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The tipping point: interest rates and financial stability

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Abstract

This paper studies how interest rates impact bank stability in a standard banking model. There are two opposite effects. While higher rates widen banks' interest margins, they also reduce the value of their long-term assets. First, the paper characterizes conditions under which an effect dominates. Second, it shows that banking crises are triggered by interest rates crossing a tipping point. Quantitatively, I find that the effect on interest margins is dominant. Hence, low rates are the threat to bank stability. Moreover, I estimate the tipping point at 0.32% for the US economy in the decade before the Global Financial Crisis.

Keywords: Financial crisis, deposit franchise, effective lower bound, maturity mismatch.

JEL Codes: E43, E50, G21.

Non-technical Summary

The secular decline in interest rates over the last three decades and increased volatility since inflation broke out in advanced economies in 2021 have stirred a debate about the wider economic effects of the level of interest rates. Particularly, policy-makers have expressed concerns about effects of interest-rate shocks on financial stability.

This paper studies how interest rates affect the likelihood of banking crises through the lens of a standard banking model by Diamond and Dybvig (1983). It finds that interest rates have two opposite effects on the health of financial intermediaries. On the one hand, an increase in interest rates reduces the value of the long-term assets that they hold. This is detrimental to banks' health. On the other hand, banks' interest margins become wider since deposit rates do not increase one-for-one with increases in interest rates, a phenomenon also documented in Drechsler et al. (2017). This effect supports banks.

The paper has two key analytical findings. First, it characterizes the conditions under which one of the effects dominates. This boils down to comparing the average duration of banks' assets with the effective duration of bank deposits, given by the average period of time deposits remain at the bank before they are withdrawn. If the former is greater than the latter, then the asset-duration effect dominates and shocks that increase interest rates can lead to bank instability. If the opposite is true, then the interest-margin effect dominates and low interest rates threaten bank stability.

Second, the paper studies the precise relationship between interest rates and banking crises. It finds that there is a tipping point. When interest rates cross it, then banks become unstable. If the dominant effect is the interest-margin effect, then the tipping point is a lower bound on the level of interest rates that is consistent with a stable banking system. Otherwise, it is an upper bound.

Finally, I carry out a quantitative exercise, calibrating the model's parameters to match characteristics of US data. This analysis reveals that the interest-margin effect is the dominant force. For this not to be the case, the average duration of bank assets has to be four times as high as the 4 years that we observe. Hence, the economy's tipping point is a lower bound on interest rates. In the calibrated model, a short-term risk-free nominal interest rate below 0.3% leads to a banking crisis. The existence of such tipping point could explain why central banks typically operate subject to an effective lower bound. Interestingly, longer-term assets on banks' balance sheets push down the tipping point and therefore stabilize the banking system in this economy. This points to side effects of liquidity regulation in terms of banks' exposure to interest-rate risk.

1 Introduction

Interest rates in advanced economies have been in secular decline for the last three decades (Holston et al., 2017; Del Negro et al., 2019). Moreover, since inflation broke out in 2021, the volatility of interest rates has increased greatly. These economic developments have kindled an interest in the effects of interest rates on the stability of the financial system. In particular, policy-makers have expressed concerns about financial stability, pointing to low rates of interest as well as recent increases in interest rates as potential threats (Committee on the Global Financial System, 2018; European Central Bank, 2021; International Monetary Fund, 2022).

Economists hold contrasting views about the effect of interest rates on the health of financial intermediaries. A traditional view focuses on the maturity mismatch between the assets and liabilities of banks as the key measure of their exposure to interest-rate risk (Kaufman, 1984; Basel Committee, 2016). According to this view, low rates are a boon for banks. Since bank assets typically have a longer duration than their liabilities, low interest rates feed into banks' funding costs faster than they lower their asset returns and thus benefit banks. By this channel, low rates boost the equity value of banks in macro-finance models that study interest-rate policies (Gertler and Karadi, 2011; Akinci et al., 2022). More recently, a different view, which emphasizes the exposure of banks' interest margins to interest-rate risk, has affirmed itself. A large literature documents a positive association of banks' interest margins with interest-rate levels (Driscoll and Judson, 2013; Yankov, 2014; Borio et al., 2017; Claessens et al., 2018). This is explained by banks' greater ability to exploit monopoly power in the deposits market when interest rates are high (Drechsler et al., 2017). Drechsler et al. (2021) show empirically that both views carry weight. That is, both a bank's maturity mismatch and the interest-rate sensitivity of its interest margins are needed to account empirically for changes in the value of bank equity after changes in interest rates.

This paper's contribution is to show that the two channels discussed in the literature above also naturally feature in a standard model of banking crises. Banks' maturity mismatch as well as the sensitivity of their interest margins determine how interest rates affect bank stability. Two key analytical results emerge from the model. First, a simple parametric condition tells us which effect dominates and hence whether high or low rates of interest threaten bank stability overall. Second, interest rates lead to bank instability when they cross a tipping point, for which the model yields a closed-form solution. Under a parametrization that matches characteristics of US data, the model can be used to determine quantitatively the dominant effect and tipping point. I find that bank stability is threatened by low rates, not high rates, and that the tipping point for the nominal short-term risk-free rate of interest is 0.32%.

I base my study of bank stability on the canonical model developed in Diamond and Dybvig (1983). Households are exposed to idiosyncratic liquidity shocks, which make them unsure about the timing of their consumption. Banks offer them deposit contracts, which provide insurance against this liquidity risk. However, because of the demandable nature of deposits, banks are crisis-prone. If too many depositors withdraw at once, banks fail. In this setting, banking crises can be self-fulfilling or driven by bad fundamentals. Following Allen and Gale (1998), I let the existence of credible deposit insurance forestall panics and thus I focus on fundamental-driven crises. In other words, banking crises take place in this paper when banks are insolvent.

In this framework, banks have two characteristics that interact with changes in interest rates: they hold long-term assets and their interest margins depend positively on the level of the interest rate. In the model, long-term bonds are issued by firms and held by banks. Since there is perfect competition in the bonds market, bonds pay an interest rate that is equal to the return on firms' capital. By a standard asset-revaluation effect, outstanding long-term bonds go down in price when interest rates go up, so that their return adjusts to the new prevailing interest rate. This harms banks. To study long-term bonds in a way that is tractable and can be mapped into empirical objects such as the average duration of bank assets, I extend the canonical model's time horizon to infinity and model long-term bonds as perpetuities with a decaying coupon (Woodford, 2001).

Banks' interest margins are interest-rate sensitive because of two of the model's core features. First, households' liquidity risk is greater when interest rates are higher. Liquidity risk is a household's risk of being hit by a shock that forces it to make expenditures early on and therefore forgo future returns on its wealth. High interest rates imply better returns on wealth and thereby make a liquidity shock costlier. This pushes up households' demand for liquidity-risk insurance from their bank. Setting interest margins is how banks effectively implement the insurance, since interest margins transfer resources away from households that are not hit by the liquidity shock at any given point in time to households that are. It follows that, if interest rates are higher and therefore demand for liquidity-risk insurance is stronger, the optimal deposit contract features wider interest margins. In other words, interest rates on bank deposits optimally move less than one to one with other interest rates. Second, the model features a lower bound on the interest rate paid on bank deposits. This is an incentive-compatibility constraint on the bank, which is due to households being able to store resources outside the banking system. Banks do not set deposit rates below such lower bound, lest they trigger mass withdrawals. Once this constraint is binding, reductions in interest rates lead to reductions in banks' interest margins.

Interest rate haves two competing effects on bank solvency. An increase widens interest margins, but also represents a negative windfall for banks holding long-term assets. As a first key result, I derive a sufficient condition for dominance of one of the two effects. It turns out that dominance depends on a comparison of the duration of bank-held assets with the probability of liquidity shocks hitting households. Since the probability of liquidity shocks also determines how long the average bank deposit stays in the bank before it is withdrawn, the comparison is equivalently between the duration of bank assets and the effective duration of bank deposits. This is a simple comparison conceptually and one that can be matched to empirical counterparts.

The second key result uncovers the precise relationship between interest rates and bank solvency. The bank becomes insolvent and a crisis ensues once the economy's interest rate crosses a tipping point. This tipping point depends on characteristics of the banking system and has a simple analytical formula. Frequent liquidity shocks and short-term bank assets make the banking system more vulnerable to interest-rate shocks if the interest-margin effect dominates. The opposite is true if the asset-revaluation effect dominates. Interestingly, in the former case liquidity regulation imposed on banks with the aim of reducing their maturity mismatch exacerbates their exposure to interest-rate risk.

The model's parameters, which determine the dominance condition and tipping point, are easily matched with empirical counterparts. Therefore, I can use bank data and results from the empirical banking literature to quantify them. Using US data from the decade before the Global Financial Crisis, the paper's quantitative exercise indicates that the interest-margin effect is dominant. Hence, it is low interest rates that may lead to a banking crisis. Indeed, the model's bank requires an average assets duration around four times as high as the observed 4.5 years to be completely hedged against the potent reduction in interest margins caused by reductions in interest rates. As for the tipping point, I find a value of 0.32%. A shock that reduces the nominal short-term risk-free interest rate permanently below 0.32% makes the banking system insolvent. This quantification has many limitations: banks are highly stylised in the model, real-world shocks do not hit the interest rate in a vacuum and are not permanent. Nevertheless, it illustrates the ease with which bank data and the empirical literature can be mapped into the parameters of the model.

Related literature. In the footsteps of the seminal paper by Diamond and Dybvig

(1983), I focus on bank instability that is inherent to the supply of liquid demandable deposits. I wish to study the effect of a real shock, an interest-rate shock, on the probability of a crisis. However, this exercise is complicated by the fact that the model features multiple equilibria.[1] Allen and Gale (1998) solves the issue by focusing exclusively on fundamental-driven crises, i.e. crises that cannot be solved merely with better coordination of depositors. An alternative option to pin down uniquely the conditions for a banking crisis is given by global-games techniques, as in Goldstein and Pauzner (2005). I adopt the former approach and justify it with the presence of deposit insurance ruling out panics in equilibrium. Baron et al. (2021) show empirically that bank panics are not necessary for banking crises to have severe economic consequences.

Mine is not the first paper in the literature to model banking crises in a fully dynamic economy. Some adopt an overlapping-generations framework (Bencivenga and Smith, 1991; Ennis and Keister, 2003). Others model infinitely-lived agents (Gertler and Kiyotaki, 2015; He and Manela, 2016; Segura and Suárez, 2017; Mattana and Panetti, 2021). The latter approach, which I also adopt, improves the match between model outcomes and data, and therefore allows for better calibration.

Other papers focus on the relationship between interest rates and bank stability. Recently, Akinci et al. (2022) studied this question in a model that features rich macroeconomic dynamics. However, banks' interest margins do not depend on the level of the interest rate in their model. Hence, only high rates can bring about bank instability by pushing down the value of banks' long-term assets. Hellwig (1994) finds that issuance of demandable deposits exposes banks to excessive interest-rate risk relative to the socially optimal level and characterizes the optimal deposit contract under interest-rate risk. I take a positive outlook, studying the effect of interest-rate shocks under a standard deposit contract. Interestingly, I find that an appropriate level of maturity mismatch can hedge a bank's interest-rate exposure even in this case. Di Tella and Kurlat (2021) embed monopolistic banks, whose profitability is increasing in the level of the interest rate, in a macroeconomic model. They find that these banks have an incentive to hold a maturity mismatch to hedge their exposure to interest rates.

The effect of the interest-rate level, particularly of low levels of interest, on bank profitability has been studied by a large empirical literature. Using a comprehensive dataset of banks, Borio et al. (2017) find that the effect of interest rates on bank profitability is positive and stronger at lower levels of the interest rate. Claessens et al. (2018)

[1] A strand of the literature studies the effect of financial crises on economic outcomes, embracing equilibrium multiplicity (Gertler et al., 2016; Quadrini, 2017; Gertler et al., 2020). I study the effect of real shocks on the likelihood of a crisis.

find that the positive effect on banks' net interest margin of an interest-rate increase is twice as large when rates are low. Bats et al. (2020) and Ampudia and Van den Heuvel (2022) document that in low-rate environments interest-rate increases lead to smaller reductions, and even increases, in bank valuations, particularly for banks more heavily funded with deposits. These results are in line with my quantitative findings.

The wider implications of low interest rates have attracted a flurry of papers. Many find that financial institutions react to low interest rates by lending to riskier counterparties (Maddaloni and Peydró, 2011; Dell'Ariccia et al., 2014; Jiménez et al., 2014; Di Maggio and Kacperczyk, 2017; Martinez-Miera and Repullo, 2017; Heider et al., 2019; Basten and Mariathasan, 2020). Others focus on the quantity of bank lending and investment in the economy (Amzallag et al., 2019; Eggertsson et al., 2019; Ulate, 2021; Abadi et al., 2022; Altavilla et al., 2022). My paper abstracts from risk-taking and credit-supply considerations. Nonetheless, it finds quantitatively that low rates of interest can harm bank stability.

Paper outline. In the next section, I describe preferences and technologies. In section 3, I illustrate the model's frictions and the agents' optimization problems. In section 4, I define the equilibrium and solve for it under perfect foresight. Section 6 studies the effects of an interest-rate shock. In section 7, I compare the model's outcomes to data and conduct a quantitative exercise. Proofs of propositions and corollaries are in appendix A. Appendices B and C contain extensions of the model.

2 Preferences and Technology

A unit mass of households inhabits the economy. They are born at date zero and are infinitely-lived. They enjoy consumption according to felicity function u, which features standard properties $u' > 0$ and $u'' < 0$ and a constant coefficient of relative risk aversion $1/\alpha > 1$. Each household enjoys consumption only at one point in time from date 1 on. This point in time θ is random and idiosyncratically distributed according to a geometric distribution with success rate $\phi \in (0, 1)$. At time t, I call households with $\theta = t$ impatient. A household, which has not consumed already, has a probability ϕ of turning impatient and enjoying consumption at any given date. Since the timing of households' desire to consume is idiosyncratic, by the law of large numbers the share of impatient households at any given date is deterministic. In particular, a share ϕ of those households that have not consumed yet turns impatient at a given date. The household's once-in-a-lifetime desire to consume can be interpreted as the random occurrence of an unusually large expense, which the household finances by withdrawing from its bank account. This type of utility was introduced by Diamond and Dybvig (1983), albeit with a finite horizon, and has become standard in the theoretical banking literature. The household's expected utility is given by

$$\sum_{t=1}^{+\infty} (1-\phi)^{t-1} \cdot \phi \cdot u(C_t). \tag{1}$$

The economy features a single good that can be consumed or invested. There are two investment technologies: the productive technology and the storage technology. For any good invested in it, the productive technology produces $1+\rho$ goods at the following date, with $\rho > 0$. Throughout the paper, I refer to ρ as the economy's interest rate, since by arbitrage it will also be the return on bonds. As for the storage technology, a stored good today gives one unit of the good tomorrow.

3 Economy

The economy features four key frictions. First, liquidity risk, i.e. the risk of becoming impatient relatively early and therefore having little time to accumulate wealth before consuming, is insured by bank-issued deposit contracts rather than directly among households. The micro-foundation for this is that the individual liquidity shock is not publicly observable and therefore not contractible. Banks emerge as a way to insure liquidity risk by means of an incentive-compatible contract.

Second, banks must offer a deposit contract as we observe them in reality. That is, deposits are not contingent on the state of the world and households have the right to withdraw their deposit balance unless the bank is bankrupt.[2]

Third, households can only invest in their bank's deposits or in storage. Direct finance is ruled out. This can be justified, for instance, with households' inability to monitor firms. Moreover, the deposit contract is exclusive. That is, once a household has opened an account at one bank, it cannot switch to another one. This can be rationalized with a sufficiently high switching cost. Jacklin (1987) emphasizes that restrictions to households' investment opportunities are essential for banks to provide liquidity-risk insurance in this class of models. If households could costlessly invest directly or defect to another bank, then the insurance mechanism would unravel and with it the role of banks in the economy.[3]

[2] As shown by Peck and Shell (2003), it is possible to relax these assumptions and still study banking crises. Nonetheless, I adopt the most standard set up.

[3] There is scope to relax this assumption with, for example, a cost for households to directly participate

Fourth, banks do not invest directly in investment technologies. It is the role of firms to invest in the productive technology. Banks lend to firms in the form of long-term bonds with a fixed duration. This relationship between banks and firms is empirically relevant and has been theoretically justified as a means to minimize banks' costs of monitoring firms (Gale and Hellwig, 1985). It is worth stressing that there is no cost in this model associated with early liquidation of long-term bonds by banks. Appendix B shows that the introduction of a liquidation cost for long-term bonds does not change the paper's main results.

The rest of this section describes the problems solved by the agents of the economy: firms, households and banks.

3.1 Firms

Firms borrow in a competitive bond market and invest in the productive technology. The bonds are perpetuities that pay a decaying coupon, as in Woodford (2001). A new bond issued at time t pays a coupon δ^{j-1} at every date $t+j$ for $j \geq 1$, with $\delta \in (0, 1]$. Notice that, as of time t, an outstanding bond issued at time $t-2$ is equivalent in terms of remaining cashflows to δ outstanding bonds issued at time $t-1$. This keeps the model tractable by allowing us at a given time t to only keep track of the quantity of bonds issued at time $t-1$ that would generate the same cashflows as the firm's actual outstanding bonds. I denote this quantity B_t^f. With $\delta \to 0$, the bond is effectively a one-period bond. In this case, outstanding bonds of the firm have zero duration, defined as the average maturity of the coupons weighted by the present discounted value of the coupon paid at each maturity. If the short-term bond is outstanding at a given date t, it means that the only coupon is due immediately at time t. The general formula for the duration of outstanding bonds is $\delta/(1+\rho-\delta)$. I use this formula in the quantitative exercise in section 7.

Taking price q_t of bonds issued at time t as given, the firm chooses how much to borrow B_{t+1}^f and how much to invest in the productive technology K_{t+1} in order to maximise the discounted value of profits

$$\sum_{t=0}^{+\infty} \left(\frac{1}{1+\rho}\right)^t \cdot \Pi_t \tag{2}$$

in financial markets (Diamond, 1997).

subject to budget constraints

$$\Pi_t + K_{t+1} + (1+\delta\cdot q_t)\cdot B_t^f = q_t \cdot B_{t+1}^f + (1+\rho)\cdot K_t \quad \text{for all } t \geq 0, \tag{3}$$

initial conditions

$$B_0^f = K_0 = 0, \tag{4}$$

and a boundary condition[4]

$$\lim_{T\to+\infty} \left(\frac{1}{1+\rho}\right)^T \cdot \left[K_{T+1} - q_{T+1}\cdot B_{T+1}^f\right] = 0. \tag{5}$$

Arbitrage by the firm between bonds and capital implies that $1+\rho = (1+\delta\cdot q_{t+1}^*)/q_t^*$ in equilibrium. Together with a condition that rules out equilibrium bubbles

$$\lim_{T\to\infty} q_T^* \neq \pm\infty, \tag{6}$$

this pins down the price of new bonds as

$$q_t^* = \frac{1}{1+\rho-\delta} \quad \text{for all } t \geq 0. \tag{7}$$

When the interest rate falls, the price of a new bond goes up.

3.2 Households and banks

At time 0, banks competitively offer deposit contracts to households. A deposit contract specifies a contingent stream of deposit rates $\{r_t\}_{t=0}^{+\infty}$. Moreover, it allows households to withdraw at any point in time any amount up to their deposit balance D_t. Households choose the deposit contract that maximizes their expected utility.

After they accepted a deposit contract, households' optimal withdrawing behaviour is as follows. Patient households, with $\theta \neq t$, do not withdraw their deposits at time t, as long as the return on deposits r_t is larger than the zero return on storage. Households that turn impatient, with $\theta = t$, withdraw all their deposits immediately at time t.

By perfect competition in the supply of deposit contracts at date zero, the prevail-

[4]The boundary condition is given by the combination of a Ponzi condition and a transversality condition.

ing deposit contract maximises households' expected utility

$$\sum_{t=1}^{+\infty} (1-\phi)^{t-1} \cdot \phi \cdot u(D_t). \tag{8}$$

As long as the incentive-compatibility constraints

$$r_t \geq 0 \quad \text{for all } t \geq 0 \tag{9}$$

hold, households only withdraw once they turn impatient. Hence, the law of motion of patient households' deposit balances is given by

$$D_{t+1} = (1 + r_t) \cdot D_t \quad \text{for all } t \geq 0. \tag{10}$$

At time zero, households deposit their unit endowment, which implies

$$D_0 = 1. \tag{11}$$

Since the bank has no own resources, namely

$$B_0 = 0, \tag{12}$$

the bank buys bonds at time zero with the households' endowments. That is, the bank's time-zero budget constraint is

$$q_0 \cdot B_1 \leq 1. \tag{13}$$

The bank's budget constraints at the following dates are

$$q_t \cdot B_{t+1} + \phi \cdot (1-\phi)^{t-1} \cdot D_t \leq (1 + \delta \cdot q_t) \cdot B_t \quad \text{for all } t \geq 1, \tag{14}$$

and the boundary condition is

$$\lim_{T \to +\infty} \left(\frac{1}{1+\rho} \right)^T \cdot q_T \cdot B_{T+1} = 0. \tag{15}$$

The bank's problem can be broken down in infinitely many subgames indexed by a starting time $j \geq 0$. Subgame 0 corresponds to the bank's problem as described above. In subgame $j > 0$, the bank maximises $\sum_{t=j}^{+\infty} (1-\phi)^{t-1} \cdot \phi \cdot u(D_t)$ subject to the incentive-compatibility constraints (9) and the intertemporal budget constraint

$$\phi \cdot \sum_{s=j}^{+\infty} \left(\frac{1-\phi}{1+\rho}\right)^{s-j} \cdot \prod_{k=j}^{s-1} (1+r_k) \cdot (1-\phi)^{j-1} \cdot D_j \leq (1+\delta \cdot q_j) \cdot B_j, \tag{16}$$

obtained by combining budget constraints (10) and (14) and the boundary condition (15). The initial conditions B_j and D_j are given. Subgames are useful because they clarify how state-contingency of the deposit contract works. If at a given time t the economy is hit by a shock, the bank's response to the shock solves subgame t. Importantly, the bank cannot immediately change the amount of outstanding deposits or the quantity of bonds it holds, because they are pinned down by initial conditions. But it is free to respond to the shock by changing the deposit rate. A focus on subgames of the bank's problem is valid, because a solution to the bank's problem is also a solution to every subgame of the bank's problem, as we will see in the next section.

Bank failure. If at a date t there is no sequence of deposit rates that is incentive compatible and feasible (i.e., it satisfies the bank's intertemporal budget constraint), then the bank fails at time t.

On the other hand, if the bank can offer a feasible and incentive-compatible sequence of deposit rates, then the bank is solvent and it does not fail. To ensure that solvent banks do not fail, I assume that the economy features deposit insurance, which makes households whole for their deposit balance in case of bank failure. Thus, patient households do not have an incentive to withdraw even if they observe other patient households withdrawing. With this assumption, I restrict the analysis to fundamental-driven failure, as in Allen and Gale (1998). This focus is justified by the empirical observation that a strong safety net for banks exists in advanced economies, both in terms of deposit insurance and of lender-of-last-resort policies. These are very effective at deterring runs on solvent banks in this class of models. Indeed, solvent banks can be made run-proof costlessly. To the contrary, the rescue of an insolvent bank involves an equilibrium disbursement of resources. This disbursement is costly as it reduces resources available for alternative uses, such as the provision of public goods by the government (Allen et al., 2018). Nonetheless, I carry out a broader analysis, which encompasses panic-driven bank failure, in appendix C. I find that the presence of bank panics leaves the main theoretical and quantitative conclusions of the paper unchanged.

As of time zero there is always a feasible and incentive-compatible deposit contract that the bank can offer. For instance, it can set $r_t = \rho$ for all $t \geq 0$. This is feasible and, since $\rho > 0$, it is also incentive compatible. This implies that in an equilibrium with perfect foresight the bank never fails.

At the heart of the paper are the consequences of a shock hitting the economy at a later date $t \geq 1$. To study these, we must analyse subgame t of the bank's problem.

Proposition 1. *At time $t \geq 1$, the bank does not fail in equilibrium if and only if condition*

$$(1 + \delta \cdot q_t^*) \cdot B_t \geq \frac{\phi \cdot (1+\rho)}{\phi + \rho} \cdot (1-\phi)^{t-1} \cdot D_t \tag{17}$$

holds. We call condition (17) the solvency condition.

The solvency condition tells us whether the bank can offer a feasible and incentive-compatible deposit contract in subgame t, as a function of initial conditions B_t and D_t, equilibrium price q_t^* and parameters. The relevant interest rate ρ is the one prevailing from time t on. Intuitively, the value of the bank's assets must be high enough relative to its outstanding deposits. Notice that a higher interest rate allows the bank to hold fewer bonds relative to outstanding deposits without failing. As we will see in detail in section 5, this is because the bank makes higher intermediation profits when the interest rate is high.

4 Equilibrium

In equilibrium, the representative firm and bank solve their optimization problems. As soon as they turn impatient, households withdraw their deposits. As long as they are patient, households do not withdraw unless storage offers a better return than deposits. The market for bonds clears.

Definition 1. *A perfect-foresight equilibrium is a sequence $\{B_t^f, B_t, D_t, K_t, q_t, r_t, \Pi_t\}_{t=0}^{+\infty}$ such that:*

1. *Given $\{q_t\}_{t=0}^{+\infty}$, the representative firm chooses $\{B_t^f, K_t, \Pi_t\}_{t=0}^{+\infty}$ to maximise its value (2) subject to budget constraints (3), initial conditions (4) and boundary condition (5).*

2. *Given $\{q_t\}_{t=0}^{+\infty}$, the representative bank chooses $\{B_t, D_t, r_t\}_{t=0}^{+\infty}$ to solve every subgame of its problem of maximising household expected utility (8) subject to incentive-compatibility constraints (9), initial conditions (11) and (12), budget constraints (10), (13) and (14), and a boundary condition (15), if such solution exists.*

3. *If a subgame starting at time t of the representative bank's problem does not admit a solution, then the bank sets $B_{s+1} = 0$ for all $s \geq t$ and households immediately receive their deposit holdings paid out in full from deposit insurance.*

4. *Prices $\{q_t\}_{t=0}^{+\infty}$ ensure that $B_{t+1}^f = B_{t+1}$ for all $t \geq 0$ and are subject to no-bubble condition (6).*

I denote perfect-foresight equilibrium values with stars.

I assume that bank deposits are fully covered by deposit insurance. While this assumption is important because it rules out panic runs, it is not necessary to specify formally how deposit insurance is financed. That is because bank failure does not take place along the perfect-foresight equilibrium. Nonetheless, we can think that deposit insurance is backed by unlimited taxation power.

Equilibrium outcomes under perfect foresight are characterised by the following proposition.

Proposition 2. *Perfect-foresight equilibrium implies that*

$$1 + r_t^* = (1+\rho)^\alpha \quad \text{for all } t \geq 1, \tag{18}$$

$$(1 + \delta \cdot q_t^*) \cdot B_t^* = \frac{\phi \cdot (1+\rho)^{1-\alpha}}{(1+\rho)^{1-\alpha} - (1-\phi)} \cdot (1-\phi)^{t-1} \cdot D_t^* \quad \text{for all } t \geq 1 \tag{19}$$

and q_t^* is given by (7).

Equation (18) corresponds to the equation that determines the ratio between late types' consumption and early types' consumption in the standard three-date Diamond-Dybvig model. As in that model, the equilibrium deposit rate is lower than the return on bank assets and is decreasing in households' coefficient of relative risk aversion, given by the inverse of α. In the polar case with infinite risk aversion (i.e., $\alpha \to 0$), the bank pays a zero deposit rate. With a zero deposit rate, households are certain about their level of consumption, regardless of the timing of their consumption. For infinitely risk-averse households, this is optimal as it eliminates all liquidity risk. Equation (19) describes the value of bank assets relative to outstanding deposits along the equilibrium path. It plays a similar role to the equation that determines the deposit rate between time 0 and time 1 in the standard three-date Diamond-Dybvig model. In fact, it can be alternatively viewed as determining how many deposits the bank creates in equilibrium. More deposits improve insurance across households, since they increase the consumption of households that become impatient relatively early. Since more risk-averse households value this insurance more, the quantity of outstanding deposits is increasing in the coefficient of relative risk aversion. Notice that a bank that creates many deposits will necessarily pay a low deposit rate on them. This is how the two equations are connected and it follows directly from the solvency constraint.

On the perfect-foresight equilibrium path, the incentive-compatibility constraints are not binding, as we can see in equation (18). Since incentive compatibility is never violated, bank failures do not occur in equilibrium. Indeed, looking at equation (19) we can confirm that the solvency condition, (17), is satisfied at every point in time.

At time 1, the economy effectively reaches a steady state, in which important endogenous variables are stable over time. In fact, both the deposit rate and the value of bank-held bonds per unit of outstanding deposits do not change. This is a surprising finding, since elements of the model, such as the number of impatient households in a given period, have a trend. This characteristic of the model helps in terms of analytical tractability. Moreover, it is easier to find empirical counterparts to objects that are stable.

5 Deposit-franchise interpretation

This section provides novel economic intuition for the results found in the previous sections. In particular, it focuses on solvency condition (17) and on the model's perfect-foresight equilibrium conditions, summarized in proposition 2. The economic intuition sheds light on the nature of the banking contract and on the mechanism that leads to bank failure. Moreover, it helps to map some of the model's parameters into empirical objects. This is key for the quantitative exercise in section 7.

Let us introduce two new definitions. First, the bank's interest margin m_t, given by

$$1 + m_t = \frac{1+\rho}{1+r_t}. \tag{20}$$

This is approximately the difference between the return that the bank earns on its bonds and the interest rate that it pays to its depositors. Second, we introduce the per-unit deposit franchise $f\left(\{m_j\}_{j=t}^{+\infty}\right)$. This is defined as the difference between the present discounted value of cashflows associated with the average unit of bank deposits outstanding at a given time $t \geq 1$ and the deposits' face value of -1. For the bank, a unit of deposits outstanding at time t engenders an immediate outflow of one unit of the consumption good with probability ϕ, an outflow of $1 + r_t$ units of the consumption good at time $t+1$ with probability $\phi \cdot (1-\phi)$, an outflow of $(1+r_t)\cdot(1+r_{t+1})$ at time $t+2$

with probability $\phi \cdot (1-\phi)^2$, and so on. Therefore, we can write the per-unit deposit franchise as

$$f\left(\{m_j\}_{j=t}^{+\infty}\right) = -\phi \cdot \sum_{j=t}^{+\infty} \left(\frac{1-\phi}{1+\rho}\right)^{j-t} \cdot \prod_{s=t}^{j-1}(1+r_s) - (-1) = 1 - \phi \cdot \sum_{j=t}^{+\infty} \frac{(1-\phi)^{j-t}}{\prod_{s=t}^{j-1}(1+m_s)}. \tag{21}$$

The concept of deposit franchise captures the discrepancy between the actual value of an outstanding deposit for the bank and the deposit's face value. Notice that the deposit franchise is zero if $\phi \to 0$, in which case deposits are immediately withdrawn and therefore the bank has no time to earn an interest margin, or if the bank's interest margin is zero. As long as the bank earns a positive interest margin and households keep their deposits in the bank for some time (i.e., $\phi > 0$), then the deposit franchise is strictly positive. Intuitively, a deposit's face value does not account for the intermediation profits that the bank earns over time on the average unit of deposits. These are captured by the deposit franchise.

The deposit franchise is a concept with a long tradition in the banking literature and it is widely studied as a driver of bank value, for instance in Hutchison and Pennacchi (1996) and recently in Egan et al. (2022). While there are papers, such as Hellmann et al. (2000), that discuss the relationship between a bank's deposit franchise and its risk-taking behaviour, to the best of my knowledge this paper is the first to identify the important role that a bank's deposit franchise plays in the standard model used to study bank stability.

Solvency. Solvency condition (17) lays out the minimum value of the bank's assets such that the bank does not succumb to failure at a given point in time. From the mathematical expression, it is clear that with a higher interest rate ρ the bank can, all else equal, get by with fewer assets. We can use the deposit-franchise interpretation of the model to better understand this effect. Let us use the definitions above to rephrase the condition as follows.

Corollary 1. *At time $t \geq 1$, the bank does not fail in equilibrium if and only if condition*

$$(1+\delta \cdot q_t^*) \cdot B_t + f\left(\{\rho\}_{j=t}^{+\infty}\right) \cdot (1-\phi)^{t-1} \cdot D_t \;\geq\; (1-\phi)^{t-1} \cdot D_t \tag{22}$$

holds.

The bank does not fail as long as the value of its bond portfolio plus the bank's deposit franchise are enough to fully cover the face value of its outstanding liabilities. In this sense, we can think of the deposit franchise as an additional asset that the bank can use to pay off its debts as they come due. For the purposes of bank solvency, the deposit franchise is evaluated at the maximum incentive-compatible interest margin ρ. That is, what counts is the deposit franchise assuming that the bank pays a zero deposit rate to its depositors forever. This is the relevant notion of the deposit franchise for solvency considerations because, if a bank is hit by a shock at a given time t and it must either reduce the deposit rate or fail, then, as according to its state-contingent deposit contract, it reduces the deposit rate. Remember that the bank re-optimizes the deposit contract at every point in time, given the state of the world. This kind of state contingency of the deposit contract is optimal from the perspective of households, given that their outside option is storage and therefore bank failure is costly for them. But incentive-compatibility constraints set a limit to the extent to which deposit rates can be lowered. At most deposit rates can go down to zero. If that is not enough to keep the bank solvent, then the bank fails.

From equation (22), we can see the economic channel through which a higher interest rate ρ makes banks more stable, all else equal. It improves the bank's ability to earn interest margins. And interest margins, reflected in a larger deposit franchise, can be used in the same way as the bank's bond portfolio to pay back deposits as they are withdrawn.

Equilibrium. Proposition 2 describes the economy's perfect-foresight equilibrium. In the language of the deposit franchise, the equilibrium conditions can be rewritten as follows.

Corollary 2. *Equilibrium implies that*

$$1 + m_t^* = (1+\rho)^{1-\alpha} \quad \textit{for all } t \geq 1, \tag{23}$$

$$(1+\delta \cdot q_t^*) \cdot B_t^* + f\left(\{m_t^*\}_{j=t}^{+\infty}\right) \cdot (1-\phi)^{t-1} \cdot D_t^* \;=\; (1-\phi)^{t-1} \cdot D_t^* \quad \textit{for all } t \geq 1 \tag{24}$$

and q_t^* is given by (7).

Along the perfect-foresight equilibrium path, the bank earns a constant and strictly positive interest margin, as indicated in equation (23). Moreover, equation (24) shows that the value of the bank's bond portfolio plus the equilibrium deposit franchise is exactly equal to the face value of outstanding deposits. Once we realize that the deposit franchise is effectively an asset of the bank, this balance-sheet condition makes sense. It simply says that at any point in time the bank's assets match its liabilities.

It is worth zooming into the equilibrium value of the deposit franchise. Plugging equilibrium values in equation (21), we find that it is given by

$$f\left(\{m_j^*\}_{j=t}^{+\infty}\right) = \frac{1-\phi}{\phi + m_t^*} \cdot m_t^*. \tag{25}$$

This is approximately the product of the average time before a deposit is withdrawn times the interest margin earned by the bank in every unit of time. It is strictly positive since $\phi > 0$ and $m_t^* > 0$. In section 7, I use equation (25) to calibrate ϕ using the observed interest margin and empirical estimates of banks' deposit franchise.

6 The Tipping Point

Consider an economy on the equilibrium path. Suppose the economy's interest rate changes unexpectedly and permanently from ρ to $\hat{\rho}$ at a given time $t \geq 1$. Does the bank fail?

With corollary 1, we know that the necessary and sufficient condition for the bank's survival is

$$(1 + \delta \cdot \hat{q}_t) \cdot B_t^* + f\left(\{\hat{\rho}\}_{j=t}^{+\infty}\right) \cdot (1-\phi)^{t-1} \cdot D_t^* \;\geq\; (1-\phi)^{t-1} \cdot D_t^*, \tag{26}$$

where $\hat{q}_t = 1/(1 + \hat{\rho} - \delta)$. First, notice that the quantity of bonds B_t^* and outstanding deposits $(1-\phi)^{t-1} \cdot D_t^*$ are predetermined variables. Hence, they do not respond contemporaneously to the shock. It follows that the shock has two effects on the condition. For illustration purposes, let us consider a fall in the interest rate. A revaluation effect increases the value of the bank's bonds, that were priced at the previous higher interest rate. This effect makes the condition likelier to hold and hence the bank likelier to survive the shock. The longer the duration of the bank's bond portfolio as captured by δ, the stronger this first effect is. On the other hand, a low interest rate weakens the bank's ability to earn interest margins, given the presence of a zero lower bound on the deposit rate. This margin compression is reflected in a lower deposit franchise f and makes the bank, all else equal, less likely to survive the shock. Interestingly, the two effects are competing.

Given the existence of two opposite effects, it is unclear whether bank insolvency is brought about by a negative or by a positive shock to interest rates. As it turns out, this depends on parameters.

Proposition 3. *Consider an economy on the equilibrium path that is hit at time $t \geq 1$ by a shock that changes the interest rate to $\hat{\rho} > -\phi$ permanently. We can distinguish three parametric regions:*

1. *Given $\delta < 1 - \phi$, the bank fails if and only if $\hat{\rho} < \rho^{tp}$.*
2. *Given $\delta > (1-\phi) \cdot (1+\rho)/(1+m_t^*)$, the bank fails if and only if $\hat{\rho} > \rho^{tp}$.*
3. *In the intermediate region, there is no admissible shock that makes the bank fail.*

The critical interest rate, which I call the tipping point, is given by

$$\rho^{tp} = m_t^* - \delta \cdot \frac{(\rho - m_t^*) \cdot (\phi + m_t^*)}{(1-\phi) \cdot (1+\rho) - \delta \cdot (1+m_t^*)}. \tag{27}$$

There are two key parts in this proposition: the condition for dominance of the effects and the value of the tipping point. With respect to dominance of the asset-revaluation or interest-margin effects, there is a simple parametric condition that determines it. The condition is in terms of δ, which governs the duration of the bank's bond portfolio and therefore its interest-rate sensitivity, and ϕ, which effectively governs the duration of bank deposits and therefore the interest-rate sensitivity of the deposit franchise. A low δ, which indicates that the bank is holding a short-term bond portfolio, and a low ϕ, which indicate that deposits are held in the bank for a relatively long time, expose a bank to failure when interest rates fall, not when they rise. The opposite is true for a high δ and a low ϕ.

Once we know which effect dominates and thus whether it is a reduction or an increase in the interest rate that leads to bank failure, we can study the smallest shock that tips the bank into bankruptcy. Equation (27) gives us an analytical solution for this tipping point. If the interest-margin effect dominates, then any shock that brings the interest rate below the value ρ^{tp} leads to bank failure. In the extreme case of zero bond duration (i.e., $\delta = 0$), there is no revaluation effect at all. In this case, the tipping point is equal to the perfect-foresight interest margin. If the interest rate falls below the interest margin that prevailed before the shock, then the bank's interest margin falls with it since the bank is unable to set its deposit rate in negative territory. Without any offsetting revaluation effect, this is enough for the bank to fail. As δ increases and the revaluation effect becomes stronger, the bank is able to withstand larger and larger negative interest rate shocks. Once δ is so large that the revaluation effect is dominant, then the bank has the opposite problem. In this parametric region, sufficiently large increases in the interest rate devalue the bank's bond portfolio more than they boost its deposit franchise. Hence, they lead to failure. In this case, as δ increases and strengthens the revaluation effect, smaller and smaller positive interest-rate shocks bankrupt the bank. Let us take the extreme case in which $\phi \to 1$. In this case, the bank has no deposit franchise and therefore there is no improvement in interest margins to offset the devaluation of bonds following an increase in the interest rate. Then, the tipping point is simply ρ. That is, any shock that increases the interest rate leads to bank failure.

7 Quantitative Exercise

This section calibrates the model's parameters to shed light quantitatively on the relationship between interest rates and financial stability. In particular, it studies whether increases or reductions in the interest rate represent a risk for financial stability and quantifies the tipping point. To calibrate the model, I use bank data from the US economy in the decade before the Global Financial Crisis, as summarised in table 1.

The first of the model's variables that I match to an empirical counterpart is interest rate ρ. I take the fed funds rate to be the short-term and safe rate in the US economy. Its average value in the period from 1997Q3 to 2007Q2 was 3.81%.[5]

Second, the deposit rate is an important endogenous object of the model and it can be observed. According to the US Call Reports, the average interest rate paid by the commercial banking sector on core deposits, the sum of checking, savings, and small time deposits, in the period 1997Q3-2007Q2 is 2.39%.

Third, the parameter δ, which regulates the speed at which the perpetuity's coupon decays, is linked to the duration of bank assets. In fact, the average maturity of bank assets weighted by the present value of coupons, the definition of duration, is given by $\delta/(1+\rho-\delta)$, as discussed in section 3. According to English et al. (2018), who use data from the US Call Reports, the average repricing time of bank-held assets in the period 1997Q3-2007Q2 is 4.46 years. I use this value for bank-asset duration and carry out a robustness analysis later in the text.

The fourth and least immediate of the model's variables with an empirical counterpart is the bank's deposit franchise. Sheehan (2013) focuses directly on estimating the deposit franchise. Exploiting data from large US commercial banks on deposit balances and deposit rates starting in December 1996 and ending in December 2003, the paper

[5] The interest rate ρ is real and the fed funds rate is nominal. In principle, the empirical counterpart is the real return on fed funds. However, the storage technology, which stands for currency in the model, should then offer a real return given by the negative of the rate of inflation. With both changes in place, the resulting tipping point is the same.

Table 1: Data.

Model	Empirical counterpart	Value	Source
ρ	Average fed funds rate	3.81%	FRED
r^*	Average interest rate on core deposits	2.39%	US Call Reports
$\delta/(1+\rho-\delta)$	Average bank-asset repricing time (years)	4.46	English et al. (2018)
f^*	Average per-unit deposit franchise	20.2%	Sheehan (2013)

Table 2: Model parameters.

Parameter	Description	Value
ρ	Short-term interest rate	3.81%
$1/\alpha$	Coefficient of relative risk aversion	1.58
δ	Common ratio of coupons' progression	84.8%
ϕ	Household's probability of turning impatient	5.13%

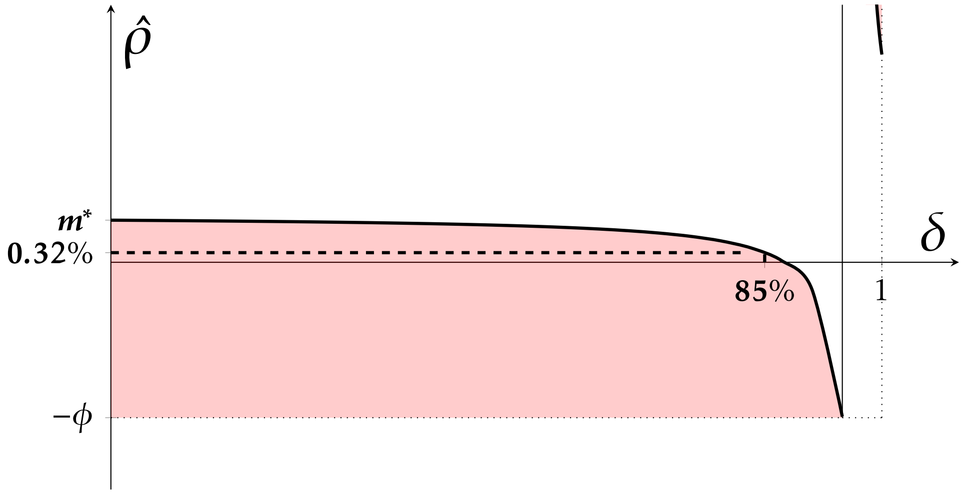
finds that deposit franchises are large. The average value across deposit types and banks is 20.2%. This means that a bank would pay at most 79.8 cents to free itself of liabilities in the form of core deposits with a face value of one dollar.

The model contains four parameters and in this exercise I match four variables from the data. Hence, I identify all of the parameters uniquely and report the results in table 2. Noticeably, the resulting coefficient of relative risk aversion is well within the range commonly used in calibrated models and supported by micro-econometric evidence (Mehra and Prescott, 1985). As for the households' probability of turning impatient, the only other attempt in the literature to calibrate it within a similar model, with the exception of purely illustrative exercises, is in Mattana and Panetti (2021). They set the parameter to match the quantity of liquid assets held by banks and find a value of 2%, which is within the same order of magnitude as my finding.

With all parameter values in place, I can first check the parameter condition specified in proposition 3 and see whether banks are vulnerable to an increase or a reduction in the interest rate. I find that the calibration points strongly to a dominant interest-margin effect. In particular, dominance of the asset-revaluation effect would imply either a duration of bank assets of at least 18.5 years or a deposit franchise of at most 7.09% of deposits' face value. Both are one order of magnitude away from the empirical evidence.

Second, I can plug the parameter values in the formula for the tipping point, (27). This yields a quantified tipping point of 0.32%. This means that the banking sector can withstand a permanent fall in the interest rate down to 0.32%. Past this level, the

Figure 1: Robustness with respect to δ.



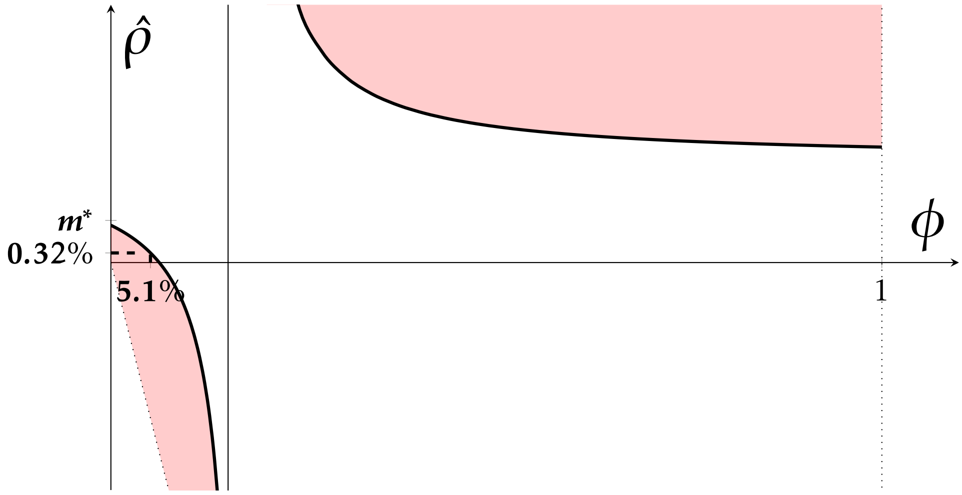
Note: In the red-shaded region $(\delta, \hat{\rho})$, the bank fails. The vertical line represents parametric condition $\delta = 1 - \phi$. The solid line represents the tipping point.

windfall from the revaluation effect is insufficient to compensate for the reduction in the interest margin and the banking sector fails.

Robustness. A robustness analysis is warranted with regard to values for δ, since the average bank-asset repricing time is only a proxy for bank-asset duration. In fact, it is likely to overestimate the true average duration of bank assets, since prepayment is common for mortgages. Figure 1 shows how the tipping point, tracked by the solid line, varies with different values for δ. In the region shaded in red, the bank fails. A δ of zero implies no revaluation effect and therefore a tipping point equal to the perfect-foresight interest margin m^*. As δ increases, the tipping point goes down, since the revaluation effect becomes stronger and stabilizes the bank. A tipping point of 1% implies average bank-asset maturity of 2.26 years. A tipping point of zero implies average bank-asset maturity of 6.90 years. For very large values of δ, the revaluation effect is dominant and, as predicted by proposition 3, the logic is inverted with high interest rates causing bank failure. In this region, the average bank-asset maturity is at least 18.5 years, which is far from the value in the data.

As already discussed, there is little guidance in the literature on a value for ϕ and the empirical literature that quantifies the deposit franchise is thin. Hence, a robustness analysis is in order with respect to the value that we use. Figure 2 shows how the tipping point, tracked by the solid line, varies with different values for ϕ. In the region shaded in red, the bank fails. The tipping point goes down as ϕ increases. This is because a

Figure 2: Robustness with respect to ϕ.



Note: In the red-shaded region $(\phi, \hat{\rho})$, the bank fails. The vertical line represents parametric condition $\delta = 1 - \phi$. The solid line represents the tipping point.

large ϕ, which means deposits are short-lived, implies a small deposit franchise that is interest-rate insensitive. Hence, only large negative interest-rate shocks, which strongly compress interest margins, can make the bank insolvent. A tipping point of 1% implies ϕ equal to 2.26% and a corresponding deposit franchise of 37.2% of deposits' face value. A tipping point of zero requires ϕ equal to 7.71% and a corresponding deposit franchise of 14.1% of deposits' face value. If ϕ is large enough, then the deposit-franchise effect is so weak relative to the revaluation effect that the tipping point logic is inverted, as predicted by proposition 3. For this to be the case, ϕ must be at least 15.2%, implying a deposit franchise of 7.09% of deposits' face value.

8 Conclusion

This paper studies the effect of interest rates on bank stability. It marries a tradition in finance, which focuses on banks' maturity mismatch and thus sees high interest rates as dangerous for banks (Kaufman, 1984), with a more recent view that sees low rates as a threat to bank profitability (Borio et al., 2017). Both effects are at play in this paper's model: an increase in the interest rate pushes down the value of bank assets, but also widens banks' interest margins.

The paper contains two key analytical findings. The first result is that we must compare the average duration of bank assets with the effective duration of bank liabilities, determined by the frequency of liquidity shocks on depositors, to find out which effect dominates, the asset-revaluation effect or the interest-margin effect. Second, I find that interest rates create bank instability once they move beyond a critical level, which I call the tipping point. I provide a closed-form solution for this quantity. A quantitative exercise finds that the effect via interest margins is dominant. This implies that low rates are the key menace to the stability of the banking sector. The role of maturity mismatch is to soften the effects of low rates on banks. In fact, a larger maturity mismatch pushes the tipping point down, making the bank more resilient.

The insurance role of banks' maturity mismatch speaks to the long-standing question in finance of why deposit taking and long-term lending are conducted under one roof (Kashyap et al., 2002). It is because long-term assets hedge the risk that low interest rates pose to bank profitability, as also found empirically in Drechsler et al. (2021). In fact, this paper's model turns the question on its head: why is the maturity mismatch of banks not large enough to fully hedge interest-rate risk? This question is left to future research. According to the quantitative analysis in this paper, full insulation from interest-rate risk would require an increase in bank-asset duration by a factor of four.

The analysis is conducted through the lens of the canonical model of banking crises developed in Diamond and Dybvig (1983) with time horizon extended to infinity. The extension allows for the introduction of long-term assets into the model in a tractable way. Moreover, the extension has the merit of bringing out in the equations the deposit franchise as the measure of bank profitability that matters for bank stability. The deposit franchise is the value to a bank of its deposit base. It is the product of the bank's average interest margin over time, the expected lifetime of a deposit and total deposits. On its own terms, this insight represents a contribution to the literature. Also, it provides a useful connection of model parameters to a bank characteristic, which has been estimated in the empirical banking literature. I use estimates of banks' deposit franchise to discipline the paper's quantitative exercise.

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A Proofs

Proof of Proposition 1. Consider a subgame of the bank's problem starting at time $t \geq 1$. Using budget constraints (10) and (14), boundary condition (15) and anticipating equilibrium condition (7), I can write the intertemporal budget constraint

$$\phi \cdot \sum_{s=0}^{+\infty} \left(\frac{1-\phi}{1+\rho} \right)^s \cdot \prod_{j=0}^{s-1} (1 + r_{t+j}) \cdot (1-\phi)^{t-1} \cdot D_t \leq (1 + \delta \cdot q_t) \cdot B_t. \tag{28}$$

Rearranging, I obtain condition

$$\phi \cdot \sum_{s=0}^{+\infty} \left(\frac{1-\phi}{1+\rho} \right)^s \cdot \prod_{j=0}^{s-1} (1 + r_{t+j}) \leq \frac{(1 + \delta \cdot q_t) \cdot B_t}{(1-\phi)^{t-1} \cdot D_t} \tag{29}$$

that determines feasible deposit contracts $\left\{ r_{t+j} \right\}_{j=0}^{+\infty}$ conditional on initial values B_t and D_t and on price q_t.

First, I prove the "if" part of the proposition. For this, we verify that a deposit contract that satisfies incentive-compatibility constraints (9) and satisfies condition (29), in which we impose initial conditions satisfying condition (17), exists. Notice that for a deposit contract that pays $r_{t+j} = 0$ for all $j \geq 0$, the left-hand side of equation (29) is equal to $\phi \cdot (1+\rho)/(\phi+\rho)$. Moreover, the left-hand side of the equation is increasing in r_{t+j} and there is no upper bound on r_{t+j}. By this argument, the "if" part is proven.

Second, I prove the "only if" part of the proposition. This is equivalent to proving that the bank fails if condition (17) does not hold. The left-hand side of equation (29) is equal to $\phi \cdot (1+\rho)/(\phi+\rho)$ when the incentive-compatibility constraint is binding at every date with $r_{t+j} = 0$ for all $j \geq 0$. Since the left-hand side of equation (29) is increasing in r_{t+j}, there is no deposit contract that is both incentive-compatible and feasible if condition (17) does not hold. Hence, the bank fails and the "only if" part of the proposition is proven. □

Proof of Proposition 2. Before I turn to the bank's problem, I look at the firm's problem to pin down equilibrium q_t^*. Arbitrage by the firm implies $1+\rho = (1+\delta \cdot q_{t+1}^*)/q_t^*$. The only sequence that satisfies this condition and no-bubble condition (6) is given by equation (7).

Now, I turn to the bank's problem. First, I solve the bank's full problem starting at time zero. Then, I verify that the solution is also a solution in every subgame of the problem. Using initial conditions (11) and (12), budget constraints (13) and (14), boundary condition (15), and the equilibrium condition (7), I can write the intertemporal budget constraint

$$\phi \cdot \sum_{t=1}^{+\infty} \left(\frac{1}{1+\rho}\right)^t \cdot (1-\phi)^{t-1} \cdot D_t = 1. \tag{30}$$

Maximizing the objective function (8) subject to constraint (30) with respect to choice variables $\{B_{t+1}, D_{t+1}\}_{t=0}^{+\infty}$, I obtain a set of optimality conditions. Once combined with (10), they can be written as

$$1 + r_0^* = (1+\rho)^\alpha \cdot \frac{(1+\rho)^{1-\alpha} - (1-\phi)}{\phi} \tag{31}$$

and $1 + r_t^* = (1+\rho)^\alpha$ for all $t \geq 1$. Notice that along the optimal path the incentive-compatibility constraints (9) are always slack. Hence, I can safely ignore them in this case. Re-arranging and combining initial conditions (11) and (12), budget constraints (13), (10) and (14), and the equilibrium condition (7), I can write a law of motion for the variable of interest given by

$$\frac{(1+\delta \cdot q_1) \cdot B_1}{(1-\phi)^0 \cdot D_1} = \frac{1+\rho}{1+r_0}, \tag{32}$$

$$\frac{(1+\delta \cdot q_{t+1}) \cdot B_{t+1}}{(1-\phi)^t \cdot D_{t+1}} = \frac{1+\rho}{(1-\phi)\cdot(1+r_t)} \cdot \left[\frac{(1+\delta \cdot q_t) \cdot B_t}{(1-\phi)^{t-1} \cdot D_t} - \phi\right] \quad \text{for all } t \geq 1. \tag{33}$$

Substituting in the optimal path $\{r_t^*\}_{t=0}^{+\infty}$, I confirm

$$\frac{(1+\delta \cdot q_t^*) \cdot B_t^*}{(1-\phi)^{t-1} \cdot D_t^*} = \frac{\phi \cdot (1+\rho)^{1-\alpha}}{(1+\rho)^{1-\alpha} - (1-\phi)} \quad \text{for all } t \geq 1. \tag{34}$$

Finally, to verify that the solution above is also a solution to every subgame of the problem, take a subgame starting at time $t \geq 1$ with initial conditions B_t and D_t that satisfy condition (34). The bank's optimality conditions imply that $1 + r_j^* = (1+\rho)^\alpha$ for all $j \geq t$. This proves the proposition. □

Proof of Corollary 1. Take the definition of the deposit franchise (21) and notice that

$$f(\{\rho\}) = \frac{(1-\phi)\cdot\rho}{\phi+\rho}. \tag{35}$$

With this, we can confirm that proposition 1 implies this corollary. □

Proof of Corollary 2. Applying the definitions of interest margin (20) and deposit franchise (21), we can confirm that proposition 2 implies the corollary. □

Proof of Proposition 3. Consider a subgame of the bank's problem starting at time $t \geq 1$. The interest rate is $\hat{\rho} > -\phi$. Using proposition 1, we can conclude that the bank does not fail as long as

$$\frac{(1+\delta \cdot \hat{q}_t)B_t}{(1-\phi)^{t-1} \cdot D_t} \geq \frac{\phi \cdot (1+\hat{\rho})}{\phi + \hat{\rho}} \tag{36}$$

with $\hat{q}_t = 1/(1+\hat{\rho}-\delta)$ for any values (B_t, D_t). Since the economy was running along its perfect-foresight equilibrium path before the time-t shock that changed the interest rate to $\hat{\rho}$, the initial conditions B_t and D_t satisfy

$$\frac{B_t}{(1-\phi)^{t-1} \cdot D_t} = [1 - f_t(\{m_t^*\})] \cdot \frac{1+\rho-\delta}{1+\rho}, \tag{37}$$

as per corollary 2, where the perfect-foresight deposit franchise is given by equation (25). Subbing in these initial conditions, we can write the necessary and sufficient condition under which the bank does not fail as

$$[1 - f_t(\{m_t^*\})] \cdot \frac{1+\rho-\delta}{1+\rho} \geq \frac{\phi \cdot (1+\hat{\rho}-\delta)}{\phi+\hat{\rho}}. \tag{38}$$

Let us first study the values of $\hat{\rho}$ where such condition holds in the parameter region $\delta < 1-\phi$. The left-hand side of equation (38) is not a function of $\hat{\rho}$ and is strictly larger than ϕ for these parameters. The right-hand side is continuous, tends to infinity for $\hat{\rho} \to \phi^+$ and tends to ϕ for $\hat{\rho} \to +\infty$. By the intermediate value theorem, there is at least one point ρ^{tp} at which left-hand side and right-hand side are equal. Since the right-hand side is strictly decreasing in $\hat{\rho}$, ρ^{tp} is unique. To the left of ρ^{tp} the right-hand side is larger than the left-hand side of the equation. Hence, the bank fails. To the right of ρ^{tp}, the bank does not fail. Solving for $\hat{\rho} = \rho^{\text{tp}}$ such that the left-hand side and the right-hand side are equal and substituting in equation (25) gives the tipping point

$$\rho^{\text{tp}} = m_t^* - \delta \cdot \frac{(\rho - m_t^*) \cdot (\phi + m_t^*)}{(1-\phi)\cdot(1+\rho) - \delta \cdot (1+m_t^*)}. \tag{27}$$

Second, let us study the parameter region $\delta \in [1-\phi, 1)$. If I study the left-hand side of the inequality, I notice that it is larger than ϕ for $1-\phi \leq \delta \leq (1-\phi)\cdot(1+\rho)^\alpha$. It is strictly smaller than ϕ for $(1-\phi)\cdot(1+\rho)^\alpha < \delta < 1$. If I study the right-hand side for $\delta \geq 1-\phi$, I find that it is continuous, it tends to minus infinity for $\hat{\rho} \to \phi^+$ and tends to ϕ^- for $\hat{\rho} \to +\infty$. Moreover, it is strictly monotonically increasing. This implies that the right-hand side is smaller than the left-hand side for any $\hat{\rho}$, whenever $1-\phi \leq \delta \leq (1-\phi)\cdot(1+\rho)^\alpha$. Hence, there is no shock such that the bank fails in this case. For $(1-\phi)\cdot(1+\rho)^{\alpha} < \delta < 1$, there is a unique ρ^{tp} at which right-hand side and left-hand side are equal. For $\hat{\rho} \leq \rho^{\mathrm{tp}}$, the left-hand side is larger than the right-hand side. Hence, the bank does not fail. For $\hat{\rho} > \rho^{\mathrm{tp}}$, the bank fails. Again, solving for $\hat{\rho} = \rho^{\mathrm{tp}}$ such that the left-hand side and the right-hand side are equal and substituting in equation (25) gives the tipping point (27). This concludes the proof. $\square$

B Asset liquidation

The assumption that the bank can sell any amount of bonds frictionlessly is innocuous for the paper's results. In this appendix, I confirm this.

At any given time t, the bank holds the equivalent of B_t new bonds.[6] Hence, it receives coupons amounting to B_t. Households withdraw in total $\phi \cdot (1-\phi)^{t-1} \cdot D_t$. The bank sells bonds if the coupon is insufficient to cover the withdrawals.

Definition 2. *If at time t*

$$\frac{B_t}{(1-\phi)^{t-1} \cdot D_t} < \phi, \tag{39}$$

a bank sells bonds.

Whether a bank sells bonds is per se irrelevant for equilibrium outcomes in this paper's model, since bond selling is frictionless. Nonetheless, it is in principle interesting to introduce a liquidation cost in the bond market and study its effect on economic outcomes. This friction is theoretically compelling as a result of information asymmetries (Eisfeldt, 2004) and is emphasized in the literature on financial crises. For example, Diamond and Dybvig (1983) posit that assets have a higher per-period return if held for two periods rather than liquidated after one period, reflecting a liquidation cost.

I find that the introduction of a liquidation cost changes none of the results of this paper. Asset liquidation by the bank never happens, unless the bank is insolvent. In other words, the bank sells its bonds exclusively as a consequence of failure. It cannot become insolvent because of the poor terms at which it must sell bonds. Hence, the size of the shock that makes the bank insolvent does not depend on the liquidation cost.

Proposition 4. *Consider an economy with $\delta \leq 1-\phi$. If in equilibrium the bank does not fail at time $t \geq 1$, then the bank does not sell bonds at any time $s \geq t$.*

[6] The bank may not actually hold only new bonds but also older vintages of bonds. What matters is the new-bond-equivalent quantity of bonds it holds. For example, it may hold a bond issued at time $t-2$. This pays a coupon of δ at time t and is equivalent to δ new bonds issued at time $t-1$, as explained in section 3.

Proof. By definition 1, if a bank does not fail at time $t \geq 1$, there exists a sequence $\left\{B_{j+1}, D_{j+1}, r_j\right\}_{j=t}^{+\infty}$ that, given initial conditions B_t and D_t, satisfies incentive-compatibility constraints (9), budget constraints (10) and (14), and the boundary condition (15). The subsequence $\left\{B_{j+1}, D_{j+1}, r_j\right\}_{j=s}^{+\infty}$ for $s \geq t$, given initial conditions B_s and D_s belonging to the above sequence, also satisfies incentive-compatibility constraints (9), budget constraints (10) and (14), and the boundary condition (15). Hence, there exists a solution to the subgame of the bank's problem starting at any time $s \geq t$. Again by definition 1, this implies the bank does not fail at any time $s \geq t$. Hence, by proposition 1 we have that the solvency condition

$$\frac{(1+\delta \cdot q_s) \cdot B_s}{(1-\phi)^{s-1} \cdot D_s} \geq \frac{\phi \cdot (1+\rho)}{\phi + \rho} \tag{40}$$

holds for any $s \geq t$. Using equation (7) to substitute out q_s, we can verify that, under parametric condition $\delta \leq 1 - \phi$, the solvency condition contradicts inequality (39) in every period. □

Take the perfect-foresight equilibrium path described in proposition 2. This is the path the economy takes in equilibrium if it is never perturbed by a shock from time 0 on. Along this path the bank never fails and never sells any bonds. It meets withdrawals entirely with the coupons at every point in time. It follows that a liquidation cost would play no role in the economy's equilibrium outcomes. The above proposition is more general: even if an economy is hit by shocks, the bank never sells any bonds along the equilibrium path as long as it survives the shocks. In other words, a solvent bank never sells its bonds.

C Panics

In this section, I introduce bank panics and show that they change neither the main theoretical results of the paper nor the quantitative results.

Panics are instances in which banks fail although they are solvent. They fail because so many households withdraw at a given point in time that the banks' resources are exhausted. To allow for bank panics in the model, it is sufficient to assume an imperfect deposit insurance that upon bank default pays out $\theta \cdot D_t < D_t$ to those depositors who did not withdraw. This provides households with an incentive to try to get paid off in full by the bank directly right before it defaults rather than waiting for deposit insurance, since the latter does not pay the deposits fully. For example, this captures the fact that deposit insurance may take time to disburse the funds. Depending on the balance-sheet conditions of the bank, an incentive arises even for patient households to withdraw if everyone else does.

Proposition 5. *At time $t \geq 1$, the equilibrium probability of bank failure is zero if condition*

$$(1 + \delta \cdot q_t^*) \cdot B_t \geq (1 - \phi)^{t-1} \cdot D_t \tag{41}$$

holds. If solvency condition (17) does not hold, the equilibrium probability of bank failure is one. Otherwise, it is $\sigma \in [0, 1]$.

Proof. Take the bank's intertemporal budget constraint at a given time $j > 0$ given by equation (16) and let all patient households withdraw at time j along with the impatient ones. This obtains condition $(1 - \phi)^{j-1} \cdot D_j \leq (1 + \delta \cdot q_j) \cdot B_j$. So, if and only if condition (41) holds, then the intertemporal budget constraint holds regardless of how many households withdraw. The solvency condition remains the same as in the model without panics since the bank's budget constraints, from which it is derived, remain the same. If solvency condition (17) fails to hold, then the bank fails for sure. Since there is no element in the model to pin down the probability of a panic in the intermediate region between the solvency condition and condition (41), I choose an exogenous parameter σ for this. □

Noticeably, the condition for the absence of bank failure is more stringent in the version of the model with panics. However, the condition below which banks are certain to fail remains the same.

It turns out that the bank chooses to be run-prone in the perfect-foresight equilibrium. This is a standard result in the literature, already found in Diamond and Dybvig (1983). It is optimal for banks to be run-prone, because this is the only way for banks to insure households against the risk of turning impatient early.

Proposition 6. *For σ small enough, perfect-foresight equilibrium implies that*

$$1 + r_t^* = [(1 + \rho) \cdot (1 - \sigma)]^\alpha \quad \textit{for all } t \geq 1, \tag{42}$$

$$(1 + \delta \cdot q_t^*) \cdot B_t^* = \frac{\phi \cdot [(1 + \rho) \cdot (1 - \sigma)]^{1-\alpha}}{[(1 + \rho) \cdot (1 - \sigma)]^{1-\alpha} - (1 - \phi)} \cdot (1 - \phi)^{t-1} \cdot D_t^* \quad \textit{for all } t \geq 1, \tag{43}$$

q_t^* is given by (7) and the probability of bank failure is σ.

Proof. Let us guess and then verify that the bank chooses a deposit contract such that it is run-prone. Hence, the equilibrium probability of default at every date is σ. At a given time $t \geq 1$, default happens with probability $(1-\sigma)^{t-1} \cdot \sigma$ and the remaining $(1-\phi)^{t-1}$ households withdraw, receive their deposits D_t and store them until they turn impatient. With probability $(1-\sigma)^t$ the bank does not default and only the impatient households $(1-\phi)^{t-1} \cdot \phi$ withdraw at a given date $t \geq 1$. Thus, the households' time-0 expected utility is given by

$$[(1-\sigma) \cdot \phi + \sigma] \cdot \sum_{t=1}^{+\infty} [(1-\phi) \cdot (1-\sigma)]^{t-1} \cdot u(D_t). \tag{44}$$

A bank that maximizes this objective function subject to incentive-compatibility constraints (9), initial conditions (11) and (12), budget constraints (10), (13) and (14), and boundary condition (15) sets r_t^* according to (42). With this result, we can use the bank's budget constraints to confirm (43). We can also check that the bank does not choose to be run-proof by comparing the time-zero expected utility of households under $r_t = \rho$ for all $t \geq 0$, which ensures the bank is run-proof, with the time-zero expected utility of households under the deposit contract described in the proposition. For $\sigma \rightarrow 0$, it is easy to confirm that this is the case. □

The presence of bank panics changes the deposit contract offered by banks in equilibrium. In particular, the bank offers a lower deposit rate. The bank has an incentive to increase the extent to which it provides liquidity-risk insurance to ensure that, if a panic takes place early, depositors get a high payout. The flip side of this is that the bank must earn a larger interest margin over time in order to be solvent.

The key theoretical results of the paper, contained in proposition 3, remain the same *conditional on the perfect-foresight interest margin*. In fact, the proof of proposition 3 makes use of the bank's solvency condition, which is unchanged with panics as shown in proposition 5, and is valid for general initial conditions (B_t^*, D_t^*) and a general perfect-foresight interest margin m_t^*.

If one knows the perfect-foresight interest margin, one can quantify the tipping point regardless of the probability of bank panics. This is important for the quantitative results in section 7 because banks' interest margins are indeed observable. As it turns out, the quantitative exercise remains almost entirely valid with bank panics. The only parameter that needs a new calibration is the coefficient of relative risk aversion $1/\alpha$. This is unimportant because the parameter does not appear either in the condition on the dominance of the effects or in the tipping point formula. Nevertheless, I find that a model with panics needs a lower degree of relative risk aversion to generate the same average interest rate and deposit rate.

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