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Simulating fire sales in a system of banks and asset managers

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Abstract

We develop an agent-based model of traditional banks and asset managers to investigate the contagion risk related to fire sales and balance sheet interactions. We take a structural approach to the price formation in fire sales as in Bluhm et al. (2014) and introduce a market clearing mechanism with endogenous formation of asset prices. We find that, first, banks which are active in both the interbank and securities markets may channel financial distress between the two markets. Second, while higher bank capital requirements decrease default risk and funding costs, they make it also more profitable to invest into less-liquid assets financed by interbank borrowing. Third, asset managers absorb small liquidity shocks, but they exacerbate contagion when their voluntary liquid buffers are fully utilised. Fourth, a system with larger and more interconnected agents is more prone to contagion risk stemming from funding shocks.

Keywords: fire sales, contagion, systemic risk, asset managers, agent-based model
JEL: C6, G21, G23

Non-technical summary

Shadow banks have been gaining prominence in recent years as an alternative intermediation channel in the financial system. They do not replace traditional banking but coexist as an important and growing market, providing financing to the economy and shaping the liquidity conditions in the financial markets. We focus on interactions between balance sheets of banks and asset managers, which represent a significantly growing segment of the shadow banking market. We are interested in addressing the following research question: is the traditional banking system more susceptible to shocks stemming from banks or from non-banks? What are channels of transmission of shocks between regulated and unregulated sectors? What structures in the traditional and non-bank financial system are more vulnerable to funding shocks?

We develop an agent based model of profit maximising traditional banks and asset managers, the first funded via equity and deposits, the latter via participations. Both types of institutions hold liquid and illiquid assets, and the risk free rate of return in the interbank lending market and the price of investment security are endogenously determined. Traditional banks are constrained by capital and liquidity requirements. In addition, both banks and asset managers hold discretionary liquidity buffers on top of the regulatory ones. All institutions are indirectly connected through holdings of similar securities, while traditional banks are also interlinked via interbank lending. These two channels determine the dynamics of contagion. In such a set-up we show how an idiosyncratic liquidity shock propagates through the system affecting both interbank market and, if sizable enough, can trigger a fire sale of the less-liquid security.

When the market for the security opens, banks and asset managers can buy or sell their assets. The interplay of endogenous supply and demand determines the price of the security. Its market price is then used to mark-to-market asset values on banks' and asset managers' balance sheets. This can trigger further rounds of portfolio adjustments both in the interbank lending market and in the security market, as banks need to adjust their balance sheets to meet the regulatory requirements as asset managers as they face redemptions due to drops in the security price. Finally, banks default when they are not able to meet either their interbank obligations or the regulatory requirements, even if they use cash generated in the fire sales.

We have several findings. First, requirements have an ample effect on the contagion spreading following a funding shock. Tighter liquidity regulation immunises the system from liquidity shocks, but higher capital requirements make it also more profitable to invest into less-liquid assets financed by interbank borrowing. Second, the speed of contagion depends on which sector the initial shock hits first. Contagion instigated by an asset manager's funding problem initially develops slowly but, as time evolves, it can have a higher impact than that for the initial shock affected a bank. Monitoring of the asset management sector activities is crucial to assess fire-sale risk. Third, fire sales are fuelled by imbalances between demand and supply for securities. This is a clear externality of the fire sales that can be mitigated by the central banks providing liquidity liquidity to the system. Fourth, asset managers have potential to amplify the fire sales, even if the initial shocks hit banks only. Fifth, business models of banks, their heterogeneity in sizes and interconnectedness matter for the magnitude of losses under funding stress conditions.

1. Introduction

Non-bank financial institutions (Constâncio, 2014) have recently been gaining prominence as an alternative source of intermediation in the financial system. They do not replace traditional banking but coexist as an important, growing market, providing credit to the economy and shaping the liquidity conditions in the financial markets. In this paper we focus on the asset management sector, which has been significantly growing in recent years, as reported by the Financial Stability Board (FSB, 2017).

The emergence and growth of shadow banks is a result of large liquidity pools that investors have ventured to allocate in safe assets other than banks' insured deposits. This raises relevant questions on the potential imbalances building up in the financial and related to the non-bank financial institutions (or, in short, non-banks). As the share of assets of non-banks in the total assets of the financial sector increases, concerns about financial stability implications mount. Ari et al. (2016), using a general equilibrium set-up with shadow bank endogenous entry, find that the sector has a natural tendency to grow until it ferments systemic risk. The FSB emphasizes that the growth of the asset management sector is realized by increasing holdings of less-liquid assets, traditionally held by banks. Therefore, the following questions arise. Is the traditional banking system more susceptible to shocks stemming from banks or from non-banks? What are the transmission channels of shocks between regulated and unregulated sectors? What structures of the traditional and non-bank financial systems are more vulnerable to funding shocks?

Regulators have been trying to address some of these risks by designing risk-sensitive capital requirements, haircuts on non-centrally cleared securities financing transactions, and limits to the reuse of collateral. The FSB (2017) lays out some policy recommendations about potentially useful instruments to manage risks building up in the financial system because of activities in the asset management sector. The recommendations suggest the use of a system-wide stress test to account for interactions between market participants. However, research supporting decisions on relevant and reliable regulatory tools and on their actual implementation is yet to advance. With our model we contribute to the discussion on the interactions between banks and asset managers, as these interactions may increase the vulnerabilities in the financial system.

In addition, since the data on shadow bank activities are still scarce, some simulation-based models, like agent-based models, can be a valuable approach to gain insight into the interplay between banks and non-banks and into potential sources of contagion. Agent-based models of the financial market help to analyse interactions between agents in a complex system of financial institutions. For instance Bookstaber et al. (2017) analyze the dynamic interactions of a network of financial agents and show that the extent of losses in the system relies more on the reactions to the initial shock than on the shock itself. We follow this avenue in our paper and develop an analytical framework to study contagion risk involving traditional banks and asset managers, including the emergence of fire sales, i.e., forced liquidation of assets at a price below the fundamental value of the assets. The asset prices are an endogenous outcome of strategic interactions between agents in the model. We then study the effectiveness of capital and liquidity requirements in containing fire-sale and contagion risks. We also show that asset managers may amplify fire sales. They depress asset prices when they are forced to liquidate securities after redemptions associated to asset managers' performance.

In our agent-based framework, we reflect bounded rationality of agents and the existing regulatory regime. Both types of agents in our model, traditional banks and asset managers, maximise profits based on their beliefs about the profitability of investment opportunities. As a result, they are funded via equity and deposits or via participations and hold liquid and less-liquid assets. Additionally, if they see good investment opportunities, they may borrow money on the interbank market and increase leverage. By increasing leverage and exploiting investment opportunities related to less-liquid assets, they expose themselves to fire-sale risk.

Fire-sale externalities are the key shock transmission mechanism in our model. The term *fire sale* was first used to refer to a custom of selling goods damaged by fire or smoke at discounted prices. By analogy, it is now commonly used to indicate a coerced sale of assets at a depressed price. The sale, usually forced by the need for liquidity to pay creditors, takes place at a price lower than the current market price. Shleifer and Vishny (2011) describe these externalities in detail. Moreover, as banks are marking to market, the temporary loss in the asset value induces balance sheet adjustments to the neighboring institutions holding the same asset, which – in turn – might cause additional sales and, ultimately, downward spirals that spread financial distress across institutions. A high leverage of banks only deteriorates fire-sale externalities.

Additionally, leveraging via interbank borrowing increases interconnectedness of banks in the market. Traditional banks are constrained by capital and liquidity requirements. To face liquidity shocks, they keep a discretionary liquidity buffer on top of the regulatory one. However, should the buffers be insufficient, they can call back interbank lending or sell less-liquid assets at a fire-sale price. In so doing they do not internalize the effects of their management actions, but rather react in a behavioral and mechanistic way.

We solve the model by determining the equilibrium in the interbank lending market and in the securities market. Then, we run simulations whereby an idiosyncratic liquidity shock hits individual banks or asset mangers and observe how the shock propagates across the system. We measure the magnitude of contagion by the fraction of banks that are non-compliant with the regulatory requirements, and by the decline in the security price.

We contribute to three strands of literature on financial stability: (i) systemic risk with contagion channels, (ii) fire-sale externalities and (iii) the interaction of banks and non-banks. The relevance of contagion channels is studied by Allen and Gale (2000); Freixas et al. (2000) (direct channels); Gai et al. (2011); Battiston et al. (2012) and Clerc et al. (2016) (indirect channels). We explore the role of heterogeneity in the system for amplification of funding shocks, indicated, for example, by Caccioli et al. (2012).

A relatively new stream of literature has looked at fire sales in other ways: to explain and reproduce the fire-sale dynamics as cascade effects (Greenwood et al., 2015; Shleifer and Vishny, 2011) or with complex financial networks (Caballero and Simsek, 2013; Cecchetti et al., 2016), to quantify vulnerability of the system to fire-sale spillovers (Hałaj and Kok, 2013; Duarte and Eisenbach, 2013; Greenwood et al., 2015), to study the role of diversification and asset commonality (Caccioli et al., 2015, 2014; Hałaj, 2018; Bichuch and Feinstein, 2019), to assess the role of capital and liquidity regulation, Cifuentes et al. (2005); Bluhm and Krahnen (2011, 2014), or to consider the role of a central bank (Bluhm et al., 2014).

Another strand of literature that our work is related to, deals with interactions of banks with non-banks that may cause systemic risk (Luttrell et al., 2012; Bank of England, 2019). Importantly, behaviors of investors and asset managers can amplify fire-sale contagion. Goldstein et al. (2017) show that funding sources of funds holding illiquid assets are more sensitive to poor performance of those funds. Moreover, information asymmetry exacerbates runs if investors have different beliefs about an asset manager's performance (Chen et al., 2010). Additionally, empirical studies show that redemptions are related to past performance (Sirri and Tufano, 1998; Barber et al., 2016). Asset managers may build cash buffers against redemptions to minimise the fire-sale price impact on the market (Zeng, 2019). However, anticipation of investors' withdrawals may induce asset managers to preemptively build cash buffers (or 'hoard cash') through asset liquidations (Morris et al., 2017), contributing to fire sales.

As far as fire sales are concerned, our paper is most related to the recent papers by Greenwood et al. (2015) and Cont and Schaanning (2017). However, there are some distinctive features of our approach that bring new results to the literature on the fire-sale risk. Cont and Schaanning (2017) and Greenwood et al. (2015) assume that the fire sale generates a price impact, while we endogenize it via a price formation process. Moreover, Greenwood et al. (2015) assume that banks target their leverage ratio when they liquidate assets after a shock, while we assume that banks do it only if voluntary buffers and interbank exposures are exploited and that they target regulatory requirements rather than leverage ratios.

We have several findings. First, following a funding shock, requirements have an ample effect on the contagion spreading. Tighter liquidity regulation immunizes the system from liquidity shocks, but higher capital requirements may spur fire sales and related contagion losses. The argumentation is the following: higher capital decreases default risk and funding costs and makes it more profitable to invest into less-liquid assets financed by interbank borrowing. Second, the speed of contagion depends on which sector the initial shock hits first. Contagion instigated by an asset manager's funding problem initially develops slowly but, as time evolves, it can have a higher impact than that for the initial shock affecting a bank. Monitoring of the asset management sector activities is crucial to assess fire-sale risk. Third, fire sales – the largest causes of contagion losses – are fuelled by imbalances between demand and supply for securities. This is a clear externality of the fire sales that can be mitigated by liquidity injections from a central bank. Fourth, asset managers have the potential to amplify the fire sales, even if the initial shocks hit banks only. Fifth, business models of banks, their heterogeneity in sizes, and their interconnectedness matter for the magnitude of losses under funding stress conditions. Therefore, identification of systemic institutions, i.e., potential plague spreaders, is warranted.

The paper is organized as follows: in Section 2 we explain the underlying theoretical model, the behavior of banks and asset managers, the equilibrium in the interbank lending market, the creation of the network of banks, the simulation of shocks, their propagation, and the dynamics of the system. In Section 3 we present the results of the simulations, and in Section 4 we conclude.

2. The Model

2.1. Overview

We build an agent-based model with heterogeneous interacting agents. The agents are banks, which are endowed with deposits and equity, and asset managers, which are funded with participations.

The agents are able to determine their optimal asset allocation by choosing among the investment opportunities: they can keep cash or invest in a less-liquid security. In addition, banks can access the interbank lending market to lend to or borrow from other banks. Moreover, traditional banks invest half of their assets in loans, which are long-term investments that cannot be liquidated in the short term. Importantly, balance sheets of banks and asset managers are linked by common exposure to securities S. The overlapping portfolios open the amplification channel of fire sales.

The returns on assets are influenced by interactions among the agents. The returns on the less-liquid security are heterogeneous, since agents purchase the security at different points in time at different prices and have different beliefs about their future returns. The expected return on the interbank lending is instead homogeneous across banks and is determined in equilibrium that minimises the excess demand for money. Due to counterparty risk, banks charge a premium for interbank lending such that the expected return is equal to the equilibrium risk-free rate.

We use simulations to study dynamics of the system hit by a funding shock. For any selected set of parameters, the institutions define their optimal individual balance-sheet structures by maximizing their profits. Banks' maximization is constrained by the liquidity and capital requirements. Asset managers are constrained by the self-imposed liquidity requirement. Consequently, the asset allocations of banks, the interbank lending market, and the market structure are determined.

There are three types of actions that an institution can take to adjust the balance sheet after a shock: use its cash reserves, call back interbank lending or, as a last resort, sell the less-liquid security. The set of available actions of any institution depends on the individual allocation of assets.

We assume that, once the funding liquidity shock hits the banks, the corresponding liquidity remains in the system and it is allocated across the non-shocked banks on a pro-rata basis. The resulting dynamics unwind in a closed system, i.e., there is no exogenous outflow or inflow of liquidity. Nevertheless, as a robustness check we run simulations with the liquidity withdrawn from the system. The modification does not affect the dynamics of the system but accelerates the cliff effect of the impact of the fire sale.

The initial shock is spread across the system through two channels of contagion: the direct linkages among banks on the interbank market, and the indirect linkages through overlapping portfolios.

We also run simulations assuming that an initial liquidity shock hits an asset manager, i.e., part of the asset manager's participations is redeemed. After the shock, an asset manager can reduce its cash holdings or sell the security. In the latter case, it also leads to the opening of the securities market.

Furthermore, banks periodically check if they meet capital and liquidity regulatory requirements and, in case they do not, they take actions to meet the requirements again; in particular, they may deleverage.

The following subsections provide a mathematical formulation of the model.

2.2. Agents' optimisation problem

2.2.1. Set-up of the financial system

The financial system in the model is composed of $N_B + N_{AM}$ financial institutions (agents) from a set of banks ($\mathcal{B}$) and a set of asset managers ($\mathcal{AM}$). We index the agents with $i \in \mathcal{B}$, where $\mathcal{B} = \{1, 2, \ldots, N_B\}$, and with $m \in \mathcal{AM}$, where $\mathcal{AM} = \{1, 2, \ldots, N_{AM}\}$. The two sets of institutions are differentiated in two main aspects: the investment opportunities and the regulatory constraints.

Bank i balance sheet

Assets	Liabilities
Cash (C_i)	Deposits (D_i)
Bank lending (BL_i)	Bank borrowing (BB_i)
Less-liquid assets (S_i)	Equity (E_i)
Loans (L_i)	

Asset Manager m balance sheet

Assets	Liabilities
Cash (C_m)	Participations (P_m)
Less-liquid assets (S_m)	

At time $t = 0$, each bank i is endowed with deposits ($D_{i,0}$) and equity ($E_{i,0}$), and each asset manager m with financial participation ($P_{m,0}$). All agents may invest in a security with random, agent-specific returns denoted r_i^s for banks and r_m^s for asset managers. The heterogeneity of returns is justified as follows. First, in reality, agents buy and sell assets in the secondary market at various prices, which define their internal rate of return or unrealised gains or losses. Second, agents may have different beliefs about future returns.

The interbank lending market is initialised with a common interest rate r^f, drawn randomly. As the market evolves, a tatônnement process ultimately determines the value of r^f in equilibrium, minimizing the gap between realised demand and realised supply for interbank lending (see Subsection 2.3).

2.2.2. Banks' optimization problem

Each bank maximizes profits, choosing the optimal amounts of bank-borrowing, cash, bank-lending and security to hold. The profit function of a bank i is defined as the difference between revenues from investments and cost of funding. More specifically, the profits of bank i at time t are defined as

$$\pi_{i,t} = \pi_{i,t}^{investing} + \sum_{j \neq i} \pi_{ij,t}^{bank\_lending} - cost_{i,t}^{bank\_borrowing},$$

with

$$\pi_{i,t}^{investing} = r_i^s \cdot S_{i,t} \tag{1}$$

$$cost_{i,t}^{bank\_borrowing} = (r^f + r_i^{PD}) \cdot BB_{i,t} \tag{2}$$

$$\pi_{ij,t}^{bank\_lending} = \begin{cases} BL_{ij,t} \cdot (r^f + r_{j,t}^{PD}), & \text{with probability } (1 - PD_{j,t}) \\ (1-\xi) BL_{ij,t} \cdot (r^f + r_{j,t}^{PD}), & \text{with probability } PD_{j,t} \end{cases} \tag{3}$$

where $S_{i,t} = lla_{i,t} \cdot p_t^{mkt}$ is a book value of the $lla_{i,t}$ units of the less-liquid asset that bank i holds at time t, at the current market price p_t^{mkt}; $r_{i,t}^{PD}$ is a risk premium determining borrowing costs of bank i, and it depends on the default probability of the bank. ξ is the loss given default, fixed across all banks and constant through time. $BL_{ij,t}$ is a lending volume from bank i to bank j. Finally, $PD_{i,t}$ is bank i's default probability, defined in Subsection 2.2.4.

Even though the profit functions of banks depend on other banks' default probabilities and amounts of lending to other banks, the expected profit optimised by banks only depend on their own parameters. Since the expected profits from interbank lending are the same as what banks would earn on a risk-free outside option, $\mathbb{E}(\pi_{ij,t}^{bank\_lending}) = BL_{ij,t} \cdot r^f$, the expected net profit of a bank i reads[1]:

$$\mathbb{E}(\pi_{i,t}) = r^f \cdot BL_{i,t} + r_i^s \cdot S_{i,t} - r^f \frac{1}{1 - \xi PD_i} \cdot BB_{i,t} \tag{4}$$

where $BL_{i,t} = \sum_{j \neq i} BL_{ij,t}$ is the aggregate volume of interbank lending of bank i at time t. In Section 2.3 we define the interbank lending market with bilateral volumes of interbank deposits emerging in equilibrium.

The profit maximisation of a bank is subject to structural, regulatory, and self-imposed constraints.

[1] For a complete derivation of the equation, please refer to Appendix A.

The structural balance sheet identity holds, i.e.

$$C_{i,t} + BL_{i,t} + S_{i,t} + L_{i,t} = D_{i,t} + BB_{i,t} + E_{i,t} \tag{5}$$

and we assume that half of the bank's assets is invested in sticky assets, i.e., loans,

$$L_{i,0} = \frac{D_{i,0} + BB_{i,0} + E_{i,0}}{2} \tag{6}$$

The loans L are sticky, i.e., they cannot be liquidated in the short horizon of the assumed funding shock, consistent with some other studies (see, e.g. Hałaj (2016)). These sticky loans usually constitute a significant part of the balance sheet. Moreover, they increase risk-weighted assets and need capital given regulatory constraints.

2.2.3. Regulatory constraints

Following the Basel III regulatory principles[2], banks are constrained by a liquidity requirement to cover potential liquidity shocks. Also, banks are bound by a capital regulatory ratio that requires them to hold capital to cover at least a fraction γ_{min} of their risk-weighted assets. Banks periodically check if they comply with the regulatory requirements and proceed with the necessary adjustments.

Capital requirement. We define the equity ratio $\gamma_{i,t}$ of bank i at time t as

$$\begin{aligned} \text{Equity Ratio}_{i,t} = \\ \gamma_{i,t} &= \frac{\text{Total Capital}}{\text{Risk Weighted Asset}} \\ &= \frac{C_{i,t} + S_{i,t} + BL_{i,t} + L_{i,t} - D_{i,t} - BB_{i,t}}{\chi_S \cdot S_{i,t} + \chi_{BL} \cdot BL_{i,t} + \chi_L \cdot L_i} \end{aligned} \tag{7}$$

where χ_S, χ_{BL}, and χ_L are the risk weights for securities, interbank deposits, and loans.

Bank must keep the equity ratio higher than the minimum γ_{min} set by the regulating authority, i.e.,

$$\gamma_{i,t} \geq \gamma_{min} \tag{8}$$

The capital requirement is equal for all banks and set to 7% in the baseline parameterization of the model.

Liquidity requirement. We assume that banks must hold liquid assets to cover at least a fraction α_{min}% of deposits. Moreover, each bank has an "appetite for liquidity", β_i, which determines the size of the discretionary liquidity buffer.

The parameter α_{min} is common to all banks and has to be met, while β_i is heterogeneous and self-imposed, and the corresponding discretionary liquidity buffer can be used when the bank needs liquidity or sees profitable investment opportunities. Moreover, a fraction of the interbank borrowing needs to be covered by liquid assets (i.e., bank lending or cash).

[2] `http://www.bis.org/publ/bcbs238.pdf`

Defining $\alpha_{i,t} = \frac{C_{i,t}}{D_{i,t}}$, the liquidity constraint reads:

$$\alpha_{i,t} \geq \alpha_{min} \quad \text{or} \quad C_{i,t} \geq \alpha_{min} D_{i,t} \tag{9}$$

while the bank's discretionary constraints are:

$$C_{i,t} \geq (\alpha_{min} + \beta_i) \cdot D_{i,t} \tag{10}$$

and

$$C_{i,0} + BL_{i,0} \geq (\alpha_{min} + \beta_i)(D_{i,0} + BB_{i,0}) \tag{11}$$

With two liquidity constraints, we try to reflect the principles of a sound and prudent liquidity management. First, banks must abide by the regulatory reserve requirements. Second, they apply a precautionary buffer to cover potential funding outflows.

All in all, each bank chooses the optimal amounts of bank lending, securities, and bank borrowing maximizing the expected profits,

$$\max_{BL_{i,0}, BB_{i,0}, C_{i,0}, S_{i,0}} \mathbb{E}(\pi_{i,0}) = r^f \cdot BL_{i,0} + r_i^s \cdot S_{i,0} - r^f \frac{1}{1 - \xi PD_i} \cdot BB_{i,0} \tag{12}$$

subject to the balance-sheet structural constraints (Eq.6 and Eq.5), the regulatory constraints (Eq.8 and Eq.9), and the discretionary constraint (Eq.10 and Eq.11).

The optimization problem belongs to a well-studied class of linear programming problems, solved for instance with a simplex method (Dantzig, 1963) implemented in any programming language.[3]

A voluntary buffer is introduced only for liquidity, while it is not introduced for the excess capital buffer. This is because only one of the two requirements can bind at the same time, so the excess liquidity buffer implies also the excess capital buffer. This happens because the two requirements are linked via the risk weight on the liquidity instruments in the denominator of the capital ratio.

2.2.4. Endogenous default probability

To determine the default probabilities of banks, which influence interbank borrowing costs, we solve the following fixed point problem. Default probabilities depend on banks' balance-sheets. However, a given bank's balance-sheet structure depends on the interbank borrowing cost, which is a function of the default probabilities assumed by creditors of banks. In the rest of the subsection we show how to consistently choose the default probabilities.

We assume that the risk of assets of bank i is proportional to the initial balance-sheet size $D_i + E_i$ with a proportionality parameter ψ, for simplicity, common across banks. We omit the time subscript for brevity of notation. Returns on securities and interbank lending are normally distributed and the expected return on illiquid loans is 0 but risky. In fact, loans are constant but affect the optimization program through the liquidity and solvency constraints. Let us suppose that the default probability of bank i assumed by its creditors is equal to $\bar{PD}_i$. Consequently, we define net profits of bank i as a normally distributed random variable R_i with:

- mean $r^s S_i + r^f \sum_j BL_{ij} - r^f \frac{1}{1-\xi \bar{PD}_i} BB_i$

[3] To derive a numerical solution, we use a solver implemented in Matlab.

- standard deviation $\psi(D_i + E_i)$

Let us define a default event of bank i if realised losses erode the entire capital E_i. The probability of such an event is the default probability implied by the balance-sheet structure $(S_i, BL_i, D_i, BB_i, E_i)$ and is calculated as follows:

$$PD_i = \mathbb{P}(E_i + R_i < 0) \tag{13}$$

By substituting the mean and standard deviation of the random returns and losses, and by applying standardization of random variables, we obtain

$$PD_i = \Phi^{-1}\left(\frac{-E_i + r^s S + r^f \sum_j BL_{ij} - r^f \frac{1}{1-\xi PD_i} BB_i}{\psi(D_i + E_i)}\right) \tag{14}$$

where Φ^{-1} is the inverse of the standard normal distribution.

Clearly, the defined probability PD_i may not be consistent with the assumed $\bar{PD}_i$. However, we demonstrate that there is a unique probability that is consistent, and it always exists under weak assumptions of the model (see Theorem C.1 in Appendix C). The PDs are then updated each time the rate of return on the interbank market or the return on securities changes.

2.2.5. Asset managers' optimization problem

Each asset manager m initially maximizes its profits $\pi_{m,0}$ by dividing its participations into investment in security and cash:

$$\max_{C_{m,0}, S_{m,0}} \pi_{m,0} = r^s_m \cdot S_{m,0} \tag{15}$$

and the profit maximization is subject only to a discretionary liquidity constraint. With redemption risk, asset managers choose a self-imposed precautionary liquidity buffer to face the possibility of redemptions:[4]

$$C_{m,0} \geq (\alpha_{min} + \beta_m) P_{m,0} \tag{16}$$

2.3. Equilibrium

2.3.1. Notion of equilibrium

Given α_{min}, β_i, γ_{min}, the drawn $D_{i,0}$ and $E_{i,0}$, risk weights χ, and returns on security r^s_i, the equilibrium is any set of $\{(C_{i,0}, BL_{i,0}, BB_{i,0}, S_{i,0})_{i\in\{1,\dots,N_B\}}, r^f\}$ that for any $i \in \{1, \dots, N_B\}$

1. satisfies the budget, liquidity and capital constraints,
2. maximises the bank's expected profits $\mathbb{E}(\pi_{i,0})$,
3. provides maximum clearing of the interbank market, i.e., for any alternative risk-free rate $\bar{r}^f$, the implied holdings of cash, aggregate interbank lending and borrowing volumes, and securities $\{\{(\bar{C}_{i,0}, \bar{BL}_{i,0}, \bar{BB}_{i,0}), \bar{S}_{i,0}\}_{i\in\{1,\dots,N_B\}}, \bar{r}^f\}$ satisfy

$$\left|\sum_i BL_{i,0} - \sum_i BB_{i,0}\right| \leq \left|\sum_i \bar{BL}_{i,0} - \sum_i \bar{BB}_{i,0}\right|$$

[4] See Morris et al. (2017) for the empirical evidence.

The optimal $\left| \sum_i BL_{i,0} - \sum_i BB_{i,0} \right|$ converges to 0 as the number of banks increases to infinity and banks are heterogeneous in parameters. We confirmed this observation using a simulation method by randomly generating banking systems with an increasing number of banks (see Appendix G). The exact clearing is not necessary in our analysis since we are interested in qualitative results.

The definition of the equilibrium does not include the interbank market. Interbank connections are implied by the aggregate supply (which equals the demand of interbank liquidity, by the definition of equilibrium) and any matching mechanism that distributes liquidity across banks.

The equilibrium may not be unique. The argument is standard in mathematical programming, and we present it in Appendix F. In the Appendix D, we present one possible and intuitive process of interbank rate adjustments that reaches the equilibrium allocation of banks' assets and liabilities.

2.3.2. Balance sheets in equilibrium

Each bank chooses its optimal portfolio depending on the relative cost of borrowing from other banks $r^f \frac{1}{1-\xi PD_i}$, the returns on money lending r^f, and the returns on the security r_i^s. Heterogeneity of PDs and returns on less-liquid securities imply that after optimization we can identify three categories of banks that we call "lenders," "investors," and "high leverage."

Lenders are banks characterised by $r_i^s < r^f < r^f \frac{1}{1-\xi PD_i}$, i.e., the security is less profitable than lending in the interbank market. They keep the least regulatory cash and try to lend all the remaining available liquidity.

High leverage banks are characterised by

$$\text{(i)} \quad r^f < r_i^s$$

and

$$\text{(ii)} \quad r^s > \frac{1/(1-\xi PD_i) - (\alpha_{min} - \beta_i)}{1 - \alpha_{min} - \beta_i} r^f$$

Their return on securities is larger than the sum of the cost of interbank borrowing and the cost of additional liquid assets covering liquidity risk of interbank borrowing. Therefore, the second condition follows that

$$\underbrace{(\alpha_{min} + \beta_i) r^f \cdot BB_i}_{\text{liquidity requirement}} + \underbrace{(1 - \alpha_{min} - \beta_i) r_i^s \cdot BB_i}_{\text{return on securities}} > \underbrace{\frac{r^f}{1 - \xi PD_i} BB_i}_{\text{cost of interbank borrowing}}$$

These banks will borrow money in the interbank lending market to invest in the security, keeping the least regulatory cash and the least discretionary bank lending. Consequently, the high return on securities expected by bank i creates incentives for the bank to increase risk-taking and leverage up to the regulatory minimum.

Investors are banks characterised by

$$r^f < r_i^s \leq \frac{1/(1-\xi PD_i) - (\alpha_{min} - \beta_i)}{1 - \alpha_{min} - \beta_i} r^f$$

i.e., for which the security is more rewarding than lending to other banks. However, borrowing costs are higher than the returns on the security adjusted for the cost of necessary liquid assets they hold. They keep the least regulatory cash, do not borrow from other banks, and invest the rest in security.

These three types of asset allocations characterize interactions among the financial institutions on the interbank market and through fire sales.

2.3.3. The emergence of the bank network

Once the clearing return r^f in the interbank lending market is set, banks need to be matched to clear the market.

Let us define $BL_{ij,t}$ as the liquidity lent by bank i to bank j at time t, and $BB_{ij,t}$ as the liquidity borrowed by bank i from bank j. Obviously, $BL_{ij,t} = BB_{ji,t}$. Banks are matched randomly and they exchange

$$BL_{ij,0} \colon = \min\{BL_{i,0}, BB_{j,0}\}$$

If $(BL_{i,0} - BB_{j,0}) > 0$, i.e., if bank i has residual liquidity to lend, bank j becomes inactive and bank i enters the interbank lending queue with its residual liquidity supply, and the opposite happens if $(BL_{i,0} - BB_{j,0}) < 0$. When either the supply or the demand queue has no active bank left, the market closes at the equilibrium risk-free return r^f minimizing the excess demand (see Subsection 2.3.1). In case a bank is left with unsatisfied supply (demand) due to the imperfect market clearing, banks adjust investment portfolios by re-maximizing profits subject to the amounts of bank borrowing and lending they could access on the interbank market.

Random matching of banks on the interbank market may create mutually exposed banks. Some banks borrow and lend at the same time.[5]

Since asset managers do not have access to the interbank lending market, so they are not exposed to direct interbank contagion effects, but are indirectly affected through the fire sales.

2.4. Model dynamics

After the system has been initialised and the banks have determined their optimal balance sheet, the second phase of the model starts: an exogenous funding liquidity shock hitting a bank i (or an asset manager m) is considered.

We assume that – under a shock scenario – the liquidity remains in the modelled system. The financial system is closed. This means that deposit outflow from the shocked bank i is transferred to other $N_B - 1$ banks, proportional to their initial deposits.

The initial funding liquidity shock has magnitude $\sigma \leq 1$, translated into outflow $\sigma \cdot (D_{i,0} + BB_{i,0})$. Clearly, the shock does not exceed the total value of "non-sticky" assets but can exceed the usable liquidity buffer, i.e., cash.

Two cases of funding shock versus liquidity buffers are possible:

Case I: if the funding liquidity shock is smaller than the available liquidity at time 0, i.e., $\sigma(D_{i,0} + BB_{i,0}) \leq C_{i.0}$, the bank simply pays out the corresponding liabilities;

Case II: if, on the contrary, the funding liquidity shock is larger than the available liquidity, i.e., $\sigma(D_{i,0} + BB_{i,0}) > C_{i,0}$, then the bank needs more liquidity to cover the shock.

In this second case, to face the liquidity shock, bank i will need an extra amount λ_i of cash.

Therefore

$$\lambda_{i,0} = \begin{cases} \sigma(D_{i,0} + BB_{i,0}) - C_{i,0} & \text{if} \quad C_{i,0} \leq \sigma(D_{i,0} + BB_{i,0}) \\ 0 & \text{otherwise} \end{cases}$$

[5] In the simulations, on average not more than one bank out of 60 was lending and borrowing at the same time.

We assume that bank lending is perfectly liquid. It can be called back with no discount if the corresponding borrower has enough liquidity to pay back the borrowed amount. The security is not fully liquid and can be sold on the market at any time, but at a discounted price.

We assume that banks divest their portfolio holdings following a pecking order, similarly to Aikman et al. (2009) and Hałaj (2013). Banks will first use all the cash, then they call back the interbank lending, proportionally across banks (i.e., borrowers). Finally, as a last resort, banks proceed to the sale of the less-liquid security. If a bank does not manage to face the funding liquidity shock even after selling the security, it goes into an "illiquidity" state.

A liquid bank after the initial shock periodically checks its financial ratios against the regulatory requirements. If any of the two regulatory requirements are violated, i.e.,

$$C_{i,t} < \alpha_{min} D_{i,t} \quad \text{or} \quad (\text{Total Capital}_{i,t}/\text{RWA}_{i,t}) < \gamma_{min}$$

the bank determines the necessary divestment in order to "rebuild" the adequate liquidity or capital regulatory buffer, and repeats the same cycle of actions: first, it calls back bank lending and then sells the less-liquid security. Successively, it checks the requirements again and if they are not met, the interbank exposures are updated on its counterparts' balance sheets according to the loss given default parameter. We assume that in times of distress, banks simply follow a passive portfolio management strategy.

Ergo, there is a threshold that triggers movements on the market following the initial shock, and if we define $\alpha_{i,0} = \frac{C_{i,0}}{D_{i,0}}$ as the actual total liquidity buffer of bank i held at $t = 0$, this critical loss amounts to

$$\sigma^{i*}(D_{i,0} + BB_{i,0}) = D_{i,0} \cdot (\alpha_{i,0} - \alpha_{min}) + \sigma \alpha_{min} D_{i,0}$$

$$\sigma^{i*} = \frac{(\alpha_{i,0} - \alpha_{min}) D_{i,0}}{(1 - \alpha_{min}) D_{i,0} + BB_{i,0}}$$

A decrease in the market price affects both banks and asset managers. A price drop causes a variation in the book value of both security and equity, implying a change of equity ratio for banks, and a potential trigger of redemption for asset managers. Redemption of participations happens because investors, observing a decrease in the asset value of the asset managers, try to anticipate further losses and decide to "withdraw" or simply not renew their participations.

Therefore, we introduce the concept of fundamental value $\overline{f}$ of the less-liquid security. Both banks and asset managers k have an individual noisy valuation of the fundamental value, defined as $f_k := \overline{f} + \epsilon_k$, centered in $\overline{f}$. In other words, ϵ_k is a random variable with mean 0.

A drop of the market price of securities below an asset manager's noisy fundamental value, generates panic among the depositors who redeem shares of their participations. The redemptions are based on asset managers' realised returns on assets. This assumption is consistent with the empirical findings of Sirri and Tufano (1998) or Goldstein et al. (2017) and is implemented in an agent-based framework by, e.g., Hałaj (2018). Redemptions increase exponentially with the percentage drop of the price, i.e., with $\delta_{t-1,t} = \frac{p_{t-1}^{mkt} - p_t^{mkt}}{p_{t-1}^{mkt}}$ between $t-1$ and t, with a sensitivity η:

$$redemption_{m,t}(\delta_{t-1,t}) = \mathbb{1}_{\{p_t^{mkt} < f_m\}} P_{m,t} \cdot (exp(\eta \delta_{t-1,t}) - 1) \tag{17}$$

The exponential relationship between performance and redemptions accounts for a potential convexity of fund flows as a function of asset values (Ferreira et al., 2012).

When an asset manager faces a redemption, it reacts in a similar fashion to banks with a liquidity shock. First, it uses cash. Only then does the asset manager sell the less-liquid security. The sale amounts to

$$N^S_{m,t} = redemption_{m,t}(\delta_{t-1,t}) - C_{m,t} \tag{18}$$

In case the sale of the entire portfolio of securities does not cover the redemptions, the asset manager exits the market. As we are interested in the downward dynamics of the fire sale and the simulated time span is relatively limited, we do not account for a potential price recovery of the less-liquid asset.

At any time, each bank i determines the amount $\lambda_{i,t}$ of liquidity needed to pay the liabilities called back by other banks ($BB^{calledback}_{ij}$) or withdrawn by depositors. At time 0, for the shocked bank i,

$$\lambda_{i,0} = \max\{\sigma(D_{i,0} + BB_{i,0}) - C_{i,0}, 0\}$$

In later iterations, any bank i can have positive values of $\lambda_{i,t}$, $\nu^{Liq}_{i,t}$, and $\nu^{Cap}_{i,t}$, representing the amounts of divestment needed to pay back the borrowings, to meet the liquidity and capital requirement at time t, respectively. The three quantities are defined as follows:

$$\lambda_{i,t} = \sum_j \{(BB^{calledback}_{ij,t})\} \tag{19}$$

$$\nu^{Liq}_{i,t} = \max\{0, \alpha_{min} D_{i,t} - C_{i,t}\} \tag{20}$$

$$\nu^{Cap}_{i,t} = \max\left\{0, \frac{\gamma_{min} \cdot RWA_{i,t} - E_{i,t}}{\gamma_{min}}\right\} \tag{21}$$

Once these amounts have been determined, banks know the portion of BL they need to call back and recover. If the interbank lending is called back, a request to pay back the borrowed amounts, in part or in full, is sent to the borrowing banks. When a bank or an asset manager decides to sell the less-liquid security, the market for the security opens. If the supply exceeds the demand, the price is lowered to stimulate demand: when supply is fully exhausted, the market closes and the final price – endogenously determined – becomes a new market price p^{mkt}_t. This price corresponds to securities sold by banks and asset managers, i.e., supply side $q^s_{i,t}(p^{mkt}_t)$ and $q^s_{m,t}(p^{mkt}_t)$, and purchased by other banks and asset managers, i.e., demand side $q^d_{i,t}(p^{mkt}_t)$ and $q^d_{m,t}(p^{mkt}_t)$. The new market price is used by institutions to mark to market the less-liquid security in their books. A falling security price impacts asset managers' performance and triggers investors' redemptions, where redemptions exemplify a funding liquidity shock suffered by asset managers. We describe technicalities of these call-back and market clearing processes in Appendix E.

At time $t+1$, all banks and asset managers update balance sheets with the new quantities of sold and bought securities, as well as the book value of the marked-to-market security and the equity value. Banks' capital and asset managers' participations change due to losses incurred on the securities sold below its book value and revaluation of the securities holdings. For sellers of securities, the losses stem from volumes sold below their book value and from revaluation of the remaining securities:

$$\begin{aligned}
\text{(banks)}: \quad & E_{i,t+1} = E_{i,t} + \left(p^{mkt}_{t+1} - p^{mkt}_t\right)\left(q^s_{i,t}(p^{mkt}_{t+1}) + lla_{i,t}\right) \\
\text{(asset managers)}: \quad & P_{m,t+1} = P_{m,t} + \left(p^{mkt}_{t+1} - p^{mkt}_t\right)\left(q^s_{m,t}(p^{mkt}_{t+1}) + lla_{m,t}\right)
\end{aligned}$$

For buyers of securities, losses are related to revaluation of the securities already in their portfolio at t, but also to revaluation of the newly purchased securities:

$$(\text{banks}): \qquad E_{i,t+1} = E_{i,t} + \sum_{\tau=1}^{\tau^*} \left(p_{t+1}^{mkt} - p^{\tau}\right) q_{i,\tau}^{d}(p^{\tau}) + \left(p_{t+1}^{mkt} - p_{t}^{mkt}\right) lla_{i,t}$$

$$(\text{asset managers}): \qquad P_{m,t+1} = P_{m,t} + \sum_{\tau=1}^{\tau^*} \left(p_{t+1}^{mkt} - p^{\tau}\right) q_{m,\tau}^{d}(p^{\tau}) + \left(p_{t+1}^{mkt} - p_{t}^{mkt}\right) lla_{m,t}$$

Banks then proceed with periodical checks of the regulatory requirements and the model iterates.

2.5. Parameterization

To pin down values of parameters of the model, we use data disclosed by the European Banking Authority (EBA) and the European Fund and Asset Management Association (EFAMA). EBA publishes the transparency exercise data[6] that we use to extract information about T1 capital and leverage ratios of the largest 60 banks in the EU as of December 2017. They account for almost 90% of the EU banking system in terms of capital. By dividing the capital figure by the leverage ratio, we obtain a proxy of the total balance-sheet size. A report of EFAMA (2018) contains information about the total assets under management of the European funds. Importantly, total assets calculated for the banks in the sample covered by the EBA transparency exercise disclosures account for approximately 60% of the combined assets of banks and asset managers. We use this proportion to parameterize the relative sizes of banking versus asset management sectors in the model. Moreover, we can use the implied distribution of banks' capital figures and leverage ratios to draw random samples of banks. To simulate the financial system in the model, we fit the observed distribution with a log-normal distribution.[7] For simplicity, we assume the same probability distribution for the distribution of participations.

We assume that the financial system is composed of 60 banks and 40 asset managers. The key exogenous variables used in the model are gathered in Table 1. Capital $E_{i,0}$ is drawn from a log-normal distribution

$$logN(\text{mean} = 9.88, \text{st. dev} = 0.87)$$

Then, for each bank we draw a leverage ratio lev_i from the estimated normal distribution

$$N(\text{mean} = 6\%, \text{st. dev} = 1.9\%),$$

which we truncate at a minimum observed value of the leverage ratio in the sample. The resulting lower bound for the leverage ratio is 3.5%. Deposits $D_{i,0}$ are implicitly calculated as a difference between total assets $A_{i,0} := \frac{E_{i,0}}{lev_{i,0}}$ and capital $E_{i,0}$. Participations P^m financing activities of the asset managers follow a similar log-normal distribution, $logN(\text{mean} = 9.88, \text{st. dev} = 0.87)$, which is assumed to be independent from the distribution describing banks' capital figures. In this way, the sizes of agents' balance sheets are comparable, and we can conveniently control the relative size of the banking system versus the asset managers' system by choosing the number of entities.

[6] https://eba.europa.eu/-/eba-transparency-exercise

[7] The class of log-normal distributions gives us the best fit in the following group of distributions: normal, log-normal, exponential, and Pareto.

We specify parameters determining the profitability of assets. The return on the security r_i^s follows a uniform distribution $U(0, 0.15)$. The appetite for liquidity β_i is drawn from a beta distribution $\beta(2, 40)$. It implies a mean value of the additional liquidity buffer of 4.8% of deposits, with standard deviation 3.3%. The noisy fundamental value of securities ϵ is drawn from a normal distribution $N(0, 0.1)$.

The loss given default is deterministic and set to 40%. The risk weights for securities χ_S, interbank deposits χ_{BL}, and loans χ_L are respectively set to: $\chi_S = 1$, reflecting the risk weight applied in Basel II, $\chi_{BL} = 0.2$, reflecting the actual one for OECD countries (Bluhm et al., 2014) and $\chi_L = 1$, reflecting the one for retail exposures.[8]

The sensitivity of redemptions to asset managers' performance is set to $\eta = 1.0$. This calibration means that a drop of prices from 1 to 0.99 (i.e., by 1%) implies redemptions equal to 0.6% of the initial participations.[9]

The magnitude of the initial liquidity shocks $\sigma \in [0, 0.25]$. This is justified twofold. First, only part of the deposit base is insured. A report of the European Commission (EPSC, 2015) quantifies deposits in euro-area countries that are covered by the national deposit guarantee schemes as 48% of the total deposit base. The uninsured fraction of the funding sources is more prone to the run risk. Second, we can resort to liquidity risk regulation that stipulates rules to calculate funding outflow under stress. They are used in the Liquidity Coverage Ratio (LCR) following Basel III guidelines. EBA (2019) collects information about deposit outflows rates for banks in the European Union. The outflows represent on average about 20% of the deposit base. We extend this number to a 25% maximum funding shock to account for heterogeneous banks.

3. Results

We use simulation techniques to explore the market dynamics implied by the model. The first phase of the simulations is about a random generation of the initial structures of the financial system. To this end, we define nine regulatory regimes, and for each we generate 100 different systems of banks and asset managers.[10] The regimes are defined by combinations of different levels of liquidity α_{min} and capital requirement γ_{min}, which belong to a set of the simulation parameters presented in Table 1. The values that the minimum liquidity requirement α_{min} can take are $\{8\%, 10\%, 12\%\}$, while the minimum capital requirement γ_{min} takes values of $\{6\%, 7\%, 8\%\}$. The model is then initialised for a given set of parameters $(seed, \alpha_{min}, \gamma_{min})$.

In the second phase of the model we simulate funding shocks. We select one institution, $k \in \mathcal{B} \cup \mathcal{AM}$, hit by the initial exogenous shock. Then, we select the magnitude of the shock, $\sigma \in [0, 0.25]$.

Once a funding liquidity shock is applied, we run simulations of contagion for all generated financial systems. Each simulation takes 50 iterations. Each time step can be interpreted as 1 day. During these periods, banks and asset managers can react to movements of the interbank lending market, to changes of the security price, and to redemptions, and adjust balance sheets to meet the regulatory requirements.[11] The steps of the simulation process and of the model dynamics are summarised in the flowchart in Appendix B.

[8] `http://www.bis.org/bcbs/publ/d347.pdf`

[9] We apply other parameters, i.e., 0.5 and 1.5, to verify sensitivity of the results. Qualitative results remain unchanged.

[10] To increase comparability between regimes, we use the same set of seeds in the random generator.

[11] Each simulation is thus characterised by (seed, α_{min}, γ_{min}, k, σ), with $\sigma \in [0, 0.25]$ by steps of 0.01, $k \in \mathcal{B} \cup \mathcal{AM}$, $\gamma_{min} \in \{6\%, 7\%, 8\%\}$ and $\alpha_{min} \in \{8\%, 10\%, 12\%\}$. Thus, a total of $26 * 9 * 100 * 100 = 2340000$ runs are performed.

Exogenous variables		
Equity	E_i	$logN(\text{mean} = 9.88, \text{st. dev} = 0.87)$
Leverage ratio	lev_i	$N(\text{mean} = 6\%, \text{st. dev} = 1.9\%)$
Participations	P_m	$logN(\text{mean} = 9.88, \text{st. dev} = 0.87)$
Returns on less-liquid assets	r_k^s	$U(0, 0.15)$
Noise to fundamental value of securities	ϵ_k	$N(\text{mean} = 0, \text{st. dev} = 0.1)$
Self-imposed liquidity buffer	β_k	$\beta(\text{shape}_1 = 2, \text{shape}_2 = 40)$
Loss given default	ξ	40%
Risk weight on bank lending (BL)	χ^{BL}	20%
Risk weight on security (S)	χ^S	100%
Risk weight on loans (L)	χ^L	100%
Sensitivity to redemptions	η	1
Simulation parameters		
Minimum liquidity requirement	α	$\in \{8\%, 10\%, 12\%\}$
Minimum capital requirement	γ	$\in \{6\%, 7\%, 8\%\}$
Exogenous shock	σ	$\in \{0, 0.01, 0.02, ..., 0.25\}$
Initially hit institution	k	$\in \mathcal{B} \cup \mathcal{AM}$

Table 1: Summary of variables and parameters: Exogenous variables are set to initialise the system. Then simulations are run for every possible combination of simulation parameters.
Note – $i \in \mathcal{B}$, where $\mathcal{B} = \{1, 2, \dots, N_B\}$; $m \in \mathcal{AM}$, where $\mathcal{AM} = \{1, 2, \dots, N_{AM}\}$; and $k \in \mathcal{B} \cup \mathcal{AM}$; $logN$ – log-normal distribution; N – normal distribution; U – uniform distribution; β – beta distribution.

The impact of any initial shock is measured by the number of banks breaching the regulatory liquidity and solvency requirements. These banks are referred to as non-compliant banks. We collect the number of non-compliant banks in each iteration, and at the end of the simulation, this measure allows us to compare the dynamics of the financial system across different scenarios. Market prices of assets are complementary measures of contagion effects. Therefore, we investigate how the initial exogenous, idiosyncratic shocks impact the price of securities S, in terms of both levels and dynamics.

We focus on the effects of regulation, speed of contagion, fire-sale externalities, and the role of asset managers and banks' asset allocation in shock amplification.

3.1. Regulatory requirements

We can look at implications of tightening regulatory requirements. Averaging the results over all simulations, in Fig. 1 we plot the final share of banks breaching liquidity or capital requirements (solid line) and defaulting banks (circles), i.e., with the capital ratio falling below 0%, and the market price of the securities (dotted line) for each intensity σ of the initial shock. Intuitively, bigger initial shocks correspond to lower fire-sale prices and more non-compliant institutions.

These conclusions are consistent for both measures of distress in the financial system, i.e. for capital ratios falling below the regulatory minimum threshold γ and for capital ratios below 0. For simplicity, we focus on the non-compliant banks (with either capital or liquidity requirements). It is a more conservative approach and reflects the fact that other market participants would assess banks' soundness based on distance to regulatory thresholds and that breaching those requirements would trigger supervisory actions.

We can observe a dispersion of the number of the affected banks across seeds. Different seeds of simulations determine the structure of the market and different initially shocked agents. Fig. 2 shows a distribution of non-compliant banks after 50 days and across simulations. For many of the scenarios, all the banks comply with the regulatory rules after the shock, and, only in some cases, the fraction of the non-compliant banks is positive. Clearly, the fraction also depends on the size of the shock.[12] Despite dispersion, the qualitative results about the dynamics of the non-compliant banks are robust.

Finally, we find that an increase in the minimum capital requirement might exacerbate the contagion effects. This is shown in Fig. 1 where the yellow solid line lies above the red and the blue ones. There are three reasons for this phenomenon to occur. First, capital constraints become binding quicker if capital ratios must be maintained above higher regulatory thresholds. Unlike in Greenwood et al. (2015), banks do not liquidate assets immediately after they experience a loss, since they do not target their starting point capital ratio. In our model, banks may only enter a fire-sale if they must bring their solvency ratios above the required minimum level. Some banks keep voluntary buffers that they consume to absorb shocks. In other words, in Greenwood et al. (2015), higher leverage (i.e., lower capital ratio) mechanistically induces banks to sell more. In our model, lower required capital ratio means that banks start selling later and only if their capital buffers are utilized and the capital ratios hit the boundary. In fact, the management buffers, which are built by banks in the model to optimize profits, decline as the required capital ratio increases: from 4.1pp for 6% requirement thresholds to 3.2pp for 8% threshold.

Second, under more stringent capital requirements, more banks decide to invest in risky securities. Higher capital reduces counterparty risk; it translates into a lower default probability and – consequently – lower interbank borrowing costs. Banks that were previously only granting loans to other banks may turn to the relatively more profitable security; banks which were only investing in the security may want to borrow additional money to reinvest; and highly leveraged banks may opt to borrow even more. This has implications for the network of exposures to risky and less-liquid assets, whereby more banks are exposed to the security market and the level of securities holdings in the system is higher. As a result, the system is more prone to the propagation of second-round effects related to a drop in the security prices in the fire sale.

Third, higher capital requirements may also increase investment opportunities for some banks due to easier access to interbank funding. Initially a higher capital requirement imposes a cap on risky security investments via risk-weighted assets, thus also limiting profitable leverage opportunities for banks which were borrowing money to invest in the security. Hence, a decrease in the money demand (see Fig. 4) paired with an almost unchanged money supply implies a lower risk-free rate of return on interbank lending. This may incentivize banks to increase leverage in the interbank market. Notably, the endogenous relationship between capital requirements and banks' exposures to securities and the interbank lending rate is another difference between our model and the seminal work of Greenwood et al. (2015), who do not internalise banks' behaviors on these markets.

The combination of these three dynamics translates into two channels through which a higher capital requirement exacerbates the contagion. Banks react sooner to shocks and more banks are directly exposed to fire-sales contagion.

The indirect effect of more stringent capital requirements overall reverses its direct effect, by implicitly creating a system with individually stable banks but in a more rigid and fragile system.

[12]Statistics of dispersion for the fraction of non-compliant banks are presented in Appendix G.

Whether the initially shocked agent is a bank, or an asset manager matters for contagion losses. Relatively small shocks can be absorbed by asset managers, which are not bound by regulatory constraints (in Fig. 1). For small initial shocks originated in the asset managers' sector, the number of non-compliant banks is always equal to zero; on the contrary, when originated in the banking system, a shock of the same size may lead to fire sales. This notwithstanding, a larger shock (higher than 0.2) together with a tighter capital regulation of banks would adversely impact a larger share of the banking system if the initial shock starts propagating from the asset managers' sector.

Tightening of the liquidity conditions can be observed in Fig. 3. Clearly, a higher coverage of the potential funding outflow with liquid assets insulates the banks from the funding shocks. It is mostly discernible when the initial shock hits an asset managers' funding sources. However, the contagion effects, measured in terms of either asset prices or non-compliant banks, are less sensitive to the level of liquidity requirements. As in the case of capital requirements, conclusions are valid if contagion effects are measured by the fraction of defaulting banks.

Solvency requirements may have a greater potential to stabilise the financial system vulnerable to funding shocks and fire sales. The illustration is provided in Fig. 5. We measured the fraction of banks for which solvency or liquidity becomes binding, and we make two observations. First, during the initial periods, liquidity constraints are binding for a small fraction of banks, triggering the liquidation of assets. Second, in later periods, binding solvency constraints prevail, reaching 15% of the banking system in the case of the most stringent regulatory constraints in the performed simulations (i.e., $\alpha = \gamma = 8\%$). Consequently, the model suggests that the solvency rules should be considered and applied with caution by the regulators.[13]

3.2. Speed of contagion

Shock origination and tightening of the regulatory conditions influence how fast and how broad the contagion losses spread across the system. Moving from left to right in rows of the 3×3 Fig. 6 and Fig. 7, we can see the effect of the higher capital requirement. An increase in the capital requirement puts a cap on the risky investments, thus limiting the exposure to possible contagion. On the other hand, it decreases the incentives to increase leverage by borrowing more. We plot the fractions of non-compliant banks for different magnitudes of the shock σ, and for each σ a line is drawn.[14] As a result of a shock, higher capital requirements create an excess money supply in the interbank lending market, which depresses the risk-free return rate determined through the tâtonnement process (see also Fig. 4). The limited demand for interbank borrowing together with a lower risk-free return rate on money lending make the alternative investment into the less-liquid security more appealing.

While the system ultimately reaches an equilibrium fraction similar to that of non-compliant banks, it takes longer when the shock originates in the banking sector. Fig. 6 shows a significant number of cases when a bank is hit first, and the number of non-compliant banks after 25 days (x = 25) is lower than under the scenario in which the first-hit is an asset manager (see Fig. 7). Thus, not all banks are in default. This is due to the contagion process described above: an asset manager that needs to engage in fire sales affects banks only with security holdings and high-leverage banks holding both securities and bank lending. Consequently, this asset manager also affects banks active

[13] Robustness of the results to the arbitrary choice of the fraction of banks assets invested into sticky loans L is presented in Appendix G.

[14] Additional robustness analysis is presented in Appendix G.

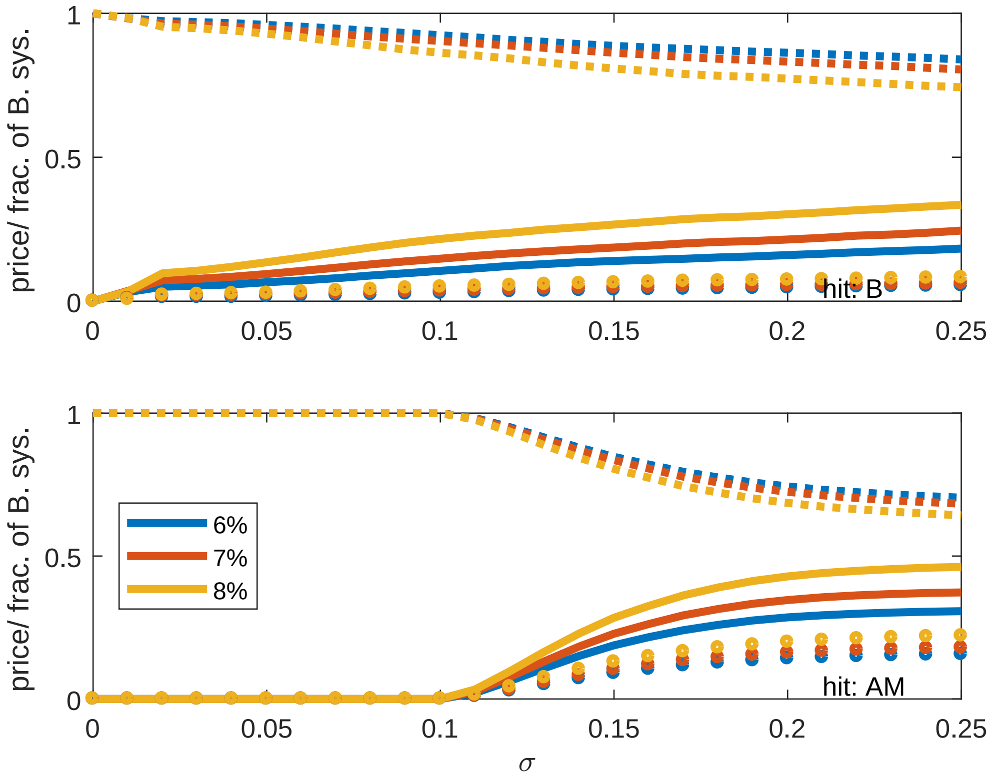

Figure 1: Impact of the funding shock on asset price and fraction of non-compliant and defaulting banks (i.e. breaching liquidity or solvency requirements) under different regimes of solvency regulation.
Note – x-axis: shock σ, i.e., share of deposits (participations) outflows for banks (asset managers); y-axis: price (dotted lines), fraction of non-compliant banks (solid lines), and fraction of defaulting banks (circles) in the banking system.
Top pane: Initial random shocks hit banks. Bottom pane: Initial random shocks hit asset managers. For all simulations, the liquidity requirement is set to $\alpha = 10\%$.

exclusively in the interbank lending market. On the other hand, a bank might affect the other banks by calling back interbank lending, but it will take longer to reach them all through this channel, or to trigger a fire sale in a later moment.

The charts in Figs 8 and 9 depict how quickly and by how much the price decreases: each subplot shows the evolution of prices during the 50 iterations of the simulation. The price falls more abruptly, more quickly, and deeper when the shock originates in the asset managers' sector. However, consistently with Fig. 6 and Fig. 7, small shocks to asset managers are absorbed with no impact on the price of the less-liquid security. No fire sale is triggered for small shocks since they can be absorbed by agents' own liquidity buffers. The difference in outcomes between traditional banks and asset managers is implied by the fact that, after using the liquidity buffers, the former group needs to re-adjust the balance sheet to meet the requirements, while the latter does not. But as soon as asset managers trigger the fire sale and the price starts to decrease, banks join the fire sale and add to the distress in the market. Moreover, for higher capital requirements the price drops more than for lower capital requirements.

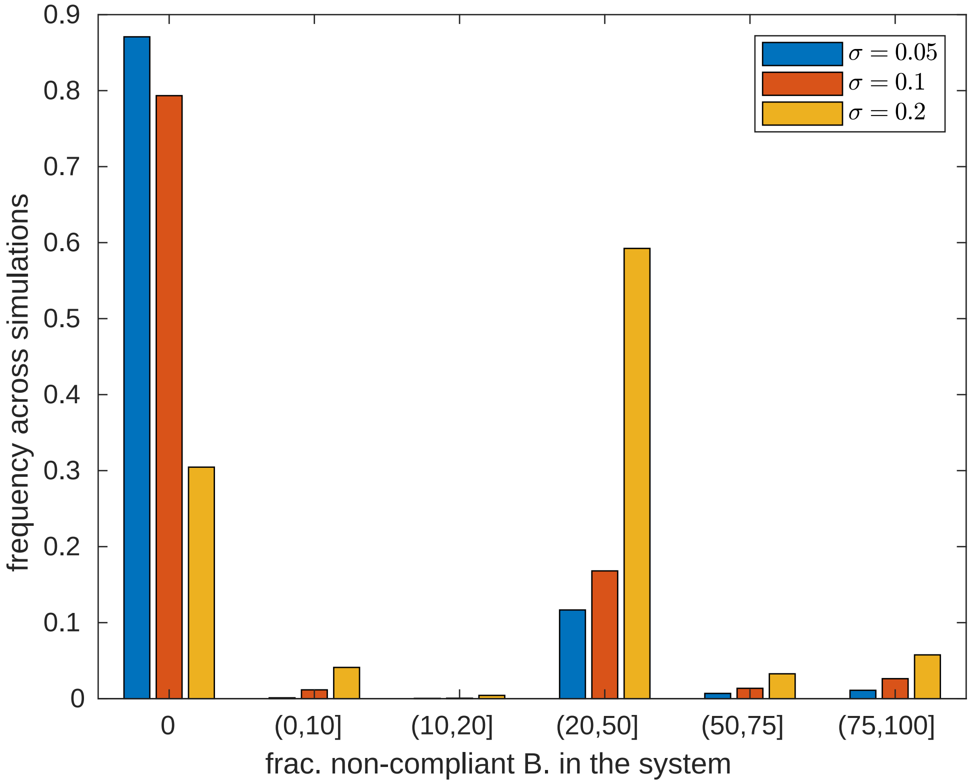

Figure 2: Distribution of non-compliant banks across scenarios and across randomly generated systems of banks and asset managers

Note – x-axis: buckets of fractions of non-compliant banks in the system (i.e., '0' means none of the banks are breaching liquidity or solvency requirements, $(a, b]$ means that a fraction of banks breaching liquidity or solvency requirements is between a% and b%); y-axis: frequency of the fractions of non-compliant banks across simulations of financial systems and across 100 shocks to individual agents. For all simulations, the solvency requirement is set to $\gamma = 7\%$ and the liquidity requirement to $\alpha = 10\%$, and we report distributions for time equal to 50 days.

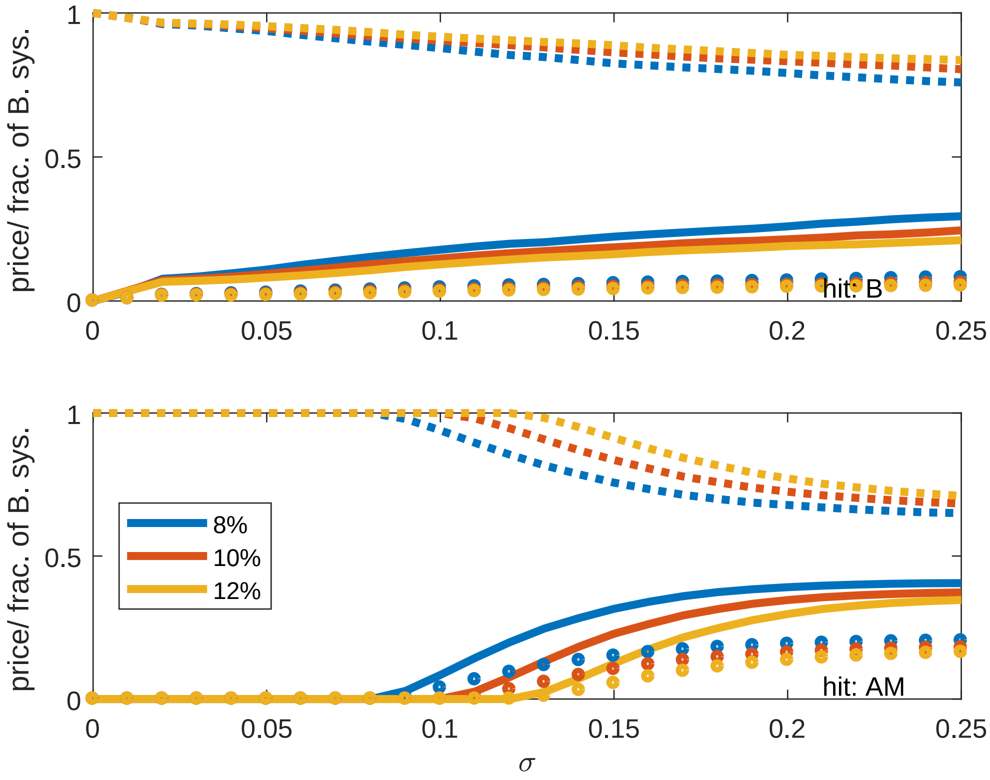

Figure 3: Impact of the funding shock on asset prices and fraction of non-compliant and defaulting banks under different regimes of liquidity regulation

Note – x-axis: shock σ, i.e., share of deposits (participations) outflows for banks (asset managers); y-axis: price (dotted lines), fraction of non-compliant banks (solid lines), and fraction of defaulting banks (circles) in the banking system.

Top pane: Initial random shocks hit banks. Bottom pane: Initial random shocks hit asset managers. For all simulations, the solvency requirement is set to $\gamma = 7\%$.

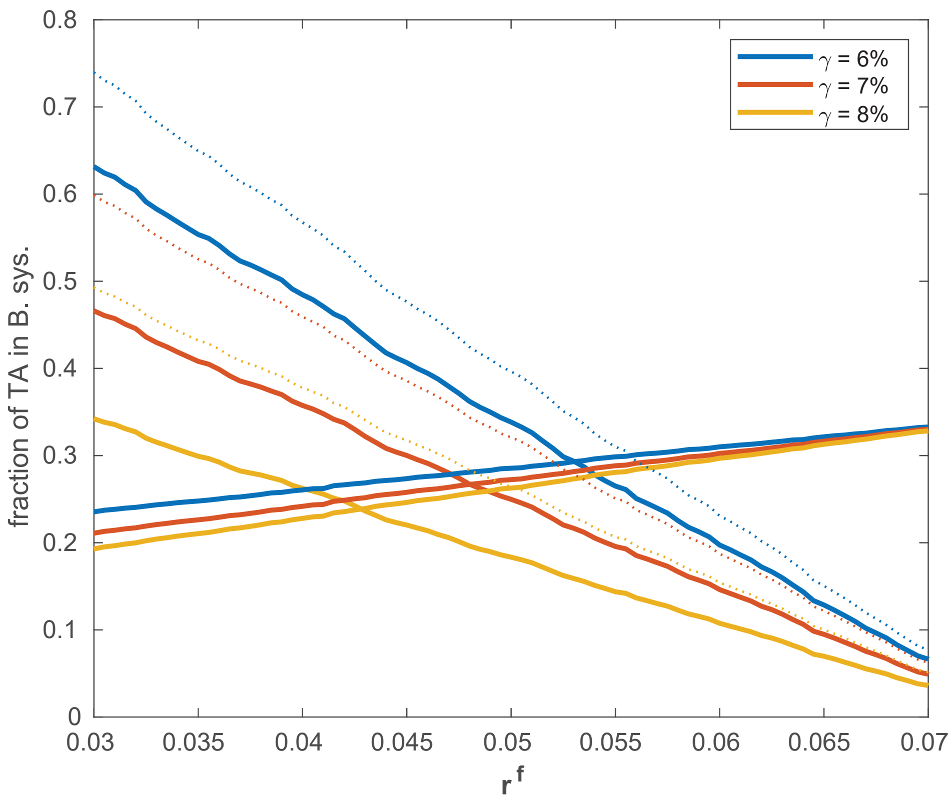

Figure 4: Money demand and money supply on the interbank lending market for different risk-free rates of return. Note – x-axis: r^f risk-free return rate; y-axis: aggregated money demand and aggregated money supply on the interbank lending market (solid line), less-liquid security demand (dotted line). For all simulations, the liquidity requirement is set to $\alpha = 10\%$. Results are averaged out over the seeds.

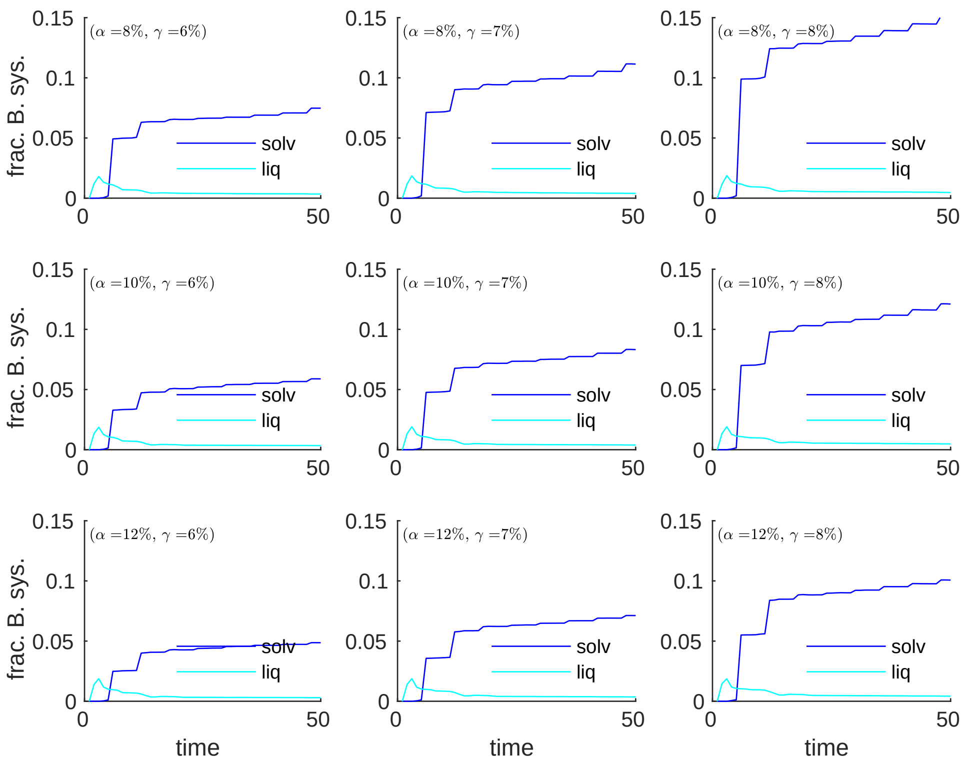

Figure 5: Fraction of banks breaching solvency versus liquidity requirements
Note – x-axis: time (elapsed days); y-axis: average fraction of banks in the banking system that breach liquidity ('liq' line) and solvency ('solv' line) requirements. Each subplot shows outcomes of simulations for a given pair of liquidity and solvency requirement (α, γ) and $\sigma = 0.05$.

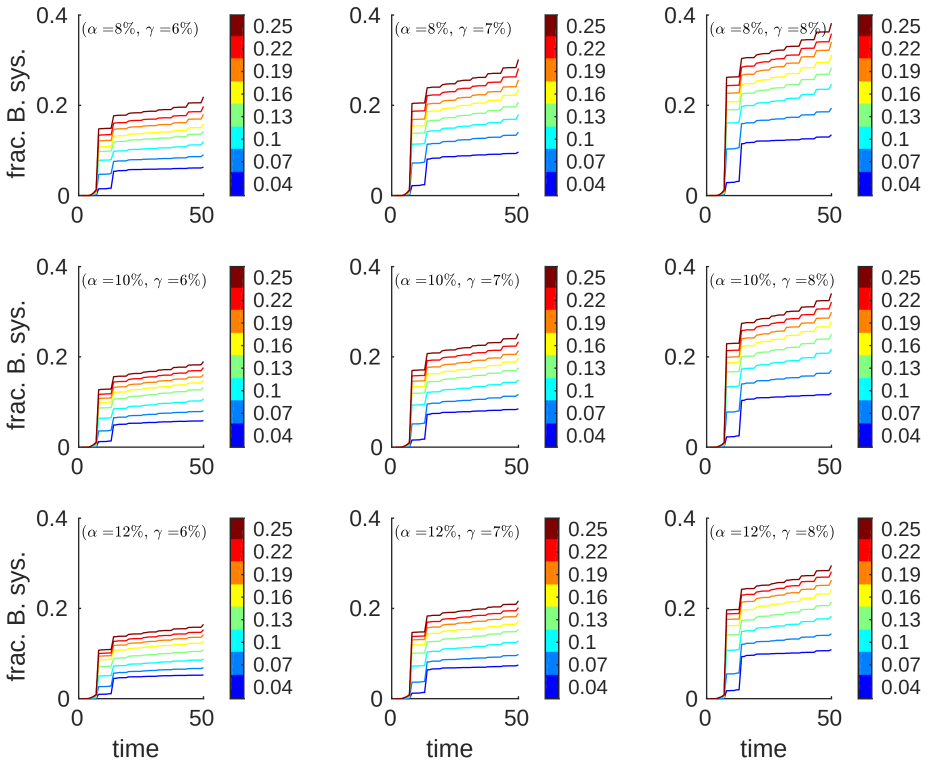

Figure 6: Dynamics of the number of banks breaching liquidity or capital requirements, when the initial random shocks hit banks.

Note – x-axis: time (elapsed days); y-axis: fraction of non-compliant banks in the banking system when the shock originates in the banking sector. Each subplot shows outcomes of simulations for a given pair of liquidity and solvency requirements, (α, γ); each line shows dynamics of the average number of non-compliant banks for a given shock σ (in rhs color bar). Parameter σ defines the outflow fraction of deposits $(D_{i,0} + BB_{i,0})$.

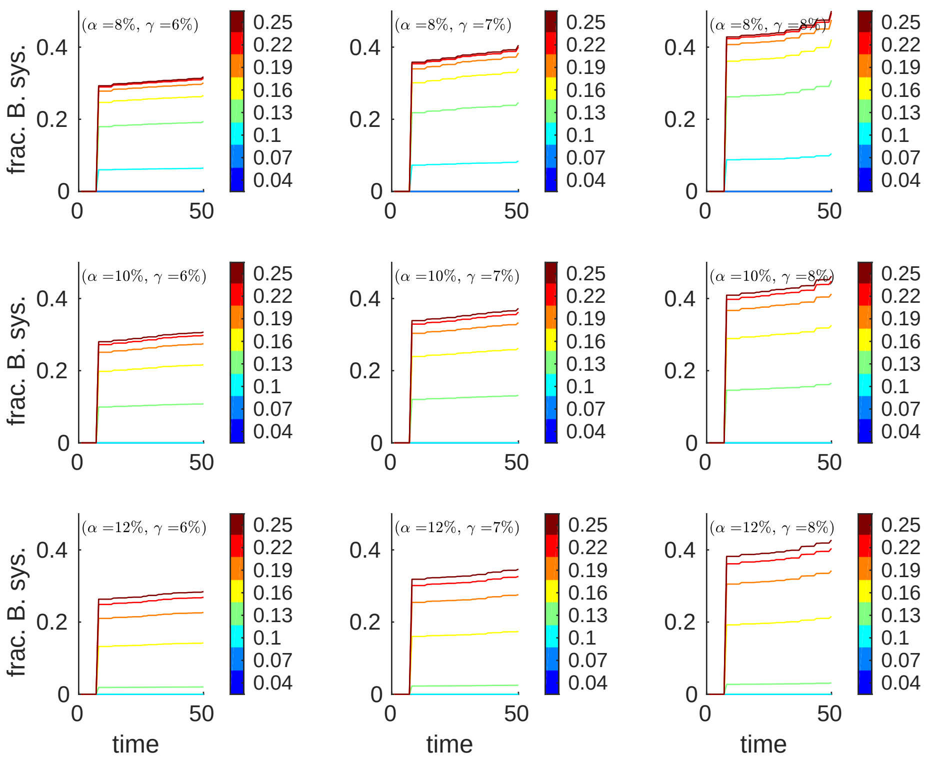

Figure 7: Dynamics of the number of banks breaching liquidity or capital requirements, when the initial random shocks hit asset managers.

Note – x-axis: time (elapsed days); y-axis: fraction of non-compliant banks in the banking system when the shock originates in the asset managers' sector. Each subplot shows outcomes of simulations for a given pair of liquidity and solvency requirement, (α, γ); each line shows dynamics of the average number of non-compliant banks for a given shock σ (in rhs color bar). Parameter σ defines the outflow fraction of participations $P_{m,0}$.

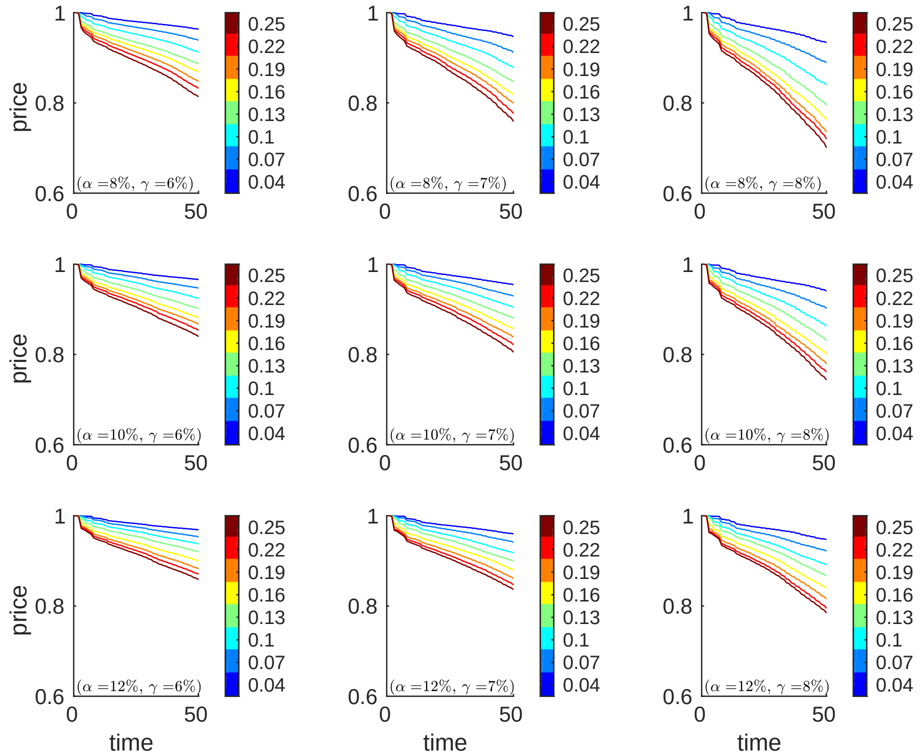

Figure 8: Dynamics of average prices of assets when the initial random shocks hit banks
Note – x-axis: time (elapsed days); y-axis: average price of S; Each subplot shows outcomes of simulations for a given pair of liquidity and solvency requirement (α, γ); each line shows dynamics of the average price of S for a given shock σ (size in rhs color bar). Parameter σ defines the outflow fraction of deposits $(D_{i,0} + BB_{i,0})$.

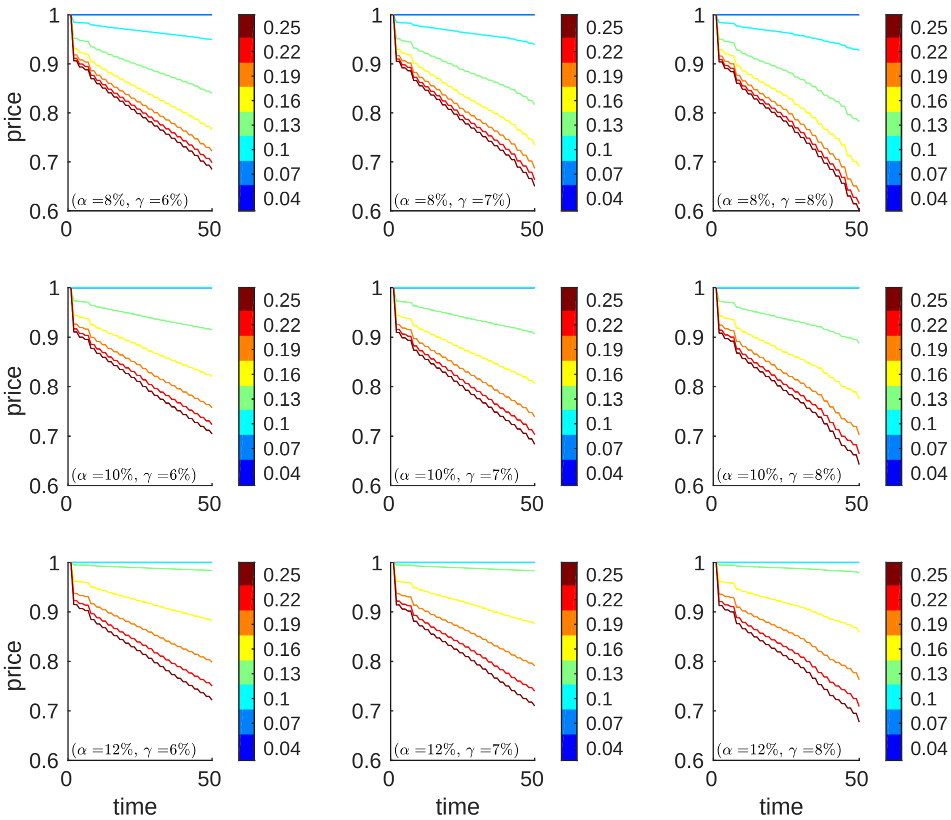

Figure 9: Dynamics of average prices of assets when the initial random shocks hit asset manager
Note – x-axis: time (elapsed days); y-axis: average price of securities S. Each subplot shows outcomes of simulations for a given pair of liquidity and solvency requirement (α, γ); each line shows dynamics of the average price of S for a given shock σ (size in rhs color bar). Parameter σ defines the outflow fraction of participations $P_{m,0}$.

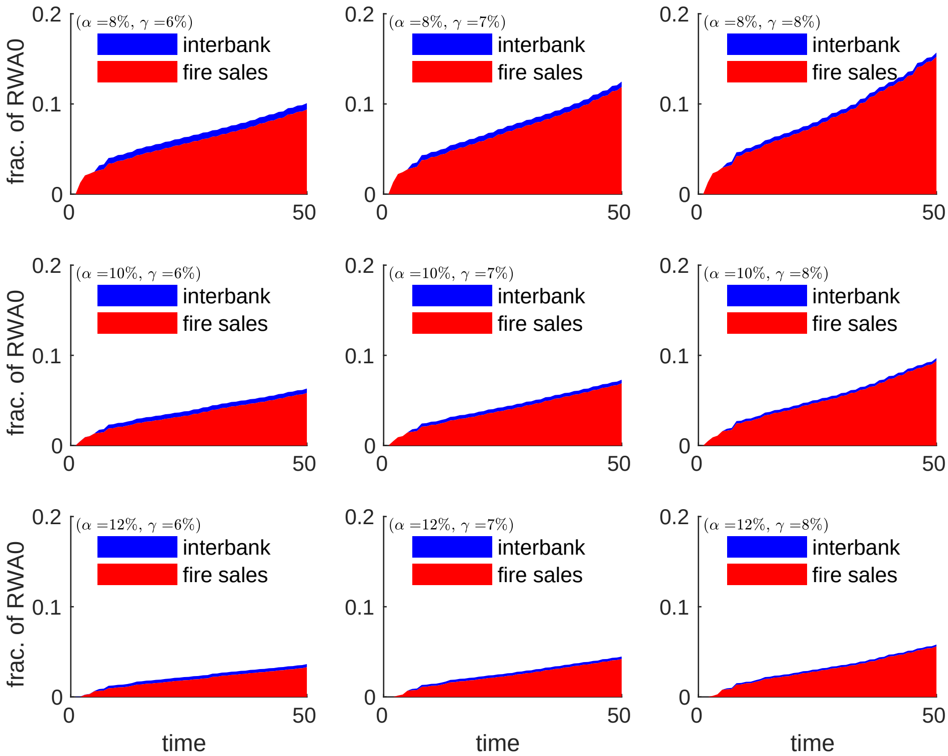

Figure 10: Drivers of contagion losses
Note – x-axis: time (elapsed days); y-axis: average losses in the system and across simulations relative to average aggregate risk-weighted assets at time=0. Each subplot shows outcomes of simulations for a given pair of liquidity and solvency requirements, (α, γ); 'fire sales' – losses related to revaluation of securities S in the contagion process, triggered by scenarios of funding shocks; 'interbank' – potential additional interbank funding cost related to changes in default probabilities and the risk-free rate r^f, following scenarios of funding shocks; funding shock parameter $\sigma = 0.2$.

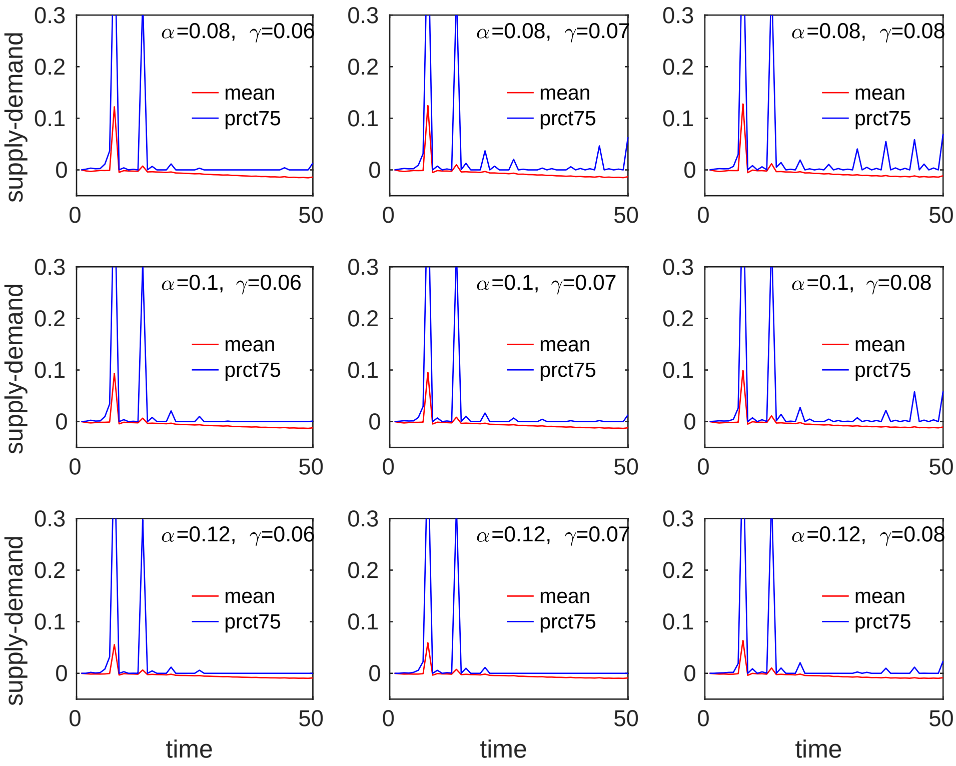

Figure 11: Supply and demand interaction
Note – x-axis: time (elapsed days); y-axis: average price of securities S. Each subplot shows outcomes of simulations for a given pair of liquidity and solvency requirements, (α, γ); each line shows the difference between the volume of securities required to restore liquidity and the volume of securities that can be purchased at a current price ('mean' and 'prct75' refer to mean of the distribution, and 75th percentile of the distribution across simulations, respectively); funding shock parameter $\sigma = 0.2$.

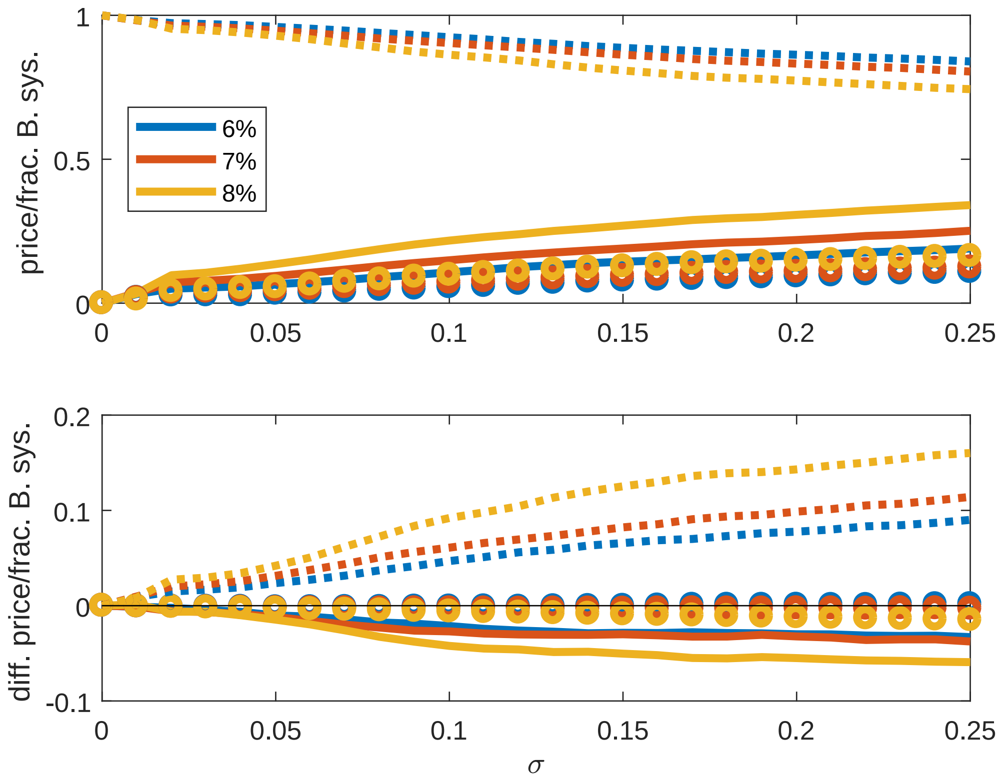

Figure 12: Impact of the asset managers' sector on the asset price and on the fractions of non-compliant and defaulting banks under different regimes of solvency regulation
Note – x-axis: shock σ, i.e., outflow fraction of deposits $D_{i,0} + BB_{i,0}$.
Top pane – y-axis: price (dotted lines), fraction of non-compliant banks (solid lines), and defaulting banks (circles) with 60 banks and 40 asset managers (i.e., the baseline set-up of the financial system). Bottom pane – y-axis: differences in price (dotted lines), in fraction of non-compliant banks (solid lines), and in fraction of defaulting banks (circles) between a system with 100 banks and 0 asset managers and the baseline system with 60 banks and 40 asset managers. For all simulations, the liquidity requirement is set to $\alpha = 10\%$ and the first-hit agent is a bank.

	average fraction of banks with PD $\geq$:					
	0.1%	0.5%	1.0%	2.0%	10.0%	95.0%
initial PD	91.2%	33.3%	7.0%	0.1%	0.0%	0.0%
post-shock PD	99.6%	66.3%	39%	25.4%	16.1%	13.0%

Table 2: Distribution of default probabilities
Note – The average default probability across simulations assuming $\alpha = 0.1$ and $\gamma = 0.07$; initial PD – distribution of consistent PDs implied by the initial optimal structure of banks' balance sheets (distributions based on 100 randomly generated initial banking systems of 60 banks); post-shock PD – distribution of consistent PDs of banks 50 days after funding shocks (distributions based on shocked systems, each system shocked with 100 scenarios of funding outflows, agent by agent).

3.3. Fire sales

Forced liquidation of securities at impaired prices is the main driver of contagion losses, and influences the dynamics of the model. To illustrate the relevance of fire-sale externalities, we aggregated losses incurred by agents due to liquidation of securities and marked-to-market revaluation of securities in portfolios of all the agents. Additionally, we calculated the change in expenses generated by the interbank borrowing when the borrowing costs increase in the impaired banking system. In the model, the cost of interbank borrowing changes due to default probabilities. Table 2 shows statistics of initial default probabilities and compares them with the default probabilities after the contagion plays out in the model. Clearly, the contagion losses are reflected in elevated default probabilities after contagion mechanism plays out compared with the initial consistent default probabilities. For instance, an increase of the default probability from 0.5% to 10%, with a loss given default equal to 40%, translates into expenses higher by 4%, i.e.,

$$\frac{1/(1 - 40\% * 10\%)}{1/(1 - 40\% * 0.5\%)}$$

Aggregation of losses is shown in Fig. 10. We report the results for one particular shock size ($\sigma = 0.2$), normalized by the initial total risk-weighted assets in the system. Fire-sale losses dominate and they increase with the regulatory solvency and liquidity thresholds. They steadily increase in time except for the most stringent regulatory environment, where a marginal increase in the fire-sale losses also occurs. This suggests an active loss amplification mechanism in that case. Conversely, the interbank expenses are the largest for lower levels of capital requirements and loose liquidity rules (subplot in the top-left corner).

Further zooming into the fire-sale mechanism, we can analyze tensions between the supply of securities following liquidation needs and the demand for the securities from other agents. Figure 11 presents a gap between the volume supplied and the volume demanded, normalised by the total holding of securities in the system at t. Positive values indicate that there is not enough demand for securities in fire sales, and the prices at $t + 1$ are further impaired compared with time t. In general, for all pairs of regulatory thresholds, there are two clusters of large and positive gaps at the beginning of the stress horizon, when the supply exceeds the demand by more then 30% of the total volume of securities. During these periods, prices are impacted the most. For stringent solvency and loose liquidity requirements ($\alpha = \gamma = 8\%$) the excess supply ripples along the stress horizon, providing an additional condition for weak security prices for time $t > 40$. The imbalances between demand and supply of securities in the stressful market conditions are the key drivers of the price impact in fire sales.

The identified imbalances on the securities market deteriorate as the capital requirements get tighter. This observation is consistent with findings presented in Subsection 3.1. To concisely reiterate the points made there, higher requirements induce more banks to sell the less-liquid securities, increasing pressure on the prices and, consequently, on banks' capital and solvency measures.

3.4. Role of asset managers

Inclusion of asset managers into our model accounts for an important contagion loss amplification channel. We illustrate the significance of asset managers for the fire-sale mechanism by changing the relative size of the asset managers' sector versus the banking sector. We assume that only banks are present on the market and set the number of asset managers to 0; to retain the size of the system unchanged, we set the number of banks to 100. Then, we compare the asset price and fractions of the banks breaching requirements with the baseline scenario, i.e., a system with 60 banks and 40 asset managers. In this way, the sizes of the two compared systems are equal. However, an increase in the number of banks in the system affects the structure of the interbank network, which may have implications for fairness of comparability. Therefore, as a robustness check we run simulations in the case with 60 banks only, assuming that asset managers are absent (see Appendix G).

The results of the comparison are shown in Fig. 12. First, and consistently across the applied regulatory solvency thresholds, asset managers systematically increase the fraction of non-compliant banks under assumed stress scenarios (see bottom pane). Second, the presence of asset managers boosts the number of non-compliant banks, to an even higher extent if banks target higher capital requirements. Third, the impact of the asset managers' fire-sale activity is even more visible through changes in assets prices. The prices are lower by as much as 15 percentage points compared to the case where only banks are present in the market. The results of the experiment underpin the significant role of asset managers in magnifying the fire-sale losses.

3.5. Asset allocation and market structure

Stricter capital requirements induce more banks to choose a high leverage investment strategy (see Fig. 13). Higher capital levels imply lower funding costs and, therefore, for a larger fraction of banks it is profitable to invest in securities. The allocation of assets in banks' portfolios follows a trade-off between profitability and regulatory requirements rather than a trade-off between risk and return, as in the typical optimal investment model of Markowitz (1952). Although the changes in asset allocation always take place within the limits imposed by the capital constraint, the capital buffers are strongly affected by the investment opportunities. As a matter of fact, banks investing only in bank lending are those (self-)endowed with the highest capital buffers, followed by those investing only in securities. Banks with high leverage portfolios prefer to exploit all the capital flexibility they have, thus keeping only regulatory capital and almost no capital buffers (see Fig. 14).

Asset allocation may be looked at through the lens of the individual banks' balance-sheet structures but also through the homogeneity of the balance-sheet sizes. The role of asset allocation in amplifying contagion effects has been extensively studied, but the conclusions are mixed and depending on the set-up of the market in the models. Greenwood et al. (2015) find that similarity in terms of asset allocation can reduce fire-sale spillovers. We give complementary evidence that similarity in balance-sheet sizes can also reduce contagion. To this end, we conduct an experiment assuming all banks to be of equal size. To allow for comparability, we generate them in such a way that the sum of all banks' balance sheets in the homogeneous case equals the sum of balance sheets generated with the calibrated log-normal assumption. The differences in the fraction of non-compliant banks between simulations run in the homogeneous system and those run in the calibrated system are negative (see lower part of Fig. 15). This means that in a more uniform system, contagion effects are smaller. A similar conclusion can be drawn from the evolution of asset prices, which drop less in the simulations with the homogeneous system. Interestingly, when the magnitude of the initial shock hitting banks increases, the favourable impact of homogeneity shrinks. Moreover, with stringent capital rules, the homogeneous system becomes even more vulnerable. This corroborates the findings of Acemoglu et al. (2015).

A more heterogeneous system is more prone to contagion risk, since it is more likely to contain systemic agents spreading the shocks. We examine two features of the institutions: size and interconnectedness. The size of a bank is measured by the total balance-sheet volume. The interconnectedness of a bank is measured by the joint in- and out-degree of lending relations of that bank on the interbank lending market.[15] We use these metrics to define – for each realisation of balance-sheet structures – a subset of large banks and a subset of the most interconnected banks. The two sets contain banks that are in the 90th percentile of the distribution (across banks) of the size and interconnectedness, respectively. Then, we use these sets to shock the system, i.e., we assume that a funding shock, $\sigma = 20\%$, hits banks from these sets, one bank after another, and average out the effects across the simulations. Results shown in Fig. 16 suggest that it matters for the magnitude of contagion whether the funding shock affects a sizeable and interconnected bank. Interconnectedness is an even stronger indicator of potentially large contagion effects than size. Intuitively, interconnectedness is relevant, since the funding shock propagates through interactions between financial institutions. Moreover, larger banks are usually more interconnected.

Our findings may be used in a policy debate on the 'too-big-to-fail' issue, suggesting that preventing large disparities in the sizes of interconnected financial institutions may reduce financial vulnerabilities.

4. Conclusions

We have built a framework that captures interactions between two kinds of agents: traditional banks and asset managers. These agents are indirectly connected through holdings of similar securities, while traditional banks are also interlinked via interbank lending. We conduct simulations that show how these two channels determine the dynamics of fire-sale contagion following an extreme liquidity stress. We investigate the role of liquidity and capital requirements in determining the price dynamics, speed, and extent of the contagion.

We find evidence of the important role that banks with high leverage play in the propagation of contagion.

First, when the liquidity shock is experienced by a holder of interbank claims, there are chances that it strands on the shores of the interbank lending market and may not spill over shocks to other segments of the financial markets through the banks with high-leverage portfolios. Conversely, when the shock hits a security-only holder it will automatically affect all security holders, and the banks forced to adjust their balance sheets due to regulatory constraints may even transmit the shock to the interbank market. This mechanism explains why shocks originating among asset managers may tend to spread and affect the banking system more quickly than those originating in the banking

[15] In-degree of bank i is the number of banks $j \neq i$ that bank i lends to. Out-degree is the number of banks that it borrows from.

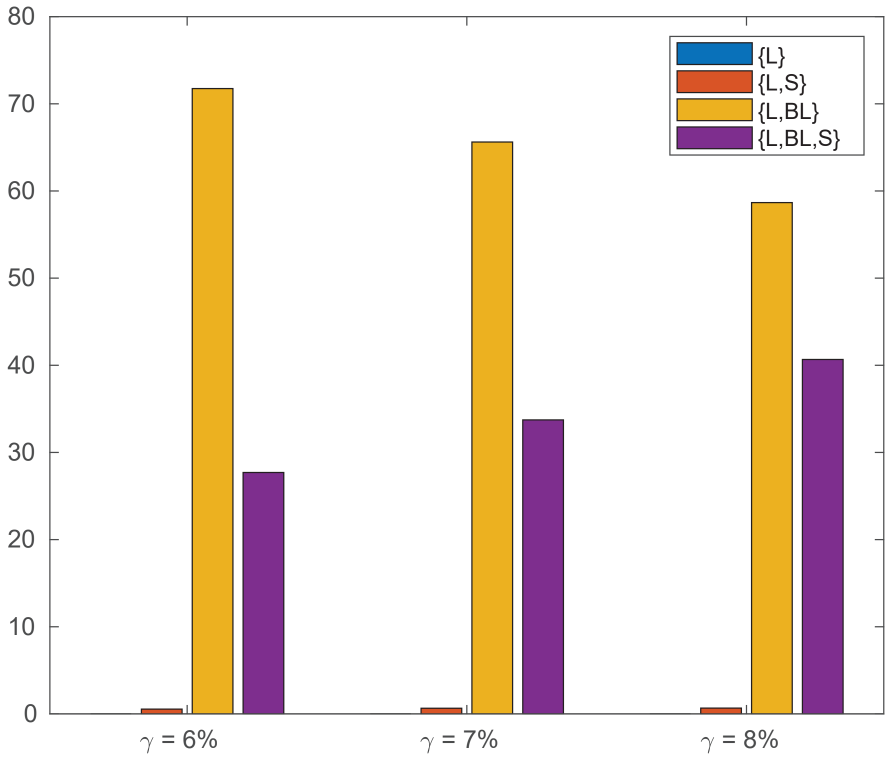

Figure 13: Fractions of banks by asset allocation choices under different regimes of solvency regulation.
Note – {L}-banks have loans only; {L, S}-banks have loans and security; {L, BL}-banks have loans and interbank lending; {L, BL, S}-banks invest in interbank loans and securities. Cash C is omitted in the labels since all banks need to hold cash because of liquidity requirements. Average fractions across simulated banks' balance sheets are shown. For all simulations, the liquidity requirement is set to $\alpha = 10\%$.

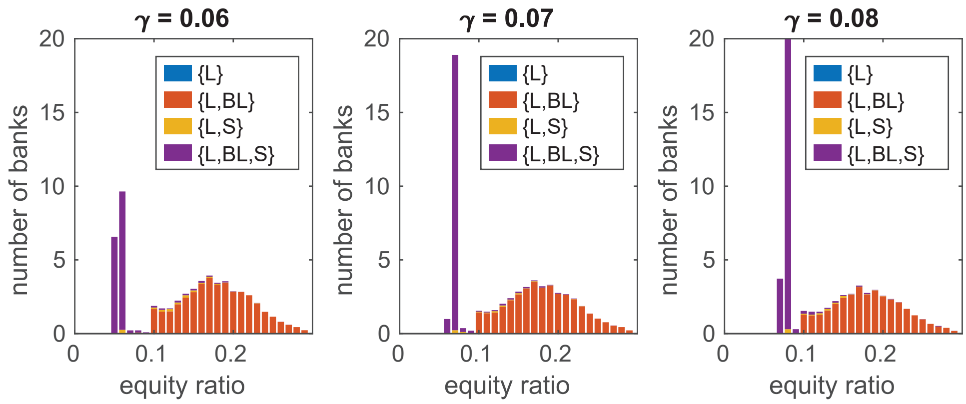

Figure 14: Fractions of banks by asset allocation choices and by actual equity ratio
Note – x-axis: equity ratio; y-axis: number of banks in the 60-bank system. Three subplots correspond to different capital requirements for which the system is generated ($\gamma \in \{6\%, 7\%, 8\%\}$). Distribution of banks across 100 simulated systems.

sector. In other words, the speed of contagion may be related to the direction in which the shock is channelled across the financial system.

Second, capital requirements may have counter-intuitive implications on the stability of the system. While higher capital requirements enhance the resilience of individual banks, they may induce banks to react to shocks quicker to abide by more stringent regulatory thresholds. Moreover, higher capital requirements make the banks' balance sheets look alike. Banks optimise their portfolios according to the risk-return principle. Initially, banks have an incentive to keep less risky assets, i.e., the interbank lending, which does not require holding as much capital as riskier assets, i.e., the less-liquid asset. However, this leads to an excess supply of interbank claims and drives down the risk-free return rate of bank lending that, as a consequence, becomes a less appealing investment for banks. The higher capital also means a lower default probability and, consequently, lower funding costs. Hence, there is a trade-off. On the one hand, the increase in the capital requirement lowers the total volume of the interbank lending market. On the other hand, the lower risk-free rate and lower default probability imply lower interbank borrowing costs. Consequently, banks want to borrow more money to invest into the riskier assets, increasing homogenization of asset allocation, and the fire-sale losses following liquidity shocks are more pronounced. Moreover, asymmetry in the sizes of banks may create conditions for the emergence of systemic institutions

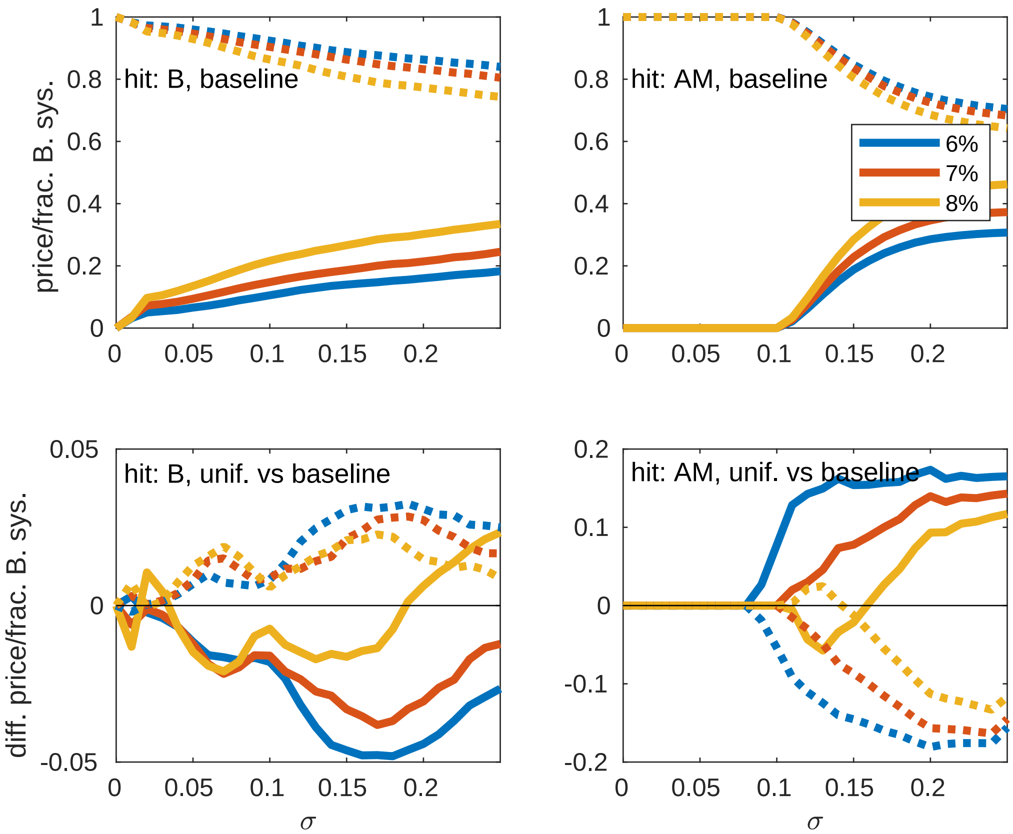

Figure 15: Homogeneous vs heterogeneous banking sectors
Note – x-axis: shock σ, i.e., outflow fraction of deposits or participation. Top panes – y-axis: price (dotted lines) or fraction of non-compliant banks (solid lines). Bottom panes – y-axis: differences in price (dotted lines) or in fraction of non-compliant banks (solid lines) between the systems with homogeneous-size banks versus the calibrated (baseline) heterogeneous banks. Left panes: initial random shocks hit banks; right panes: initial random shocks hit asset managers.
For all simulations, the liquidity requirement is set to $\alpha = 10\%$.

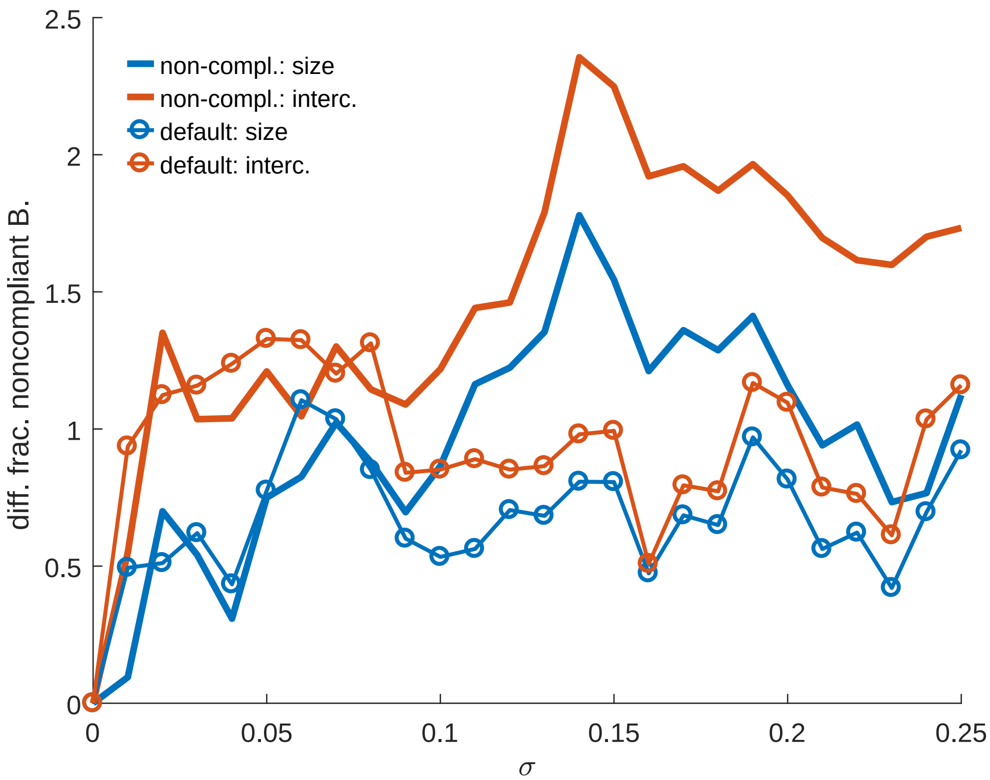

Figure 16: Differences in the average fractions of non-compliant and defaulting banks between simulations with shocks to different groups of banks
Note – the blue line ('size') represents the difference (in pp) between fractions of non-compliant banks with capital or liquidity requirements (solid line) or defaulting banks (line with circles) when the shock hits large banks versus cases when the shock hits smaller banks. Large banks are defined as those with total assets in the top 10 percentiles of the distribution of assets across banks. The red line ('interc.') represents a difference (in pp) between fractions of non-compliant banks (solid line) or defaulting banks (line with circles) when the shock hits highly interconnected banks versus cases when the shock hits less interconnected banks. Highly interconnected banks are defined as those with in- and out-degrees of connection in the interbank network in the top 10 percentiles of the distribution of in- and out-degrees across banks. Defaulting banks are those with capital below zero at $t = 50$.
x-axis: funding shock to banks as a share of deposits and interbank borrowing; y-axis: differences in average frequency of the fractions of non-compliant banks across simulations of financial systems and across 100 shocks to individual agents. For all simulations, the solvency requirement is set to $\gamma = 7\%$ and the liquidity requirement to $\alpha = 10\%$; results are reported for the final time period ($t = 50$).

that have a higher potential to spread contagion.

Third, asset managers can act as shock absorbers but only for relatively small shocks and only when they have excess liquidity. Small liquidity shocks originating in the asset managers' sector are absorbed because these institutions do not need to re-adjust their balance sheets to meet the regulatory requirements. On the downside, we demonstrate that asset managers can substantially amplify contagion losses initiated in the banking system, which is related to the mark-to-market revaluation of assets and the redemption risk.

Future work on more granular balance sheets of banks and asset managers may be needed to empirically analyze the two identified trade-offs. First, higher capital requirements may increase the resilience of the entire system by strengthening the capital position of individual banks. However, at the same time they may enhance contagion by homogenizing banks' balance sheets. Second, while non-regulated asset managers may exacerbate contagion and fire sales, they can also provide flexible buffers and absorb the adverse effects of small liquidity shocks.

Policy and regulation, like a parent with a child, should aim at the right balance between control and freedom, fostering stability and growth.

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Appendix A Derivation of profit function

The profit maximisation problem of bank i is given by

$$\max_{BL_i, BB_i, C_i, S_i} \mathbb{E}(\pi_i) = \max_{BL_i, BB_i, C_i, S_i} \mathbb{E}(\pi_i^{lending}) + \mathbb{E}(\pi_i^{investing}) - \mathbb{E}(cost_i^{borrowing}) \tag{22}$$

where – to reiterate – $\mathbb{E}(\pi_i^{lending})$ is the expected profit coming from the lending activity of bank i to other banks (BL_i), $\mathbb{E}(\pi_i^{investing})$ is the expected profit of investing S_i in the less-liquid asset, and $\mathbb{E}(cost_i^{borrowing})$ is the cost of borrowing money (BB_i) from other banks. We drop the time index for brevity.

Although the profits should also include the return on loans (L_i), together with the cost of equity (E_i) and deposits (D_i), these are not included in the maximization problem since loans are sticky and cannot be disposed of, and capital and deposits are bank-specific initial endowments; therefore these variables do not enter the maximization problem.

Interbank lending. If bank i lends BL_{ij} to bank j, the first will charge a price of $(r^f + r_j^{PD})$ to bank j for the corresponding amount, where r^f is the risk-free return rate charged solely for the transfer of funds from one bank to the other, while r_j^{PD} is internalizing the probability that the borrowing bank j will default.

Therefore, the profits derived from lending BL_{ij} are given by

$$\mathbb{E}(\pi_i^{lending_{ij}}) = (1 - PD_j)BL_{ij}(r^{rf} + r_j^{PD}) + PD_j BL_{ij}(1 - \xi)(r^{rf} + r_j^{PD}) \tag{23}$$

where ξ is the loss given default, and PD_j is the probability of default of bank j. In fact, with probability $(1-PD_j)$ bank j will pay back the amount borrowed and will also pay the corresponding fees, and with probability PD_j bank j will pay back to bank i only a fraction $(1-\xi)$ of the contractual amount.

We assume that creditors charge a fair risk premium, which implies that the expected profits in the presence of risk (that is, when charging r^{PD_j}) is equal to the profits from a risk-free loan.

$$\mathbb{E}(\pi_{ij}^{lending}) = BL_{ij} r^f \tag{24}$$

Thus, solving equations 23 and 24 for r_j^{PD}, the risk premium for bank j is given by

$$r_j^{PD} = \frac{\xi PD_j}{1 - \xi PD_j} r^f \tag{25}$$

and – consequently –

$$(r^f + r_j^{PD}) = \frac{r^f}{1 - \xi PD_j} \tag{26}$$

Substituting Eq. 26 in Eq. 23 and summing over all j's, the expected profits of bank i derived from lending activity to other banks is

$$\mathbb{E}(\pi_i^{lending}) = \sum_j \mathbb{E}(\pi_{ij}^{lending}) = BL_i r^f \tag{27}$$

Investing in security.. The second source of profits for bank i are the returns r_i^s on investments S_i in the less-liquid asset. The returns are heterogeneous among institutions and are divided by the current market price of the security, initially set to 1, to take into account the inverse relation between the market price and the yield (e.g., in the case of a bond, having a fixed interest rate). In our setting, since banks are not allowed to re-optimize in the short run, this particular specification is not determinant of the system dynamics.

$$\mathbb{E}(\pi_i^{investing}) = r_i^s S_i \tag{28}$$

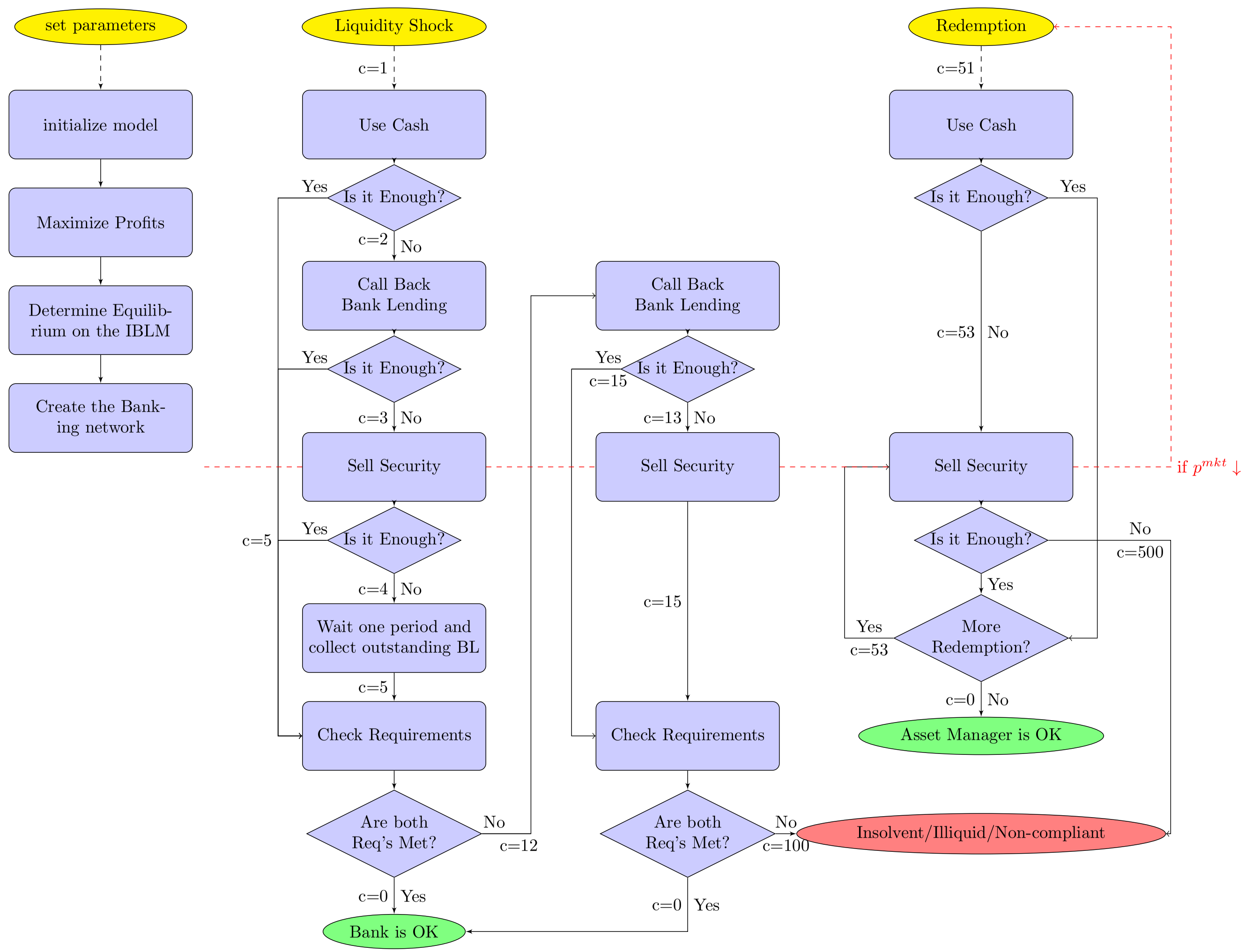
Interbank borrowing.. Symmetrically, using Eq. 24, the cost of borrowing BB_i on the interbank lending market is given by

$$\begin{aligned}\mathbb{E}(cost_i^{borrowing}) &= \sum_j BB_{ij}(r^f + r_i^{PD}) &(29)\\ &= BB_i(r^f + r_i^{PD}) &(30)\\ &= BB_i \frac{r^f}{1 - \xi PD_i} &(31)\end{aligned}$$

Taken together (Eq.23, Eq.28 and Eq.31), the expected profits of bank i are

$$\mathbb{E}(\pi_i) = r^f BL_i + r_i^s S_i - BB_i \frac{r^f}{1 - \xi PD_i} \tag{32}$$

Appendix B Flowchart of model dynamics



Appendix C Consistent default probability

Theorem C.1. *For a vector of rates*[16] $r \in \bigtimes_{d=1}^{4} \mathbb{R}_+ \cup \{0\}$*, risk weights* $\chi \in \mathbb{R}^3_+$*, capital* $e_0 > 0$*, deposits* $d_0 > 0$ *and loans* $l_0 \in [0, e_0 + d_0)$*, regulatory parameters* $\alpha_{min} > 0$ *and* $\gamma_{min} > 0$*, preference* β_0*, and riskiness of assets* $\psi \in \mathbb{R}^3_+$*, let us define an optimal profit function* $\pi : [0,1] \to \mathbb{R}$ *as follows:*

$$\pi(x) = \max_{z \in \mathcal{A}} \left\{ r_1 z_1 + r_2 z_2 - r_1 \frac{1}{1 - \xi x} \cdot z_3 \right\} \tag{33}$$

where

$$\mathcal{A} = \left\{ z \in \bigtimes_{d=1}^{4} \mathbb{R}_+ \cup \{0\} \right|$$

$$\begin{aligned} l_0 + z_1 + z_2 + z_4 &= d_0 + z_3 + e_0, & (34)\\ \frac{e_0}{\chi_1 z_1 + \chi_2 z_2 + \chi_3 l_0} &\geq \gamma_{min}, & (35)\\ z_4 &\geq (\alpha_{min} + \beta_0) d_0, & (36)\\ z_1 + z_4 &\geq (\alpha_{min} + \alpha_0)(d_0 + z_3) \Big\} & (37) \end{aligned}$$

Then, there exists a unique probability $p^* \in (0, 1]$ *such that*

$$p^* = \Phi^{-1} \left(\frac{-e_0 + \pi(p^*)}{\psi_1 (d_0 + e_0)} \right) \tag{38}$$

Remark: z_1 plays the role of interbank lending in the model, z_2 – securities, z_3 – bank borrowing, and z_4 is cash.

Proof of theorem C.1:

Let us define an auxiliary mapping $\bar{\Phi} : [0,1] \to [0,1]$ as

$$\bar{\Phi}(x) = \Phi^{-1} \left(\frac{-e_0 + \pi(x)}{\psi_1 (d_0 + e_0)} \right)$$

It is sufficient to show that there is one fixed point of $\bar{\Phi}$.

We will rely on the following simple lemma.

Lemma C.2. *Let* $g : [0,1] \to [0,1]$ *have the following properties:*

1. $g(0) > 0$*;*
2. g *is decreasing;*
3. g *is continuous.*

Then, there exists a unique fixed point x^* *of* g *and it is positive, i.e.,* $g(x^*) = x^* > 0$*.*

[16] $\bigtimes_{d=1}^{k} A_k$ is a Cartesian product of sets A_k.

Ad 1. Since we assume that $l_0 > 0$, which is not subject to optimization, assets will always be risky. Since Φ^{-1} is strictly positive, then in particular $\bar{\Phi}(0) > 0$.[17]

Ad 2. Let us take x and y such that $y > x$. First, we show that $\pi(y) \leq \pi(x)$. Let denote the corresponding optimal strategy as $z(x) \in \bigtimes_{d=1}^{3} \mathbb{R}_+ \cup \{0\}$, which is a function of $x \in [0,1]$.

We have two cases of rates to consider:

If $r_1 > r_2$, then any interbank borrowing x_3 makes the profit lower. Then, the bank does not borrow on the interbank ($z_3 = 0$), keeps cash at the minimum cash requirement (Ineq. 36), and keeps the rest in bank lending, i.e., $z_4(x) = \alpha_{min} d_0$ and $z_1(x) = d_0 + e_0 - l_0 - z_4(x)$, which is independent of x. This implies

$$\pi(x) = r_1 z_1(x) = r_1 z_1(y) = \pi(y)$$

If $r_2 > r_1$ then zero-profit cash stays at the minimum $z_4(x) = (\alpha_{min}) d_0$. Therefore, a constraint given by Ineq. 37 implies $z_1(x) = (\alpha_{min} + \beta_0) z_3(x) + \beta_0 d_0$. The access funding is invested in z_2, i.e.,

$$z_2(x) = e_0 + d_0 - l_0 - z_4(x)$$

How much funding $z_3(x)$ is optimal? That the amount $z_3(x)$ is used to finance an additional investment into securities implies some interbank lending following the liquidity requirement in Ineq. 37. We denote the additional volumes with a prime. The additional funding has to be profitable, i.e.,

$$r_1 z_1'(x) + r_2 z_2'(x) - r_1 \frac{1}{1 - \xi x} z_3'(x) > 0$$

and

$$r_1(\alpha_{min} + \beta_0) z_3'(x) + r_2(1 - \alpha_{min} - \beta_0) z_3'(x) - r_1 \frac{1}{1 - \xi x} z_3'(x) > 0 \tag{39}$$

Let us define the additional profit margin function $m^+ : [0,1] \to \mathbb{R}$ as

$$m^+(x) = r_1(\alpha_{min} + \beta_0) + r_2(1 - \alpha_{min} - \beta_0) - r_1 \frac{1}{1 - \xi x}$$

Inequality 39 implies that

a) if $m^+(x) \leq 0$, then $z_3'(x) = 0$,

b) if $m^+(x) > 0$, then $z_3'(x)$ is the maximum volume allowed by the capital constraint, i.e., it satisfies

$$\gamma_{min} = \frac{e_0}{\chi_2 z_2(x) + \chi_3 l_0 + \chi_1(\alpha_{min} + \beta_0) z_3'(x) + \chi_2(1 - \alpha_{min} - \beta_0) z_3'(x)}$$

We have two cases to consider:

(i) y is such that $m^+(y) > 0$.

Since $z_2(x) = z_2(y)$, and

$$\frac{1}{1 - \xi x} > \frac{1}{1 - \xi y}$$

[17] Remark: Condition 1. is not necessary to prove the existence and uniqueness of the consistent default probabilities but guarantees that they are non-degenerate.

$$
\begin{aligned}
\pi(x) = r_1\beta_0 d_0 + r_2 z_2(x) + r_2 z_2'(x) &\quad - \quad r_1 \frac{1}{1-\xi x} z_3'(x) \\
= r_1\beta_0 d_0 + r_2 z_2(x) + \left(r_1(\alpha_{min} + \beta_0) + r_2(1-\alpha_{min} - \beta_0)\right) z_3'(x) &\quad - \quad r_1 \frac{1}{1-\xi x} z_3'(x) \\
&\quad < \\
r_1\beta_0 d_0 + r_2 z_2(y) + \left(r_1(\alpha_{min} + \beta_0) + r_2(1-\alpha_{min} - \beta_0)\right) z_3'(y) &\quad - \quad r_1 \frac{1}{1-\xi y} z_3'(y) \\
= r_1\beta_0 d_0 + r_2 z_2(y) + r_2 z_2'(y) - r_1 \frac{1}{1-\xi y} z_3'(y) &\quad = \quad \pi(y)
\end{aligned}
\tag{40}
$$

(ii) $m^+(y) < 0 \Rightarrow z_3'(y) = 0$.

Ad 3. A proof of the continuity uses similar arguments to the proof of monotonicity, since $s(x) = \frac{1}{1-\xi x}$ is continuous in x. The only point that requires additional care is x' such that $m^+(x') = 0$ since the strategy is discontinuous from above at this point. However, for small enough $\delta > 0$, for any x_ϵ such that $x' < x_\epsilon < x' + \delta$,

$$\pi(x_\epsilon) - \pi(x') = m^+(x_\epsilon) z_3'(x_\epsilon)$$

and

$$|\pi(x_\epsilon) - \pi(x')| <= |m^+(x_\epsilon)| z_3'(x' + \delta)$$

so $x_\epsilon \to x' \Rightarrow |m^+(x_\epsilon)| \to 0 \Rightarrow \pi(x_\epsilon) - \pi(x') \to 0$.

Appendix D Determining the equilibrium interest rate r^f

$BL_{ij,t}$ is defined as the liquidity lent by bank i to bank j at time t, and $BB_{ij,t}$ as the liquidity borrowed by bank i from bank j. Furthermore, as bank i can lend to more than one bank and borrow from more than one bank, let us also define

$BL_{i,t} = \sum_{k=1}^{N_B} BL_{ik,t}$ as bank i's total lending to all other banks; and

$BB_{i,t} = \sum_{k=1}^{N_B} BB_{ik,t}$ as bank i's total borrowing from all other banks.

As in Bluhm et al. (2014), an initial value of r^f is randomly chosen from the interval $[\underline{r}^f, \overline{r}^f]$[18] to find the initial clearing price on the interbank lending market. Banks evaluate the optimal amounts of $BL_{i,0}$ or $BB_{i,0}$ for the value of r^f, then a central auctioneer determines the aggregate supply and demand of liquidity on the interbank lending market:

$$F^{supply} = \sum_{i=1}^{N_B} BL_{i,0}$$

$$F^{demand} = \sum_{i=1}^{N_B} BB_{i,0}$$

and

- if $F^{supply} < F^{demand} \Rightarrow r^f$ is increased and $r^f_{new} = \frac{r^f_{old} + \overline{r}^f}{2}$
- if $F^{supply} > F^{demand} \Rightarrow r^f$ is decreased and $r^f_{new} = \frac{r^f_{old} + \underline{r}^f}{2}$

With the new risk free return rate, r^f, new aggregate demand and supply are determined and the process is repeated. The tâtonnement stops when the excess demand is minimised.

Appendix E Call-back and equilibrium price: algorithm

Calling back the bank lending. The portion of BL that banks need to call back and recover is determined as follows:

$$N^{BL}_{i,t} = \min\left\{\max\{\lambda_{i,t} + \nu^{Liq}_{i,t}, \nu^{Cap|BL}_{i,t}\}, BL_{i,t}\right\} \quad (41)$$

where $\nu^{Cap|BL}_{i,t}$ is the amount of bank lending that needs to be called back if bank i plans to meet the capital requirements only by divesting BL, i.e.,

$$\nu^{Cap|BL}_{i,t} = \frac{\nu^{Cap}_{i,t}}{\chi_{BL}}$$

$N^{BL}_{i,t}$ is called back proportionally from all banks j that borrowed money from bank i. The amount of called-back bank borrowing is, in turn, considered by banks j as a liquidity shock, and is treated similarly to the initial funding shock. Moreover, mutual exposures are netted.

[18] In our simulations it is simply an interval $[0, 1]$.

We deliberately abstract from using a clearing interbank payment as in Eisenberg and Noe (2001) at each step of the sequence of contagion. The clearing mechanism resulting from the sequence of actions involves not only the interbank of matched banks but also the selling process of less-liquid assets (lla). The market does not wait for the emergence of the equilibrium on the interconnected interbank lending market and banks immediately deal with the calls received from their counterparties. The sequence of acting banks depends only on the initial composition of balance sheets (drawn at the beginning of simulations) and on the initial funding liquidity shock.

The second channel of propagation of the shock involves the less-liquid assets. The market is driven by those agents that need to sell less-liquid assets. They determine how many securities they want to liquidate to meet capital after calling back bank lending.

Selling the less-liquid security.

Who sells: The supply. After calling back bank lending, banks update their liquidity needs and determine the new values of $\lambda_{i,t}$, $\nu_{i,t}^{Liq}$, and the divestment of the security to meet the capital requirement:

$$\nu_{i,t}^{Cap|S} = \frac{\gamma_{min} \cdot RWA_{i,t} - E_{i,t}}{\gamma_{min}\chi_S} \tag{42}$$

The total value of the less-liquid security to be sold by bank i is

$$N_{i,t}^S = \min\{\max\{\lambda_{i,t} + \nu_{i,t}^{Liq}, \nu_{i,t}^{Cap|S}\}, S_{i,t}\} \tag{43}$$

The market opens if either a bank or an asset manager needs to sell the security. The seller sets the initial selling price at $p^\tau = p_t^{mkt}$. We define $q_{i,t}^s(p^\tau)$ and $q_{m,t}^s(p^\tau)$ as the amounts of lla that bank i and asset manager m need to sell at the price p^τ, i.e.,

$$q_{i,t}^s(p^\tau) = \min\left\{lla_{i,t}, \frac{N_{i,t}^S}{p^\tau}\right\} \tag{44}$$

$$q_{m,t}^s(p^\tau) = \min\left\{lla_{m,t}, \frac{N_{m,t}^S}{p^\tau}\right\} \tag{45}$$

The sum of individual supplies of the less-liquid security gives the aggregate supply

$$Q_t^S(p^\tau) = \sum_{i=1}^{N_B} q_{i,t}^s(p^\tau) + \sum_{m=1}^{N_{AM}} q_{m,t}^s(p^\tau) \tag{46}$$

The aggregate supply includes three kinds of selling agents: a) banks facing a liquidity shortage due to a direct exogenous funding liquidity shock or due to a shock on the interbank lending market; b) banks that need to divest to meet the regulatory requirements; c) asset managers that need to sell assets to face redemption.

If the sale of the entire security portfolio is not enough to face the liquidity shortage, an institution withdraws from the market. The institution freezes its management actions and does not contribute to a further depression of the asset market price.

Who buys: The demand. If there is at least one agent selling the less-liquid security, every other agent can buy under two conditions: it has to be *profitable* and *feasible*.

The investment is said to be *profitable* for buyer i if the price at which the security is sold in iteration τ is lower than its noisy valuation of the same security, $p^\tau < f_i$. It is *feasible* if the institution has spare liquidity. While an asset manager can use up all cash, a bank can deplete only the extra liquidity buffer within the limits posed by the regulatory liquidity and capital requirements.

Participants to the market either sell or buy: an institution selling lla does so due to a capital or liquidity shortage; consequently, buying would not be feasible.

Bank i's demand for less-liquid security $q^d_{i,t}(p^\tau)$ needs to comply with capital regulatory requirement

$$q^d_{i,t}(p^\tau) \leq \frac{E_{i,t} - \gamma_{min}\mathrm{RWA}_{i,t}}{(\gamma_{min}\chi_S - 1)p^\tau} \tag{47}$$

and with the liquidity regulatory requirement, still limited by the borrowings that need to be paid back

$$q^d_{i,t}(p^\tau) \leq \frac{C_{i,t} - \alpha_{min}(D_{i,t}) - \sum_j BB^{askedback}_{ij,t}}{p^\tau} \tag{48}$$

Instead, the demand of an asset manager m for less-liquid security $q^d_{m,t}(p^\tau)$ needs to be feasible within the limits posed by the redemptions:

$$q^d_{m,t}(p^\tau) = \frac{C_{m,t} - redemption_{m,t}\left(\frac{p^{mkt}_t - p^{mkt}_{t-1}}{p^{mkt}_{t-1}}\right)}{p^\tau} \tag{49}$$

The aggregate demand of the less-liquid security at price p^τ is then given by

$$\begin{aligned} Q^D_t(p^\tau) &= Q^D_{B,t}(p^\tau) + Q^D_{AM,t}(p^\tau) \\ &= \sum_{i=1}^{N_B} \mathbb{1}_{\{p^\tau < f_i\}} q^d_{i,t}(p^\tau) + \sum_{m=1}^{N_{AM}} \mathbb{1}_{\{p^\tau < f_m\}} q^d_{m,t}(p^\tau) \end{aligned} \tag{50}$$

The exchange of securities between sellers and buyers takes place when both supply and demand at price p^τ are positive. If demand is smaller than supply, a lower price is set to $p^{\tau+1} = p^\tau - \epsilon$, the sellers update their liquidity and selling needs, and the other institutions determine their demand for the new price. If the lower price stimulates additional demand, more exchange at the new lower price takes place; otherwise the price is lowered even further. The process iterates until the supply is fully exhausted in iteration τ^*.

If, on the contrary, supply is smaller than demand, buyers purchase proportionally to the overall demanded quantity, i.e., each bank or asset manager k will buy

$$q^d_{k,t}(p^{\tau^*}) \leftarrow \frac{q^d_{k,t}(p^{\tau^*})}{Q^D_t(p^{\tau^*})} \cdot Q^S_t(p^{\tau^*}) \tag{51}$$

When the supply is fully exhausted, the market closes and the last price p^{τ^*} becomes the new market price p^{mkt}_{t+1}.

Appendix F Non-uniqueness of equilibrium in the market

Suppose that r^{f*} is an interbank rate clearing the demand and supply on the interbank market and that for r^{f*} there exists a unique optimal composition of each bank's balance sheet. Then, since banks' optimisations are in the class of linear programming problems, the solution belongs to the set of corner points of a simplex defined by the capital, liquidity and budget constraints. Therefore, for any value r^{ϵ} in a small enough vicinity of r^{f*}, an affine hyperplane defined by the objective function with risk-free rate equal to r^{ϵ} has only one common point with the simplex. This point is the optimal structure of the balance sheets. The argument is standard in the mathematical programming literature (see e.g., Steuer (1981) for the simplex algorithm). Therefore, the optimal solution for the problem with r^{ϵ} does not change, and – consequently – the demand and supply of interbank loans remain balanced. In other words, for each bank the hyperplane can be rotated by a sufficiently small angle (corresponding to the change of the free rate from r^{f*} to r^{ϵ}) so that the rotated plane is still a supportive hyperplane.

Appendix G Additional robustness and sensitivity analysis

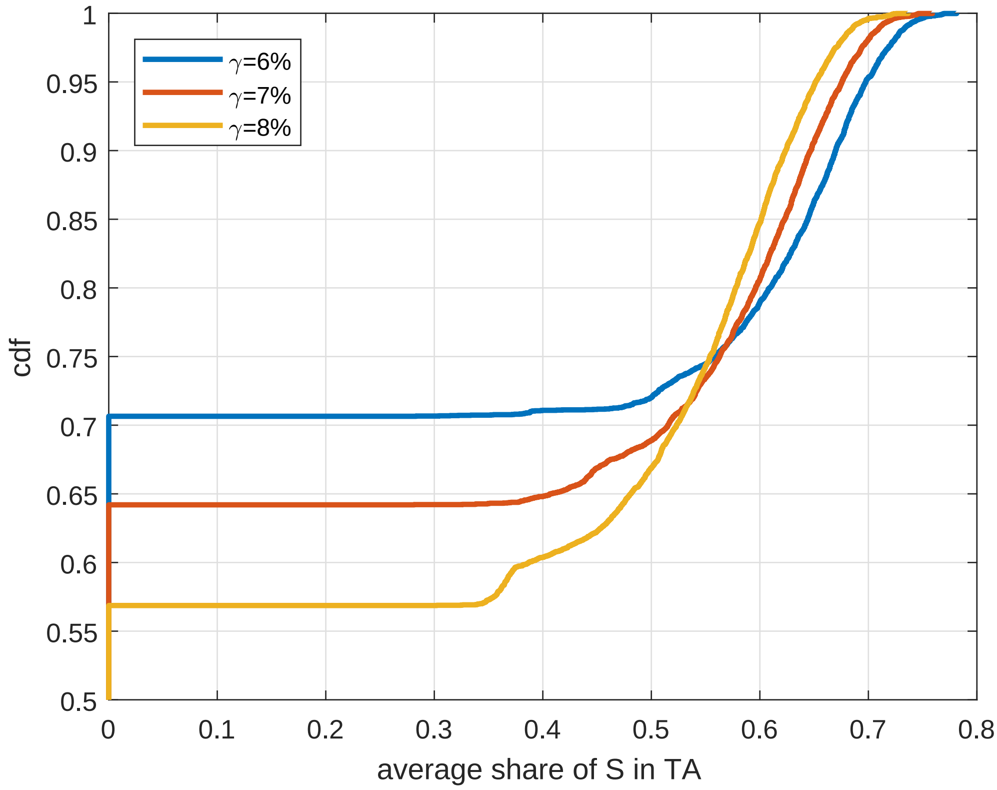


Figure 17: Share of less-liquid securities in banks' total assets for different capital requirements
Note – x-axis: share of securities S in total assets TA (i.e., S/TA) for initial, optimised balance sheets; y-axis: cumulative distribution; distribution of S/TA across simulations (seeds) and shock scenarios for $\alpha = 10\%$.

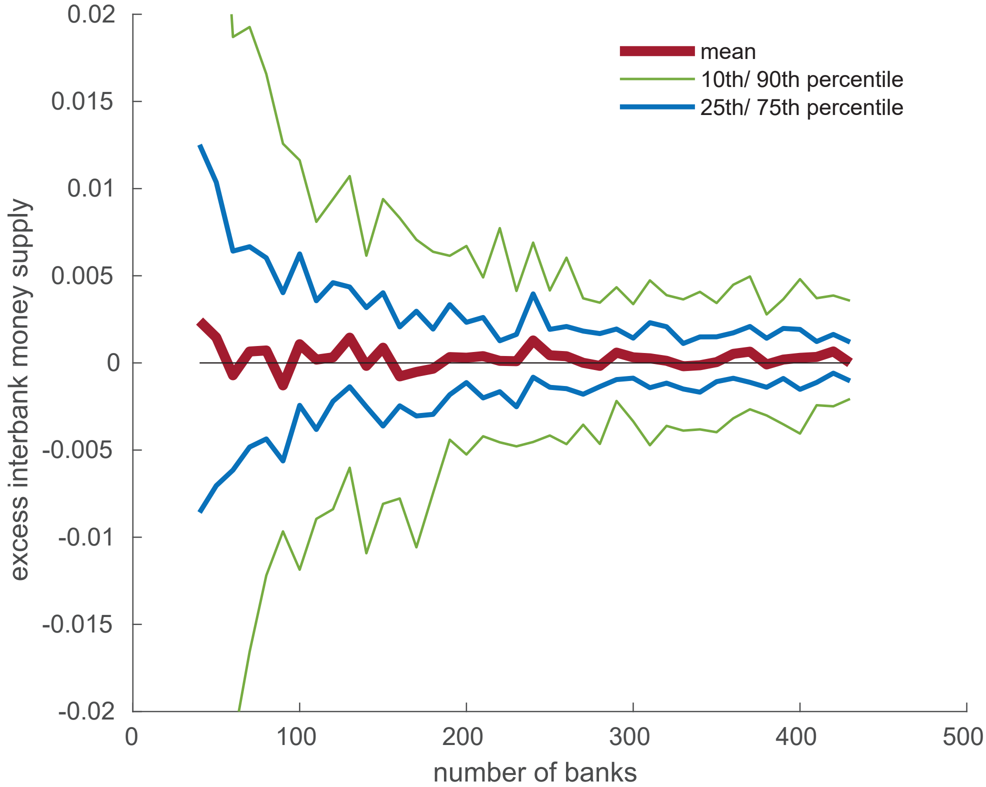


Figure 18: Clearing of the interbank money market
Note – x-axis: number of banks in the interbank market; y-axis: excess money supply in the tâtonnement process as a fraction of total assets of banks; statistics (mean and percentiles) calculated across 100 seeds determining the initial endowment of deposits D_i and capital E_i.

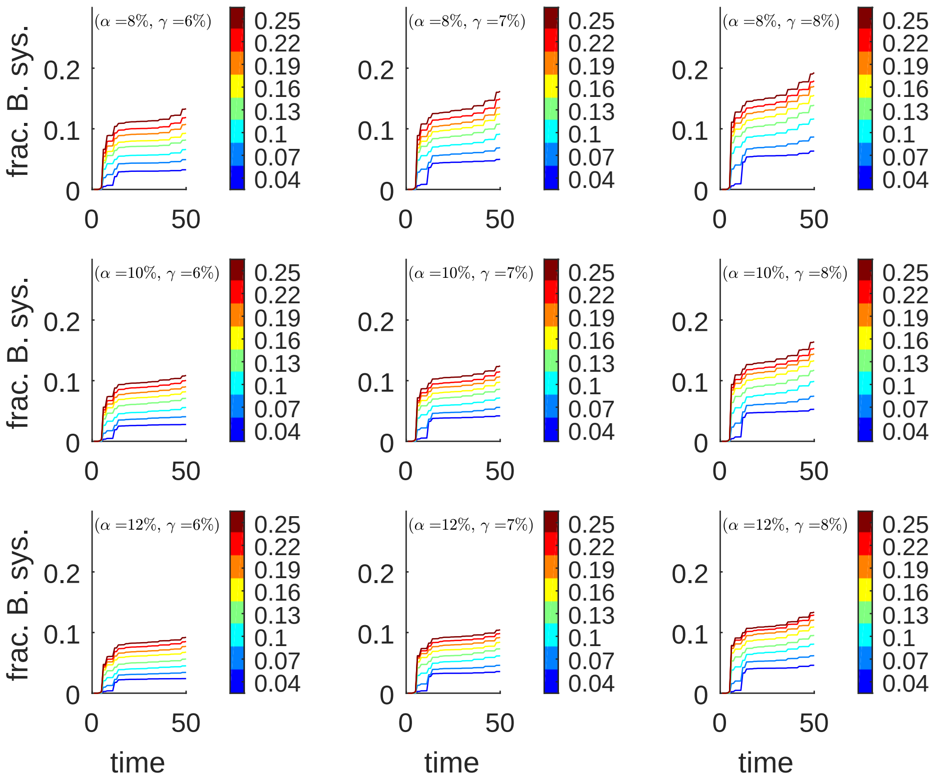


Figure 19: Dynamics of the number of defaulting banks, when the initial random shocks hit banks
Note – x-axis: time (elapsed days); y-axis: fraction of banks with capital below zero in the banking system when the shock originates in the banking sector. Each subplot shows outcomes of simulations for a given pair of liquidity and solvency requirements, (α, γ); each line shows dynamics of the average fraction of defaulting banks for a given shock σ (in rhs colo rbar). Parameter σ defines the outflow fraction of deposits $(D_{i,0} + BB_{i,0})$.

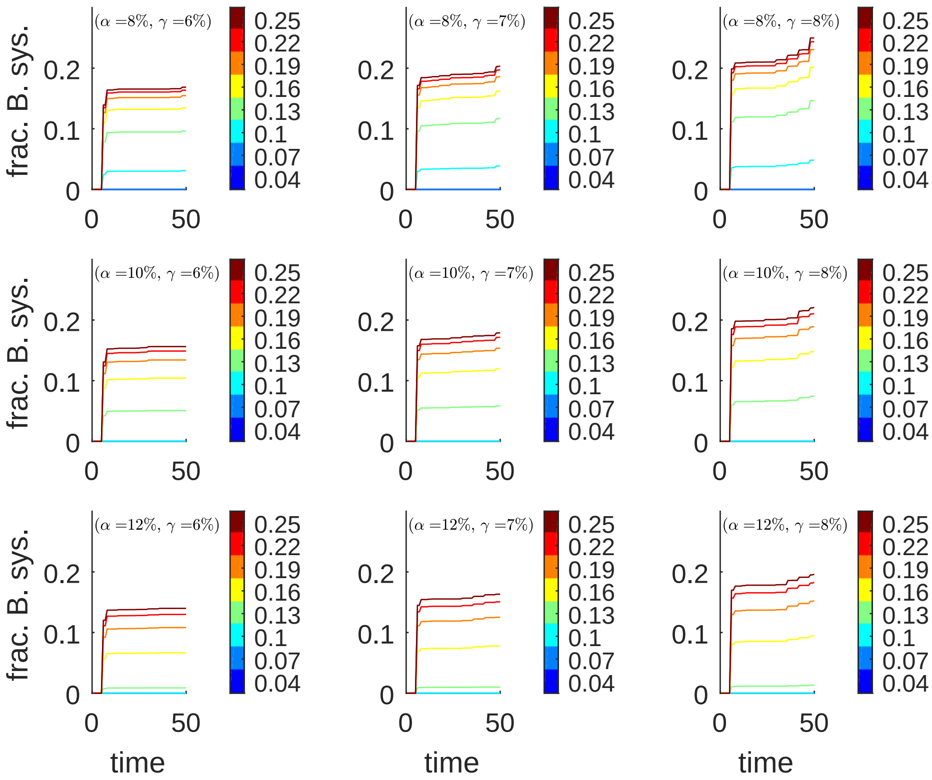


Figure 20: Dynamics of the number of defaulting banks, when the initial random shocks hit asset managers
Note – x-axis: time (elapsed days); y-axis: fraction of banks with capital below zero in the banking system when the shock originates in the asset managers' sector. Each subplot shows outcomes of simulations for a given pair of liquidity and solvency requirements (α, γ); each line shows dynamics of the average fraction of defaulting banks for a given shock σ (in rhs color bar). Parameter σ defines the outflow fraction of participations $P_{m,0}$.

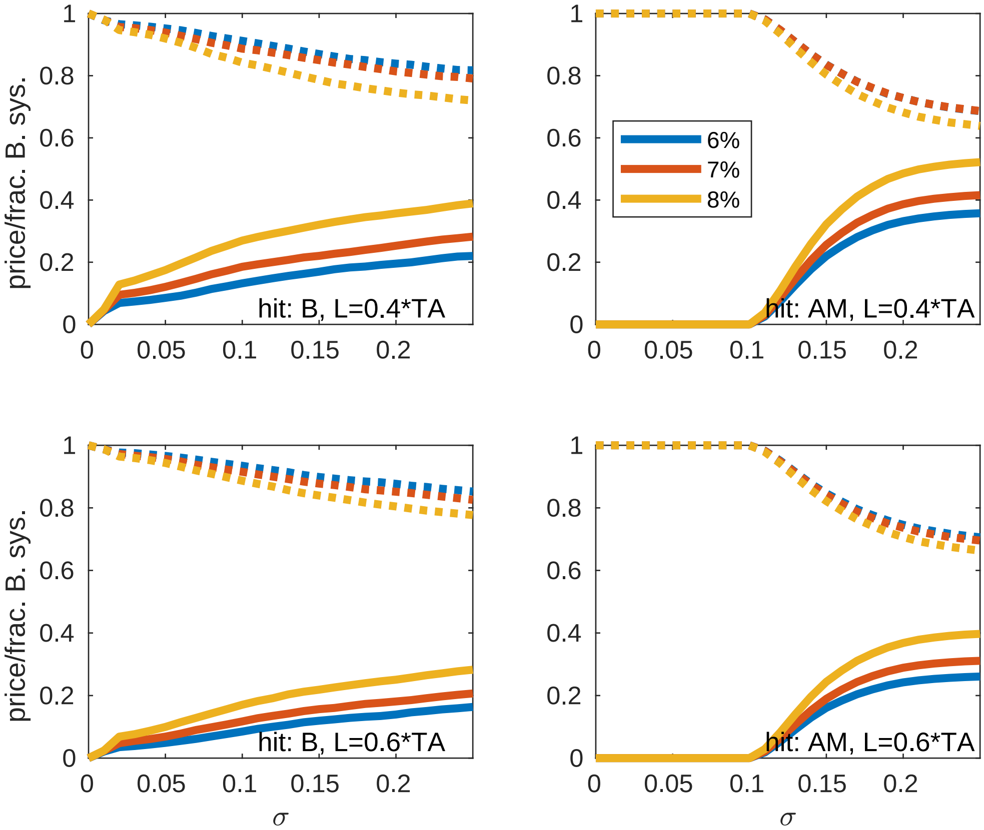


Figure 21: Impact of the funding shock on the asset price and fraction of non-compliant banks for different activity levels on the loan market and under different regimes of solvency regulation
Note – x-axis: shock σ; y-axis: price (dotted lines) or fraction of non-compliant banks (solid lines) in the banking system.
Top panes: loans (L) are equal to 40% of total assets (TA). Bottom panes: loans (L) are equal to 60% of total assets. For all simulations, the liquidity requirement is set to $\alpha = 10\%$ and TA=$D + E$.
The qualitative conclusions about the relationship between the size of the funding liquidity shock, the asset prices, and the share of non-compliant banks are not sensitive to the share of illiquid loan investments locked in banks' balance sheets. We run simulations for alternative settings where the share of loans is instead set to 40% or 60%, and we confirm that more stringent capital requirements may contribute to larger contagion effects. Additionally, we can observe that a higher share of investments into sticky and less-liquid assets has a stabilizing potential for the market. Fewer banks are adversely affected by the liquidity shock when the share of loans L in the banks' assets equals 60%. In other words, funding liquidity shocks are amplified to a lesser extent by banks focused more on the core activity of credit provisioning to the real economy, thus creating less overlap with the asset managers active in trading. This is an interesting conclusion *per se*.

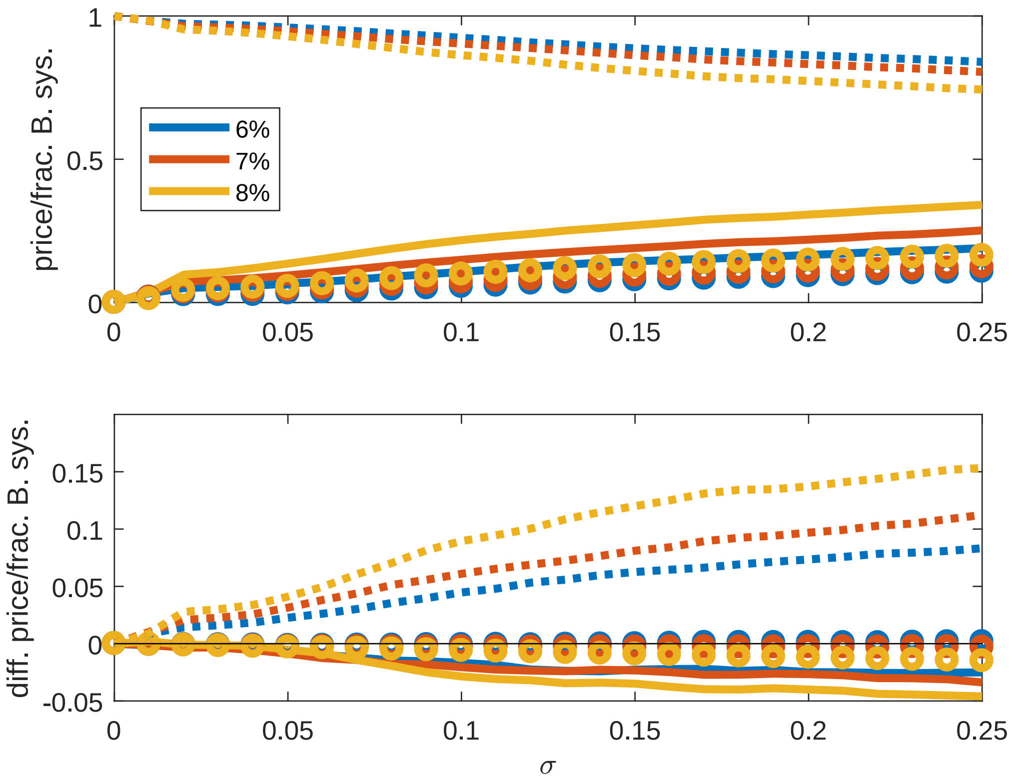


Figure 22: Impact of the asset managers' sector on the asset price and on the fractions of non-compliant and defaulting banks under different regimes of solvency regulation.
Note – x-axis: shock σ, i.e., outflow fraction of deposits $D_{i,0} + BB_{i,0}$.
Top pane – y-axis: price (dotted lines), fraction of non-compliant banks (solid lines), and defaulting banks (circles) with 60 banks and 40 asset managers (i.e., the baseline set-up of the financial system). Bottom pane – y-axis: differences in price (dotted lines), in fraction of non-compliant banks (solid lines), and in fraction of defaulting banks (circles) between a system with 60 banks and 0 asset managers and the baseline system with 60 banks and 40 asset managers. For all simulations, the liquidity requirement is set to $\alpha = 10\%$, and the first-hit agent is a bank.

σ	γ	stat	α time=25 8%	10%	12%	time=50 8%	10%	12%
0.1	6%	25th	0.0	0.0	0.0	0.0	0.0	0.0
	6%	mean	14.3	9.0	7.6	16.2	10.7	8.7
	6%	std	24.2	20.3	18.4	29.5	25.6	22.2
	6%	75th	36.1	0.0	0.0	38.8	0.0	0.0
	7%	25th	0.0	0.0	0.0	0.0	0.0	0.0
	7%	mean	19.9	12.7	11.0	23.5	14.9	12.8
	7%	std	30.4	25.5	23.5	38.6	31.6	28.9
	7%	75th	52.7	0.0	0.0	55.5	0.0	0.0
	8%	25th	0.0	0.0	0.0	0.0	0.0	0.0
	8%	mean	26.5	18.2	15.7	31.6	21.7	18.2
	8%	std	36.9	32.3	29.8	47.0	40.8	36.3
	8%	75th	66.6	2.7	0.0	72.2	2.7	0.0
0.25	6%	25th	2.7	2.7	0.0	2.7	2.7	0.0
	6%	mean	38.4	35.6	32.2	43.0	39.4	35.4
	6%	std	25.5	25.3	24.4	33.1	31.1	29.3
	6%	75th	58.3	55.5	52.7	58.3	58.3	55.5
	7%	25th	41.6	2.7	2.7	44.4	2.7	2.7
	7%	mean	49.4	44.6	40.8	57.1	50.0	44.7
	7%	std	29.0	28.1	27.6	39.5	35.3	33.0
	7%	75th	69.4	66.6	61.1	75	69.4	66.6
	8%	25th	55.5	50.0	8.3	58.3	52.7	41.6
	8%	mean	60.5	56.4	51.1	71.5	64.8	58.0
	8%	std	31.7	31.7	31.2	44.5	41.9	39.7
	8%	75th	80.5	77.7	72.2	88.8	83.3	77.7

Table 3: Dispersion of banks breaching liquidity or capital requirements across simulations (in %)
Note – statistics across seeds ('mean' – average number of banks; 'std' – standard deviation of the number of banks).

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