent, by an increase in the variance of innovations.

which matters, because the continuation value of information generates the third term in equations (12)-(13); and our model is a pure RI model, as in Sims (2003), in the sense that the attention cost is always a function of entropy reduction. Furthermore, the purpose of the two models is different. The model in Weber et al. is tailored to the findings in that paper. The purpose of our model is to understand how the treatment effect varies over time, across individuals, and across experimental designs (e.g., nature of provided information).

The above discussion is about why the average treatment effect in a population changes over time with the environment. One can also apply the model to understand why the treatment effect varies across individuals. As an example from macroeconomics, in a survey concerning monetary policy Knotek et al. (2024) find a stronger treatment effect for people who are ex-ante poorly informed and pay more attention during the survey. The interpretation based on Propositions 1 and 3 is these are low- $\gamma$  and/or high- $\mu$  individuals (individuals for whom monetary policy is relatively unimportant).<sup>13</sup>

There are many information provision experiments in microeconomics; as an example, we focus here on education economics. Researchers often find differences in the treatment effect across individuals when provided information is about returns to education or children's educational outcomes. In Bleemer and Zafar (2018) and in Dizon-Ross (2019), respondents are confronted with information that was *available* to them before the experiment (Dizon-Ross emphasizes this fact, while Bleemer and Zafar treat respondents with publicly available information). Propositions 1 and 3 imply that a strong treatment effect can come from a small  $\gamma$  (a small cost of a mistake) or a large  $\mu$  (a high opportunity cost of attention outside the survey). In Jensen (2010), respondents are confronted with information which, as the author emphasizes, was *unavailable* before the experiment. Proposition 2 (Cases 1-2) becomes the relevant theoretical prediction: a strong treatment effect can come from a large  $\gamma$  or a small  $\lambda$  (greater ability to absorb information during the survey).<sup>14</sup> A

<sup>14</sup>In Jensen (2010) and Bleemer and Zafar (2018), respondents are treated with information about returns to education. In Dizon-Ross (2019), parents are treated with information about their children's educational outcomes.

model.

<sup>&</sup>lt;sup>13</sup>Similarly, Link et al. (2023) document that firms have more accurate expectations of central bank policy rates than households, consistent with policy rates being more important to firms than to households. Link et al. then find that firms update their policy rate expectations less than households when provided with an expert forecast. Armantier et al. (2016) find that those individuals who are less uncertain about inflation (which indicates that inflation may be more important to them) respond less to the information treatment.

broader lesson is that, according to the RI model, the nature of the information—whether the relevant information was available to respondents before the survey or not—affects the interpretation of the findings.

A recurrent finding is that the treatment effect on actions is smaller than on beliefs (Yang, 2024, reviews the relevant experiments in detail). The model predicts this pattern. A small  $\gamma$  can reflect a small cost of a mistake in the action or a weak sensitivity of the optimal action to the state. Under the conditions of Proposition 1, a large treatment effect on beliefs comes from low- $\gamma$  individuals; such individuals are also expected to show a small treatment effect on actions, to the extent that their low  $\gamma$ 's reflect weak sensitivity of the optimal action to the state.

In most information provision experiments, participants are shown a given piece of information for free. See Fuster et al. (2022) for an experiment and a model in which agents have to acquire the information at a cost and have a choice of the information sources they want to see.

### 5 Conclusions

We introduced an information provision experiment into a plain-vanilla dynamic RI model. We characterized analytically how the treatment effect varies with the characteristics of the environment and the individual.

RI helps interpret information provision experiments. It provides an explanation for existing evidence. It helps us understand what survey findings imply about behavior outside the survey. We would like to argue that RI is a useful starting point for understanding the evidence, not that it will explain all of it. The model is a benchmark that can be modified in future research, including by deviating from the linear entropy-reduction cost function.

The analysis has implications for communication policy. If communication involves merely restating information that has already been publicly available, then it may have less effect among individuals who care more about being informed and at times when being informed is more important (Proposition 1). If communication involves expanding the set of publicly available information, then it may be more effective among individuals who care more about being informed and at times when being informed is more important (Proposition 2, Case 2).

# A Appendix

**Proof of Proposition 1:** The treatment effect is

$$G = 1 - \frac{\Sigma_{0|0,s}}{\Sigma_{0|0}},\tag{14}$$

where  $\Sigma_{0|0}$  is the posterior variance of the state in period t = 0 before the survey and  $\Sigma_{0|0,s}$  is the posterior variance of the state in period t = 0 during the survey. For ease of exposition, define  $x \equiv \Sigma_{0|0}$  and  $y \equiv \Sigma_{0|0,s}$ . It follows from the product rule and the chain rule that

$$\frac{\partial G}{\partial \gamma} = \frac{y}{x} \left[ -\frac{\frac{\partial y}{\partial \gamma}}{y} - \left( -\frac{\frac{\partial x}{\partial \gamma}}{x} \right) \right]. \tag{15}$$

If the inequality constraint (3) is non-binding in period t = 0 before the survey, the first-order condition for x reads

$$\gamma - \mu \frac{1}{x} + \beta \mu \frac{1}{x + \frac{\sigma_{\varepsilon}^2}{a^2}} = 0.$$

$$\tag{16}$$

It follows from the implicit function theorem that

$$\frac{\frac{\partial x}{\partial \gamma}}{x} = -\frac{1}{\mu} \left[ \frac{1}{x} - \beta \frac{x}{\left(x + \frac{\sigma_{\varepsilon}^2}{\rho^2}\right)^2} \right]^{-1}.$$
(17)

Using equation (16) to substitute for  $\frac{1}{x}$  in equation (17) yields

$$\frac{\frac{\partial x}{\partial \gamma}}{x} = -\frac{1}{\mu} \left[ \frac{\gamma}{\mu} + \beta \frac{\frac{\sigma_{\varepsilon}^2}{\rho^2}}{\left(x + \frac{\sigma_{\varepsilon}^2}{\rho^2}\right)^2} \right]^{-1}.$$
(18)

Next, if the inequality constraint (3) is also non-binding in period t = 0 during the survey and in period t = 1 after the survey, the first-order condition for y reads

$$\gamma - \lambda \frac{1}{y} + \beta \mu \frac{1}{y + \frac{\sigma_{\varepsilon}^2}{\rho^2}} = 0.$$
<sup>(19)</sup>

It follows from the implicit function theorem that

$$\frac{\frac{\partial y}{\partial \gamma}}{y} = -\frac{1}{\mu} \left[ \frac{\lambda}{\mu} \frac{1}{y} - \beta \frac{y}{\left(y + \frac{\sigma_{\varepsilon}^2}{\rho^2}\right)^2} \right]^{-1}.$$
(20)

Using equation (19) to substitute for  $\frac{\lambda}{\mu}\frac{1}{y}$  in equation (20) yields

$$\frac{\frac{\partial y}{\partial \gamma}}{y} = -\frac{1}{\mu} \left[ \frac{\gamma}{\mu} + \beta \frac{\frac{\sigma_{\varepsilon}^2}{\rho^2}}{\left(y + \frac{\sigma_{\varepsilon}^2}{\rho^2}\right)^2} \right]^{-1}.$$
(21)

Finally, equations (14), (16), and (19) imply

$$G > 0 \Leftrightarrow y < x \Leftrightarrow \lambda < \mu, \tag{22}$$

and equations (15), (18), and (21) imply

$$\frac{\partial G}{\partial \gamma} < 0 \Leftrightarrow y < x. \tag{23}$$

**Proof of Proposition 3:** The treatment effect is given by equation (14). It follows from the product rule and the chain rule that

$$\frac{\partial G}{\partial \sigma_{\varepsilon}^2} = \frac{y}{x} \left[ -\frac{\frac{\partial y}{\partial \sigma_{\varepsilon}^2}}{y} - \left( -\frac{\frac{\partial x}{\partial \sigma_{\varepsilon}^2}}{x} \right) \right].$$
(24)

The first-order condition for x is given by equation (16). Using the implicit function theorem yields

$$\frac{\frac{\partial x}{\partial \sigma_{\varepsilon}^2}}{x} = \frac{\beta}{\rho^2 \left(x + \frac{\sigma_{\varepsilon}^2}{\rho^2}\right)^2} \left[\frac{\gamma}{\mu} + \beta \frac{\frac{\sigma_{\varepsilon}^2}{\rho^2}}{\left(x + \frac{\sigma_{\varepsilon}^2}{\rho^2}\right)^2}\right]^{-1}.$$
(25)

The first-order condition for y is given by equation (19). Using the implicit function theorem yields

$$\frac{\frac{\partial y}{\partial \sigma_{\varepsilon}^2}}{y} = \frac{\beta}{\rho^2 \left(y + \frac{\sigma_{\varepsilon}^2}{\rho^2}\right)^2} \left[\frac{\gamma}{\mu} + \beta \frac{\frac{\sigma_{\varepsilon}^2}{\rho^2}}{\left(y + \frac{\sigma_{\varepsilon}^2}{\rho^2}\right)^2}\right]^{-1}.$$
(26)

Finally, equations (14), (16), and (19) imply

$$G > 0 \Leftrightarrow y < x \Leftrightarrow \lambda < \mu, \tag{27}$$

and equations (24)-(26) imply

$$\frac{\partial G}{\partial \sigma_{\varepsilon}^2} < 0 \Leftrightarrow y < x.$$
<sup>(28)</sup>

Next, we turn to the derivative of G with respect to  $\mu$  (for given  $\lambda/\mu$ ). Following the exact same steps as in the proof of Proposition 1 yields

$$\frac{\partial G}{\partial \mu} = \frac{y}{x} \left[ -\frac{\frac{\partial y}{\partial \mu}}{y} - \left( -\frac{\frac{\partial x}{\partial \mu}}{x} \right) \right],\tag{29}$$

$$\frac{\frac{\partial x}{\partial \mu}}{x} = \frac{\gamma}{\mu^2} \left[ \frac{\gamma}{\mu} + \beta \frac{\frac{\sigma_{\varepsilon}^2}{\rho^2}}{\left(x + \frac{\sigma_{\varepsilon}^2}{\rho^2}\right)^2} \right]^{-1},\tag{30}$$

and

$$\frac{\frac{\partial y}{\partial \mu}}{y} = \frac{\gamma}{\mu^2} \left[ \frac{\gamma}{\mu} + \beta \frac{\frac{\sigma_{\varepsilon}^2}{\rho^2}}{\left(y + \frac{\sigma_{\varepsilon}^2}{\rho^2}\right)^2} \right]^{-1}.$$
(31)

In equation (31) we are holding constant  $\lambda/\mu$ . See the text above Proposition 3. Equations (29)-(31) imply

$$\frac{\partial G}{\partial \mu} > 0 \Leftrightarrow y < x. \tag{32}$$

**Proof of Proposition 2:** Case 1: If the constraint  $\Sigma_{0|0,s} \leq \Sigma_{0|0}$  is binding during the survey, then  $\Sigma_{0|0,s} = \Sigma_{0|0}$ , implying  $G = 1 - \frac{\Sigma_{0|0,s}}{\Sigma_{0|0}} = 0$ . For a small enough change in  $\gamma$ , the constraint remains binding during the survey, and thus  $\frac{\partial G}{\partial \gamma} = 0$ .

Case 2: Since the inequality constraint (3) is binding before the survey and since we have assumed that the posterior uncertainty has converged to the steady-state posterior uncertainty by period t = 0, we have

$$\Sigma_{0|0} = \frac{\sigma_{\varepsilon}^2}{1 - \rho^2}.$$
(33)

For the rest of the proof, we use again the notation  $x \equiv \Sigma_{0|0}$  and  $y \equiv \Sigma_{0|0,s}$ . Note that x does not depend on  $\gamma$  and thus  $\frac{\partial x}{\partial \gamma} = 0$ . Next, since the constraint  $\Sigma_{0|0,s} \leq \Sigma_{0|0}$  is non-binding during the survey, the first-order condition for y reads

$$\gamma - \lambda \frac{1}{y} + \beta \left[ \gamma \rho^2 + \beta \gamma \rho^4 + \beta^2 \gamma \rho^6 + \ldots \right] = 0.$$
(34)

The third term in equation (34) differs from the third term in equation (19), because when the inequality constraint (3) is binding in period t = 0 before the survey, then it will also be binding in all periods t = 1, 2, ... after the survey, which changes the continuation value of information. It follows from equation (34) and the implicit function theorem that  $\frac{\partial y}{\partial \gamma} < 0$ . Equation (15) implies  $\frac{\partial G}{\partial \gamma} > 0$ .

Case 3: If the inequality constraint (3) is non-binding in period t = 0 before the survey, then the first-order condition for x is given by equation (16); and if the inequality constraint (3) is binding in period t = 1 after the survey, then the first-order condition for y reads:

$$\gamma - \lambda \frac{1}{y} + \beta \left[ \gamma \rho^2 + \beta \mu \frac{1}{y + \frac{\sigma_{\varepsilon}^2}{\rho^2} + \frac{\sigma_{\varepsilon}^2}{\rho^4}} \right] = 0,$$
(35)

when the constraint is binding for exactly one period after period t = 0, while the first-order condition for y reads:

$$\gamma - \lambda \frac{1}{y} + \beta \left[ \gamma \rho^2 + \beta \gamma \rho^4 + \beta^2 \mu \frac{1}{y + \frac{\sigma_{\varepsilon}^2}{\rho^2} + \frac{\sigma_{\varepsilon}^2}{\rho^4} + \frac{\sigma_{\varepsilon}^2}{\rho^6}} \right] = 0,$$
(36)

when the constraint is binding for exactly two periods after period t = 0, and so on. One can again use the implicit function theorem to derive the expression for  $\left(\frac{\partial y}{\partial \gamma}/y\right)$  from equation (35) or equation (36). It turns out that, since the third term in equations (35) and (36) differs from the third term in equation (19), the sign of the difference between  $\left(\frac{\partial x}{\partial \gamma}/x\right)$  and  $\left(\frac{\partial y}{\partial \gamma}/y\right)$  now depends on parameter values. Hence, the sign of  $\frac{\partial G}{\partial \gamma}$  now depends on parameter values. For example, for the parameters given in footnote 10, G is increasing in  $\gamma$  in region 3 of Figure 1; but for a sufficiently lower value of  $\lambda$ , G becomes decreasing in  $\gamma$  in part of region 3. Furthermore, at the values for  $\gamma$ at which the constraint (3) becomes binding for one more period after period t = 0, the treatment effect G is non-differentiable in  $\gamma$ .

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# Online Appendix to "Rational Inattention during an RCT"

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This Online Appendix contains additional results for "Rational Inattention during an RCT."

An analytical solution for the posterior variances  $\Sigma_{0|0}$  and  $\Sigma_{0|0,s}$ . If the inequality constraint (3) in the paper is non-binding in period t = 0 before the survey, the first-order condition for the before-the-survey posterior variance  $\Sigma_{0|0}$  is given by equation (12) in the paper. It is a quadratic equation with the unique positive solution given by

$$\Sigma_{0|0} = \frac{-\left(\gamma\sigma_{\varepsilon}^{2} - \mu\rho^{2} + \beta\mu\rho^{2}\right) + \sqrt{\left(\gamma\sigma_{\varepsilon}^{2} - \mu\rho^{2} + \beta\mu\rho^{2}\right)^{2} + 4\gamma\rho^{2}\mu\sigma_{\varepsilon}^{2}}}{2\gamma\rho^{2}}.$$

The same expression is also the solution for the steady-state posterior variance in stage 1 and in stage 3,  $\bar{\Sigma}_{t|t}$ .

If the inequality constraint (3) in the paper is non-binding in period t = 0 during the survey and in period t = 1, the first-order condition for the after-the-survey posterior variance  $\Sigma_{0|0,s}$  is given by equation (13) in the paper. It is a quadratic equation with the unique positive solution given by

$$\Sigma_{0|0,s} = \frac{-\left(\gamma\sigma_{\varepsilon}^{2} - \lambda\rho^{2} + \beta\mu\rho^{2}\right) + \sqrt{\left(\gamma\sigma_{\varepsilon}^{2} - \lambda\rho^{2} + \beta\mu\rho^{2}\right)^{2} + 4\gamma\rho^{2}\lambda\sigma_{\varepsilon}^{2}}}{2\gamma\rho^{2}}.$$

**Persistence of the treatment effect.** Rationally inattentive agents optimally decrease attention for some time after an information treatment, which reduces the persistence of the treatment effect compared with a model with exogenous imperfect information.

In the exogenous-imperfect-information model introduced in the paper: (i) outside the survey, in every period each agent updates their prior based on a realization of the signal "current state plus exogenous noise," with noise  $\psi$  drawn from a normal distribution with mean zero and variance  $\sigma_{\psi}^2$ , (ii) during the survey, each agent updates their prior based on a realization of the signal "current state plus exogenous noise," with noise  $\eta$  drawn from a normal distribution with mean zero and variance  $\sigma_{\eta}^2$ .

Figure 1 compares the impulse response of the conditional expectation of the state,  $\pi_{t|t}$ ,  $t \ge 0$ , to an innovation in the state in period t = 0 for three types of agents: (i) agents from the RI model who did not participate in the survey ("Control group"), (ii) agents from the RI model who participated in the survey ("Treatment group, rational inattention"), and (iii) agents from the exogenous-imperfect-information model who participated in the survey ("Treatment group, exogenous imperfect information"). The figure also reports the impulse response for hypothetical agents with perfect information. The parameter values are:  $\rho = 0.9$ ,  $\sigma_{\varepsilon}^2 = 1$ ,  $\gamma = 1$ ,  $\beta = 0.99$ ,  $\mu = 2/ln(2)$ ,  $\lambda = 0.75\mu$ ,  $\sigma_{\eta}^2 = 2.8954$ ,  $\sigma_{\psi}^2 = 3.9529$ . With this parameterization, both treatment groups absorb the same amount of information during the survey. Furthermore, the treatment group from the exogenous-imperfect-information model absorbs the same amount of information in each period after the survey as does the control group. That is, agents in the RI model choose  $\sigma_{\eta}^2 = 2.8954$ during the survey and  $\sigma_{\psi}^2 = 3.9529$  in the steady state; in the steady state, agents update their belief as if in each period t they observed a realization of  $x_t + \psi$ , where  $\psi \sim i.i.d.N\left(0, \sigma_{\psi}^2\right)$  and  $\sigma_{\psi}^2 = 3.9529$ .

As can be seen in Figure 1, both treatment groups start out together in period zero, closer to the perfect-information benchmark than the control group. The initial treatment effect is the same for both treatment groups. In period one, the period after the survey, the treatment group from the RI model optimally chooses to pay less attention than in the steady state. The extra information from the survey pushes the prior variance below its steady-state value, making it optimal to decrease attention. This effect reduces the persistence of the treatment effect compared with the exogenous-imperfect-information model. In period one in Figure 1, the treated agents from the exogenous-imperfect-information model are about 70 percent further from the control group than the treated agents from the RI model, which is a non-trivial difference in the persistence of the treatment effect. In this AR(1) model, convergence to the steady state is immediate in the RI model, and therefore the aforementioned effect operates only in period one. Note that in the exogenous-imperfect-information model, the prior variance is likewise below its steady-state value after the survey, which also affects the Kalman gain in that model; furthermore, convergence to the steady state is gradual.



Figure 1: Impulse response of  $\pi_{t|t}$  to an innovation in  $\pi_0$ 

Forecast treatment. During a survey respondents may be shown a forecast of a future value of the state, and they may be asked to report their forecast. Suppose that in stage 2 of the model agents are constrained to learn from a forecast of the state. The signal absorbed during the survey is a realization of  $S = \rho \pi_0 + \eta$ , with  $\rho > 0$  and  $\sigma_{\eta}^2$  chosen in the same stage-2 problem, where  $\rho \pi_0$ is the best forecast of next period's state at the time of the survey. The updating equation reads:

$$\pi_{1|0,s} = \pi_{1|0} + \frac{\Sigma_{0|0}}{\Sigma_{0|0} + \sigma_{\eta}^2/\rho^2} \left(S - \pi_{1|0}\right)$$

where  $\pi_{1|0,s}$  is the posterior (after-the-survey) mean of period-1 state, and  $\pi_{1|0}$  is the prior (beforethe-survey) mean. The treatment effect in the baseline model is  $G = \sum_{0|0} / (\sum_{0|0} + \sigma_{\eta}^2)$ . Here the treatment effect is  $\sum_{0|0} / (\sum_{0|0} + \sigma_{\eta}^2 / \rho^2)$ , which is smaller for a given  $\sigma_{\eta}^2$  than in the baseline model. However, a rationally inattentive agent chooses a lower  $\sigma_{\eta}^2$  here than in the baseline model. In fact, the agent chooses a  $\sigma_{\eta}^2$  such that  $\sigma_{\eta}^2 / \rho^2$  is equal to  $\sigma_{\eta}^2$  in the baseline. The reason is that during the survey the agent wants to pay the same amount of attention to the current value of the state as in the baseline. As a result, in equilibrium the treatment effect is equal to the treatment effect in the baseline. It does not matter if the provided information is the current realization ( $\pi_0$ ) or the forecast ( $\rho\pi_0$ ). Outside the AR(1) case, we no longer expect the current-realization and the best-forecast treatments to yield exactly the same treatment effect. Being provided with the current realization or the best forecast are no longer equivalent options for a rationally inattentive agent, and both options are suboptimal (the optimal signal is on the state vector, see Maćkowiak et al., 2018). As an example, we solved numerically the model in the case when the state follows an AR(2) process,  $\pi_t = \rho_1 \pi_{t-1} + \rho_2 \pi_{t-2} + \varepsilon_t$ , assuming  $\rho_1 = 1.2$ ,  $\rho_2 = -0.3$  with the rest of the baseline parameterization unchanged. We used the code of Afrouzi and Yang (2021) because the stage-3 problem is a special case of the problem that they study. We obtained very similar treatment effects: 0.288 for the current-realization treatment, and 0.293 for the best-forecast treatment. Coibion et al. (2022) elicit households' inflation expectations and provide different pieces of information regarding inflation during the survey. They find very similar treatment effects when the provided information is the most recent inflation rate or the FOMC's inflation forecast (or the Federal Reserve's inflation target). The inflation-forecast and the inflation-target treatments appear to result in slightly more persistent belief revisions.

"The rate of change in my cost-of-living is different from the official inflation rate." We study a version of the model where the treatment consists of showing respondents a noisy measure of the current value of the state, with noise beyond the respondents' control. To fix ideas, think of the state as inflation. Suppose the agent cares about an inflation rate that is different from the official inflation rate, because the agent's consumption basket differs from the consumption basket underlying the official inflation rate or because there is measurement error in the official inflation rate. During a survey experiment, the agent is treated with the official inflation rate.

In a model with perfect information, an agent knows all official statistics and consequently shows a treatment effect of zero, even if the official statistics are only a noisy measure of the state relevant for the agent's optimal action. In a model with RI, an agent learns directly about the state (i.e., not about official statistics). During a survey, the rationally inattentive agent reports their conditional expectation of the official statistic; during an information treatment, the agent updates their conditional expectations of the state and of the official statistic. Let us now interpret  $\pi_t$  as the inflation rate that the agent cares about. Suppose that the official inflation rate is given by  $\pi_t + z_t$ , where  $z_t$  follows a Gaussian AR(1) process independent of  $x_t$ . During the survey agents are constrained to learn from the current official inflation rate and they decide how much attention to pay to that variable. The signal absorbed during the survey is a realization of  $S = \pi_0 + z_0 + \eta$ , with  $\sigma_{\eta}^2$  chosen in the same stage-2 problem. The updating equation reads:

$$\pi_{0|0,s} = \pi_{0|0} + \frac{\Sigma_{0|0}}{\Sigma_{0|0} + \Sigma_z + \sigma_\eta^2} \left( S - \pi_{0|0} \right)$$

where  $\Sigma_z$  is the unconditional variance of  $z_t$ .<sup>1</sup> The treatment effect,  $\Sigma_{0|0} / (\Sigma_{0|0} + \Sigma_z + \sigma_\eta^2)$ , is smaller than in the baseline model because the signal during the survey is suboptimal for the agent, implying less attention during the survey compared with the baseline. We solve this version of the model numerically using the code of Afrouzi and Yang (2021). We find qualitatively the same comparative statics of the treatment effect with respect to  $\gamma$  and  $\sigma_{\varepsilon}^2$  as in the baseline model. It does not matter for the comparative statics if the agent cares about an inflation rate that is different from the official inflation rate.

Respondents are incentivized to pay attention during the survey. The result in Proposition 1 in the paper is robust to giving agents additional monetary incentives during the experiment. Suppose that the first term in the loss function in stage 2 is not  $\gamma \Sigma_{0|0,s}$  but  $(\gamma + \alpha) \Sigma_{0|0,s}$ , where  $\alpha \ge 0$  captures additional monetary incentives that are provided during the experiment (e.g., agents are given a show-up compensation and some money is deducted from this show-up compensation when the action, such as the reported posterior, is incorrect). Agents who are called into the survey and receive the information treatment then solve the following RI problem at the end of period t = 0 (during the survey):

$$\min_{\sum_{0|0,s}} \left\{ \left( (\gamma + \alpha) \Sigma_{0|0,s} + \lambda ln \left( \frac{\Sigma_{0|0}}{\Sigma_{0|0,s}} \right) \right) + \beta V_1 \left( \Sigma_{1|0} \right) \right\}$$

subject to

$$\begin{split} \Sigma_{0|0,s} &\leq \Sigma_{0|0} \\ \Sigma_{1|0} &= \rho^2 \Sigma_{0|0,s} + \sigma_{\varepsilon}^2 \\ \Sigma_{0|0} \ given. \end{split}$$

<sup>&</sup>lt;sup>1</sup>In principle, it is the prior variance of  $z_t$  at the beginning of the survey, equal to the steady-state posterior variance from stage 1, but the agent had no incentive to pay attention to  $z_t$  in stage 1. Note also that the updating equation for the official inflation rate (as opposed to the inflation rate that the agent cares about) is very similar; the numerator reads  $\Sigma_{0|0} + \Sigma_z$  instead of  $\Sigma_{0|0}$ .

The model in the paper is the special case of  $\alpha = 0$ . The statements of the RI problems in stages 1 and 3 do not change, since the additional monetary incentives are only given during the experiment.

If the inequality constraint (3) in the paper is non-binding in period t = 0 before the survey, the first-order condition for  $x \equiv \Sigma_{0|0}$  is again given by equation (16) in the paper (Proof of Proposition 1) and the expression for  $\left(\frac{\partial x}{\partial \gamma}/x\right)$  is again given by equation (18) in the paper (Proof of Proposition 1), since the additional monetary incentives during the experiment do not affect agents' choices before the experiment.

Next, if the inequality constraint  $\Sigma_{0|0,s} \leq \Sigma_{0|0}$  is non-binding during the survey and the inequality constraint (3) in the paper is non-binding in period t = 1 after the survey, the first-order condition for  $y \equiv \Sigma_{0|0,s}$  reads

$$\gamma + \alpha - \lambda \frac{1}{y} + \beta \mu \frac{1}{y + \frac{\sigma_e^2}{\rho^2}} = 0.$$
<sup>(1)</sup>

It follows from the implicit function theorem that

$$\frac{\frac{\partial y}{\partial \gamma}}{y} = -\frac{1}{\mu} \left[ \frac{\lambda}{\mu} \frac{1}{y} - \beta \frac{y}{\left(y + \frac{\sigma_{\varepsilon}^2}{\rho^2}\right)^2} \right]^{-1}.$$
(2)

Using equation (1) to substitute for  $\frac{\lambda}{\mu}\frac{1}{y}$  in equation (2) yields

$$\frac{\frac{\partial y}{\partial \gamma}}{y} = -\frac{1}{\mu} \left[ \frac{\gamma + \alpha}{\mu} + \beta \frac{\frac{\sigma_{\varepsilon}^2}{\rho^2}}{\left(y + \frac{\sigma_{\varepsilon}^2}{\rho^2}\right)^2} \right]^{-1}.$$
(3)

Equations (1)-(3) are the generalizations of equations (19)-(21) in the paper (Proof of Proposition 1) for the case of  $\alpha \ge 0$ .

Comparing the expression for  $\left(\frac{\partial y}{\partial \gamma}/y\right)$  given by equation (3) to the expression for  $\left(\frac{\partial x}{\partial \gamma}/x\right)$  given by equation (18) in the paper (Proof of Proposition 1) yields again that  $\left(\frac{\partial y}{\partial \gamma}/y\right)$  is strictly smaller than  $\left(\frac{\partial x}{\partial \gamma}/x\right)$  in absolute value. It follows from equation (15) in the paper (Proof of Proposition 1) that the treatment effect *G* is strictly decreasing in  $\gamma$ .

In sum, the presence of the additional monetary incentives during the experiment ( $\alpha > 0$ ) reduces  $y \equiv \Sigma_{0|0,s}$  because agents are paying more attention during the experiment. See equation (1). However, the additional monetary incentives during the experiment does not affect the result in Proposition 1: the treatment effect G is strictly decreasing in  $\gamma$ .

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