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Money markets, collateral and monetary policy

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Abstract

During the financial and sovereign debt crises, euro area interbank money markets underwent dramatic changes: the share of unsecured borrowing declined throughout the euro area, while private market haircuts on sovereign bonds and bank borrowing from the European Central Bank increased in the South. We construct a quantitative general equilibrium model to evaluate the macroeconomic impact of these developments and the associated policy response. Our model features heterogeneous banks and sovereign bonds, secured and unsecured money markets, and a central bank. We compare a benchmark policy – the central bank providing collateralized lending to banks at haircuts lower than the market - to an alternative policy that maintains a constant central bank balance sheet. We show that the fall in output, investment, and capital would have been twice as high under the alternative policy. More generally, the model allows the analysis of monetary policy tools beyond interest rate policies and quantitative easing.

Key words: money markets, collateral, monetary policy, central bank balance sheet
JEL code: E44, E52, E58

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Non-technical summary

Interbank money markets are vital to the liquidity management of banks. During the financial and sovereign debt crises, euro area interbank money markets underwent dramatic changes. Using a novel dataset surveying interbank money market activity, we emphasize four main observations. First, the share of unsecured interbank borrowing declined over this period throughout the euro area, with banks substituting toward secured transactions. Second, with the onset of the sovereign debt crisis in 2010, market haircuts on Southern government bonds increased substantially, while the European Central Bank (ECB) kept haircuts nearly unchanged throughout, creating a "haircut gap" between private market and ECB haircuts. Third, bank borrowing from the ECB increased eight-fold in Southern regions. Fourth, household deposits at banks remained stable.

As our main contribution, we construct and analyze a quantitative general equilibrium model to understand these developments, the policies employed by the central bank, and their impact on the overall economy. Our model features heterogeneous banks and heterogeneous government bonds, interbank money markets for both secured and unsecured credit, and a central bank that can conduct open market operations and lend to banks against collateral. We use the model to clarify the role of different interbank market frictions and to quantify the impact of central bank responses compared to a counterfactual scenario of no intervention.

In the model, there are two regions, the "North" and the "South," each with its own government bonds, but one common central bank. Each period in the model is subdivided into a morning and an afternoon. In the morning, banks choose their assets (loans, bonds, and money) and liabilities (central bank funding and deposits), subject to a leverage constraint as proposed by Gertler and Karadi (2011) and Gertler and Kiyotaki (2011). Banks are randomly selected to only hold Northern government bonds or Southern government bonds.

Central bank funding must be backed by bond collateral and is subject to a haircut imposed by the central bank. Deposit funding is uncollateralized but exposes banks to idiosyncratic liquidity shocks in the afternoon, as formulated by Bianchi and Bigio (2022). These liquidity shocks can be managed by borrowing or lending in private interbank money markets. Banks face an exogenous probability of being "connected," defined as the ability to borrow in the unsecured market in the afternoon. The remaining "unconnected" banks need to satisfy possible withdrawals either by holding sufficient reserves or by posting bonds as collateral in the secured market. Collateralized borrowing is subject to a haircut, with haircuts in the private afternoon market being potentially different in the North than in the South and, in turn, different from morning haircuts set by the central bank.

To match the developments in the euro area money markets during the financial and sovereign debt crises, we impose the condition that the probability that a bank is connected gradually and permanently falls from 42% to around 10%, while the private market haircuts on

Southern bonds rise temporarily from 3% to 25%. For the central bank, we compare a benchmark policy – the central bank providing collateralized lending to banks at haircuts lower than the market - to an alternative policy that maintains a constant central bank balance sheet.

We demonstrate that the two policies do not differ much in terms of their impact on the economy in the case of the change in the share of secured lending but they induce significantly different responses from the rise in private market haircuts. In particular, the fall in output, investment, and capital would have been twice as high under the constant balance-sheet policy compared to the benchmark. There is a rich set of interactions, which we highlight in our analysis.

More generally, our analysis shows that interbank money market frictions matter, and central bank policy tools beyond interest rate setting or quantitative easing can have a considerable impact on the economy. Our model provides a framework for studying such financial market developments and central bank policy tools, expanding our understanding compared to the existing literature. The model provides a rich laboratory in which to explore the interaction between liquidity management and leverage, interbank market frictions, and the role of central bank lending beyond the particular episode examined here.

1 INTRODUCTION

During the financial and sovereign debt crises, euro area interbank money markets underwent dramatic changes. Using a novel dataset surveying interbank money market activity, we emphasize four main observations. First, the share of unsecured interbank borrowing declined over this period throughout the euro area, with banks substituting toward secured transactions. Second, with the onset of the sovereign debt crisis in 2010, market haircuts on Southern government bonds increased substantially, while the European Central Bank (ECB) kept haircuts nearly unchanged throughout, creating a "haircut gap" between private market and ECB haircuts. Third, bank borrowing from the ECB increased eight-fold in Southern regions. Fourth, household deposits at banks remained stable.

As our main contribution, we construct and analyze a quantitative general equilibrium model to understand these developments, the policies employed by the central bank, and their impact on the overall economy. Our model features heterogeneous banks and heterogeneous government bonds, interbank money markets for both secured and unsecured credit, and a central bank that can conduct open market operations and lend to banks against collateral. We use the model to clarify the role of different interbank market frictions and to quantify the impact of central bank responses compared to a counterfactual scenario of no intervention.

In the model, there are two regions, the "North" and the "South," each with its own government bonds, but one common central bank. Each period in the model is subdivided into a morning and an afternoon. In the morning, banks choose their assets (loans, bonds, and money) and liabilities (central bank funding and deposits), subject to a leverage constraint as proposed by Gertler and Karadi (2011) and Gertler and Kiyotaki (2011). Banks are randomly selected to only hold Northern government bonds or Southern government bonds. Central bank funding must be backed by bond collateral and is subject to a haircut imposed by the central bank. Deposit funding is uncollateralized but exposes banks to idiosyncratic liquidity shocks in the afternoon, as formulated by Bianchi and Bigio (2022). These liquidity shocks can be managed by borrowing or lending in private interbank money markets. Banks face an exogenous probability of being "connected," defined as the ability to borrow in the unsecured market in the afternoon. The remaining "unconnected" banks need to satisfy possible withdrawals either by holding sufficient reserves or by posting bonds as collateral in the secured market. Collateralized borrowing is subject to a haircut, with haircuts in the private afternoon market being potentially different in the North than in the South and, in turn, different from morning haircuts set by the central bank.

We argue that we need all key ingredients of the model to understand the observed developments in the euro area money markets. We show how reduced versions without the morning leverage constraint and/or the afternoon liquidity management constraint will fail at that task. With all the key ingredients in place, frictions can arise in different places, making the analysis particularly challenging. Five inequality constraints on banks emerge as crucial: the "morning" leverage constraint, a collateral constraint vis-a-vis the central bank, and three short-sale constraints. We show that one cannot a priori require any of these constraints to bind or to be slack. On the contrary, each of these may turn on or off, and each can be crucial for the macroeconomic outcomes as we traverse the parameter space and different monetary policies. We provide the calculus for the bank's decision problem, weighing the various trade-offs. We illuminate the inner workings of the model using steady-state comparative statics while solving the dynamic calibrated model numerically, using techniques in Dynare allowing for these constraints to become occasionally binding.

In order to match the developments documented in Section 2, we impose the condition that the probability that a bank is connected gradually and permanently falls from 42% to around 10%, while the haircuts on Southern bonds rise temporarily from 3% to 25%.[1] For the central bank, we compare the benchmark "collateralized credit operations" policy or CO policy of keeping the central bank haircut unchanged at 3% to a counterfactual and alternative "constant balance-sheet policy" or CB policy, where the central bank haircut is 100%, ruling out bank-borrowing from the central bank.

We demonstrate that the two policies do not differ much in terms of their impact on the economy in the case of the change in the share of secured lending but they induce significantly different responses from the rise in private market haircuts. In particular, the fall in output, investment, and capital would have been twice as high under the constant balance-sheet policy compared to the benchmark. There is a rich set of interactions, which we highlight and illuminate in our analysis.

More generally, the analysis here shows that interbank money market frictions matter, and central bank policy tools beyond interest rate setting or quantitative easing can have

[1] Agents correctly foresee these exogenous changes in probabilities and haircuts. While it would be intriguing to provide deeper foundations and link them to, say, sovereign default risks, doing so would exceed the scope of the already rich analysis presented here.

a considerable impact on the economy. Our model provides a framework for studying such financial market developments and central bank policy tools, expanding our understanding compared to the existing literature. The model provides a rich laboratory in which to explore the interaction between liquidity management and leverage, interbank market frictions, and the role of central bank lending beyond the particular episode examined here.

Related literature. This paper is related to three strands of the literature. First, it is related to numerous papers in macroeconomics that emphasize the role of banks' balance sheet and leverage constraints for the provision of credit to the real economy and for the transmission of standard and non-standard monetary policies (see, e.g., Gertler and Karadi (2011) and Gertler and Kiyotaki (2011)). As in those papers, banks in our model face an enforcement problem and endogenous balance-sheet constraints. Additionally, they solve a liquidity management problem that further constrains their actions. Another novel feature of our framework is that we do not require the various constraints to be binding at all times (as in Brunnermeier and Sannikov (2014); He and Krishnamurthy (2019); Mendoza (2010); Bocola (2016); Justiniano et al. (2019)). In our calibrated models, the five key constraints can switch from binding to slack and vice versa, interacting in complex ways and determining the effectiveness of monetary policy.

Second, our paper is related to a strand of the literature in banking that is concerned about the interactions between capital and liquidity requirements (see Vo (2021) for a survey). For example, Cecchetti and Kashyap (2018) construct a stylized framework to capture the four key Basel III regulations - the risk-weighted capital ratio, the leverage ratio, the liquidity coverage ratio, and the net stable funding ratio - and examine which are likely to bind and how they affect different bank business models. Several papers in this literature study whether capital and liquidity requirements are complements or substitutes (e.g., Vives (2014), Kara and Ozsoy (2020)). While our framework also features multiple constraints for banks, we do not address regulatory issues and, instead, focus on building a dynamic general equilibrium model of bank liquidity management through interbank markets. In this sense, our paper is related to a large body of the literature in banking that studies the role of interbank markets in banks' liquidity management (e.g., Bhattacharya and Gale (1987); Flannery (1996); Repullo (2005); Freixas and Jorge (2008); Freixas et al. (2011); Afonso and Lagos (2015); Atkeson et al. (2015); Martin et al. (2014); Heider et al. (2015)). Relative to these studies, we focus on the macroeconomic implications of interbank frictions. Some authors such as Nguyen (2020), Nissinen and Sihvonen (2022), and Kaldorf and Röttger (2023) have investigated the relationships between the haircuts on European sovereign bonds, their use as collateral and their yields. Our theory here is broadly in line with their findings and approach.

Third - and closest to our work - is the general equilibrium literature that studies how frictions in the unsecured or secured money markets impact the macroeconomy and interact with the effectiveness of monetary policy. Bruche and Suarez (2010) show that freezes in the unsecured money market segment can cause large reallocations of capital across regions, with significant impact on output and welfare. Gertler et al. (2016) point to runs on wholesale banks as a major source of the breakdown of the financial system in 2007-2009. Our analysis is broadly consistent with that of Acharya et al. (2019), who have documented that the increased lending by the ECB via its OMT program launched in 2012 went hand-in-hand with banks building up cash reserves rather than financing real economic activity, though our interpretation and mechanism differ. Bianchi and Bigio (2022) build a model where banks are exposed to liquidity risk. Monetary policy affects lending and the real economy by supplying reserves and thus by changing banks' trade-off between profiting from lending and incurring greater liquidity risk. Arce et al. (2020) show that a policy of a large central bank balance sheet that uses interest rate policy to react to shocks achieves stabilization properties similar to those of a policy of a lean balance sheet, where QE is occasionally used when the interest rate hits the zero lower bound. Piazzesi and Schneider (2017) build a model in which the use of inside money by agents for transaction purposes requires banks to lend or borrow secured in the interbank market or use central bank reserves. As in our paper, the provision and allocation of collateral play a central role in their framework. Our paper contributes to this literature by considering both unsecured and secured funding. In our setup, frictions in the unsecured money market segment may in principle be offset by an increased recourse to private secured markets or to central bank funding. We are also interested in understanding the role of various frictions in explaining the dynamics during the euro area financial and sovereign debt crises.[2] Importantly, several recent empirical papers study the effects of the "haircut gap" channel of monetary policy, the difference between private market and central bank haircuts for the same collateral assets (e.g., Drechsler et al. (2016) and Jasova et al. (2023)). Our model explains and quantifies the

[2] Altavilla et al. (2018) provide evidence that increases in interbank rate uncertainty, as observed during 2007-2009 and again during the European sovereign crisis, generate a significant deterioration in economic activity.

effects of the "haircut gap" channel in a general equilibrium model, by comparing the impact of a collateralized lending policy whereby haircuts are more favorable relative to those in the private market with the counterfactual policy of no lending by the central bank.[3]

The remainder of the paper is organized as follows. Section 2 documents the four observations described above. Section 3 presents the model. Section 4 outlines the analysis of banks - whose behavior is key to the model - and shows that all key ingredients of the model are necessary for interbank market frictions to matter. Section 5 explains the calibration of the model. Section 6 presents a numerical analysis, focusing on understanding the dynamic impact of the interbank market developments in the euro area during the financial and sovereign debt crises. Section 7 concludes.

2 EURO AREA MONEY MARKETS 2003-2015

In this section, we document the four observations described in the introduction.

2.1 Data sources

Our observations are derived from several data sources. First, we use data on haircuts charged by the LCH.Clearnet - one of the largest clearers of repo transactions in the euro area - on government bonds of several euro area countries. We consider the four largest euro area countries - Germany (DE), France (FR), Italy (IT), and Spain (ES) - plus Portugal (PT), for which the relevant data are also available. We obtained the data from the LCH.Clearnet website, where snapshots of the haircut schedules have been made available since 2010.[4] The evolution of haircuts over time suggests substantial cross-country heterogeneity, with haircuts for German and French government bonds moving very little and haircuts on Italian, Spanish, and, in particular, Portuguese government bonds skyrocketing during the sovereign debt crisis. We summarized these data as follows. Country-level haircuts are obtained as simple averages across maturities. Country-group-level haircuts are constructed as weighted averages across the relevant countries, with weights given by the shares of the respective banking sector assets in total. For the Southern countries we consider, the weights are 49%, 44%, and 7% for IT, ES, and PT, respectively, in 2010 (see Table 1 for the time series evolution of haircuts in the

[3]Some authors debated the design features of central bank collateral frameworks and their impact on financial markets, e.g., Nyborg (2015) or Bindseil et al. (2017).

[4]These data have been used in other academic studies; see, e.g., Drechsler, Drechsel, Marquez-Ibanez, and Schnabl (2016), Boissel, Derrien, Ors, and Thesmar (2017), and Jasova, Mendicino, and Supera (2021).

Southern countries). For the Northern countries we consider, the weights are 48% and 52% for DE and FR, respectively, in 2010.

Second, we take data on secured and unsecured interbank market turnover from the ECB Money Market Survey, which was a survey run by the Eurosystem between 2003 and 2015 to gauge money market developments in the euro area. More than 100 banks from across the euro area and additional European countries participated in the survey, conducted in the second quarter of each year. Participating banks were selected to cover the biggest players in money markets of the respective countries. The survey was discontinued in 2015.[5] The ECB regularly published the survey results in aggregated form. As we are interested in cross-country heterogeneity - given the large heterogeneity in repo market haircuts we uncovered - we obtained the proprietary disaggregated Money Market Survey data, which we present at a country-group level. To the best of our knowledge, the disaggregated data have not been explored in the literature before.

Third, we rely on the ECB's Statistical Data Warehouse (SDW) to obtain country-level time series of banking sector assets as well as deposit flows and central bank borrowing. These data are available monthly. To combine this data with the Money Market Survey data - which are based on the observations from the second quarter of each year - we aggregate the monthly data to quarterly frequency by averaging over the quarter (for flows) or by taking end-of-quarter observations (for stocks).

Finally, to aggregate the series to country-group level ("North" and "South"), we weigh country-level observations for Germany and France by the size of their banking sectors in total to obtain the values for the "North" and, similarly, we apply the same weights methodology for Italy, Spain, and Portugal to obtain the values for the "South."

2.2 Four observations

Observation 1: Decline in the share of unsecured interbank borrowing. Between 2008, the year the Great Financial Crisis erupted, and 2015, the end of our sample, turnover in the unsecured money market decreased, while turnover in the secured money market increased. Figure 1 documents this development. Turnover is defined as the sum of all transactions over the second quarter of each year, reported by banks participating in the survey, in the respective money market segment. Normalizing the respective turnover in the secured and unsecured

[5] As of the second half of 2016, information on money markets has been gathered via the data collection called Money Market Statistical Reporting.

money market in 2007 to 100, we observe a negative co-movement between secured (dotted line) and unsecured turnover (bold line) as of 2008. Importantly, the co-movement is negative in both of the large Northern European countries (Germany, DE, and France, FR), in which sovereigns enjoyed a higher credit rating, and in the three Southern European countries (Spain, ES, Italy, IT, and Portugal, PT), in which sovereigns had a lower credit rating.[6]

Observation 2: Bank borrowing from the ECB increased eight-fold in the South. Figure 2 plots bank borrowing from the ECB as a share of bank total assets (solid line, left-hand-side scale) for the largest Northern European countries (DE and FR) and three Southern European countries (Italy, Spain, and Portugal). Figure 2 illustrates that there was no substantial increase until 2010 in bank borrowing from the ECB across both Northern European and Southern European countries. Bank borrowing fluctuated in a range from 0% to 3% across countries. During the sovereign debt crisis, however, bank borrowing increased about eight-fold for banks from the "South," from about 1% to about 8% of total banking sector assets in the "South".

Observation 3: Increase in haircuts on government bonds in the South. The increase in Southern countries' bank borrowing from the ECB as of 2010 (solid line in the right-hand panel of Figure 2) is accompanied by the rise in interbank market haircuts on government bonds in these countries (dashed line in the right-hand panel of Figure 2), while haircuts have stayed largely unchanged in the North.

Meanwhile, the ECB has kept its haircut schedule for central government debt instruments (so-called Category I assets) unchanged all through the Great Financial Crisis and the Sovereign Debt Crisis regardless of the country of issuance.[7] This was the case for both higher-rated assets (rated AAA to A-) and for lower-rated assets (rated BBB+ to BBB-).[8]

Observation 4: Household deposits remained stable. Figure 3 illustrates the time series evolution of the share of bank borrowing from households (aka household deposits) in total bank assets, along with the share of bank borrowing from the ECB, again as a share in

[6]This observation is suggestive of a substitution between unsecured and secured money market activity. Such substitution was documented empirically by Di Fillipo, Ranaldo, and Wrampelmeyer (2022), who analyzed individual banks' borrowing and lending in the unsecured and secured euro interbank markets.

[7]Marginal adjustments in this category were made only in January 2014.

[8]We report haircut numbers for fixed coupon central government debt as it is the same category of assets for which we report LCH.Clearnet haircuts in Table 1. We note that there was a level difference in haircuts charged by the ECB between higher-rated and lower-rated central government debt - 2.8% versus 7.8%, respectively. For simplicity, in our simulations, we keep the central bank haircuts fixed at 3%. This is not very material as what matters in the model is the haircut gap, i.e., the difference between private and central bank haircuts.

total bank assets. Figure 3 shows that the share of bank borrowing from the household sector has a broadly similar pattern across the “North” and the “South.” Importantly, this share is stable or even increases as bank borrowing from the ECB increases. This is true for IT, ES, and PT.

3 THE MODEL

In this section, we present a dynamic general equilibrium model geared toward understanding the developments described in Section 2.

The economy is a monetary union composed of two regions, N (for "North") and S (for "South"). Each region, denoted by $\gamma = N, S$, is of size s_γ, where $s_N + s_S = 1$, and is inhabited by a continuum of identical households and firms. There are two governments and a single central bank. Each government finances public expenditures by raising taxes and issuing debt. Goods and labor are perfectly mobile across regions. Deposit and credit markets are integrated, i.e., banks located in region γ can raise deposits from all households in the union and extend loans to all firms, while bond markets are segmented. In each region γ, banks trade only the bond issued by their own government, $B_{t,\gamma}$. Banks can be "connected" or "unconnected" depending on their ability to access unsecured money markets, as explained below.

Time is discrete and infinite. There is no aggregate uncertainty. In the dynamic analysis, we consider shocks,that are entirely unforeseen (“MIT shocks”). Each period is composed of a “morning” and an “afternoon.”An overview of the timing is shown in Figure 4.

In the morning, households receive nominal payments from their holdings of financial assets and allocate their wealth among money and deposits at banks. They also supply labor to firms, receiving wages in return. Each government taxes the labor income of the households, makes payments on its debt and may change the stock of outstanding debt. Banks are hit by a shock that discloses their afternoon “type,”which will soon be explained in detail.

In the afternoon, firms use labor and capital to produce a homogeneous output good that is consumed by households. Banks settle the idiosyncratic liquidity shocks with reserves acquired in the morning or with interbank loans that are either unsecured (if issued by the connected banks) or secured with government bond collateral (if issued by the unconnected banks).

Firms and banks are owned by households. Similar to Gertler and Kiyotaki (2011) and Gertler and Karadi (2011), banks are operated by managers who run a bank on behalf of their owning households. Banks pay a fixed fraction of their net worth to households as a dividend at the end of the morning in every period.

As the general equilibrium model is nominal, we will use capital letters to denote the relevant nominal variables henceforth.

3.1 The household

In each region, there is a continuum of identical households. Since the two governments impose identical taxes, households are identical in both regions, so we drop the subscript γ.

Households arrive in period t with cash, M_{t-1}^h, and bank deposits D_{t-1}^h that are remunerated at the gross return R_{t-1}^d. Households also earn wages from labor supplied, net of proportional taxes paid, $(1-\tau_t) W_t h_t$. Assets and post-tax wages are then used to finance consumption, $P_t c_t$ along with savings allocated to deposits, D_t^h, and money, M_t^h. The household's budget constraint is then given by

$$M_t^h + D_t^h \quad = \quad M_{t-1}^h + R_{t-1}^d D_{t-1}^h + (1-\tau_t) W_t h_t + \phi N_t + \Pi_t^{CP} - P_t c_t \tag{1}$$

where $\left(\phi N_t, \Pi_t^{CP}\right)$ are bank dividends and profits of the capital producers, respectively, that are both owned by the households.

The household chooses $c_t > 0, h_t > 0, D_t^h \geq 0, M_t^h \geq 0$ to maximize the objective function

$$\max \sum_{t=0}^{\infty} \beta^t \left[u\left(c_t, h_t\right) + v\left(\frac{M_t^h}{P_t}\right)\right] \tag{2}$$

subject to the budget constraint (1).[9] We assume that households derive convenience from their holdings of money, reflected as "money-in-the-utility."

3.2 Final goods firm

A representative final-good firm rents capital k_{t-1} and hires labor h_t to produce a homogeneous final output good y_t according to the production function

$$y_t = k_{t-1}^{\theta} h_t^{1-\theta}. \tag{3}$$

[9]As a functional form, we assume that, $u(c_t, h_t) = log(c_t) - \frac{h_t^{1+\varsigma}}{1+\varsigma}$, and $v(m_t^h) = \frac{1}{\chi} \log(m_t^h)$.

It receives revenues $P_t y_t$, pays wages $W_t h_t$ and pays rent $P_t r_t k_{t-1}$ to banks who are the capital owners.

3.3 Capital-producing firm

The capital-producing firm has access to an investment technology that produces capital. By making investments i_t in units of consumption goods, it produces units of capital $\Phi(i_t/k_{t-1})k_{t-1}$ that are sold at price Q_t^k to the banks, since they are the capital owners. Assuming they are price-takers, each firm solves the following problem

$$\max_{i_t \geq 0} Q_t^k \Phi\left(\frac{i_t}{k_{t-1}}\right) k_{t-1} - i_t \tag{4}$$

where $\Phi'(.)0$, $\Phi''(.) > 0$ and $\Phi(\delta) = \delta$, where δ is the depreciation rate.[10] Given the choice of i_t, capital evolves according to

$$k_t = (1-\delta)\, k_{t-1} + \Phi\left(\frac{i_t}{k_{t-1}}\right) k_{t-1}.$$

Given the price of capital, the nominal return realized at time t for investing in a unit of capital at time $t-1$ is given by

$$R_t^k \quad = \quad \frac{P_t}{P_{t-1}} \frac{Q_t^k}{Q_{t-1}^k} (r_t + 1 - \delta) \tag{5}$$

3.4 Fiscal policies

Fiscal policies are entirely mechanical. Their role is to create a supply of government debt and to calculate implied tax rates. At the beginning of period t, the government of region $\gamma = N, S$ has some outstanding debt with face value $\overline{B}_{t-1,\gamma}$ of which a fraction κ (common across the two regions) will be repaid. The government issues discount bonds with a face value $\Delta\overline{B}_{t,\gamma}$ to be added to the outstanding debt next period. Debt trades at the region-specific price Q_t^γ. The new debt provides nominal resources $Q_t^\gamma \Delta\overline{B}_{t,\gamma}$ in period t. The outstanding debt at the beginning of period $t+1$ for the government in region γ will be $\overline{B}_{t,\gamma} = (1-\kappa)\,\overline{B}_{t-1,\gamma} + \Delta\overline{B}_{t,\gamma}$. We assume that fiscal policy stabilizes the stock of debt at $\overline{B}_{t,\gamma} = s_\gamma \overline{B}^*$, where $\overline{B}^*$ is a parameter, and approaches that target at rate α,

$$\Delta\overline{B}_{t,\gamma} \quad = \quad \alpha(s_\gamma B^* - (1-\kappa)\,\overline{B}_{t-1,\gamma}) \tag{6}$$

[10] As a functional form, we assume that $\Phi(x) = a_1 x^{1-\zeta} + a_2$, $a_1 = \left(\frac{1}{1-\zeta}\right)\delta^\zeta$ and $a_2 = -\delta\left(\frac{\delta}{1-\delta}\right)$. The parameter ζ controls the degree of convexity in the capital adjustment cost function.

Total debt is $\bar{B}_t = \bar{B}_{t,N} + \bar{B}_{t,S}$ and the total debt increase is $\Delta\bar{B}_t = \Delta\bar{B}_{t,N} + \Delta\bar{B}_{t,S} = \alpha(B^* - (1-\kappa)\bar{B}_{t-1})$. Assuming $\bar{B}_{t-1,\gamma} = s_\gamma \bar{B}_{t-1}$, we find $\Delta\bar{B}_{t,\gamma} = s_\gamma \Delta\bar{B}_t$ and $\bar{B}_{t,\gamma} = s_\gamma \bar{B}_t$.

Government γ purchases the amount of goods

$$g_{t,\gamma} = s_\gamma g^* \tag{7}$$

where $g^\star$ is a parameter. Government γ receives its share of seigniorage $s_\gamma S_t$ from the central bank. For the remaining gap, the governments raise proportional taxes on labor income. As debt prices may be region specific, we assume that there are cross-region transfers $T_{t,\gamma}$, so that the labor income tax τ_t is the same in both regions. The government budget balance at time t is

$$P_t s_\gamma g^* + \kappa \overline{B}_{t-1,\gamma} = s_\gamma \tau_t W_t h_t + Q_t^\gamma \Delta\overline{B}_{t,\gamma} + s_\gamma S_t + T_{t,\gamma} \tag{8}$$

where the cross-region transfers net out to zero,

$$T_{t,N} + T_{t,S} = 0 \tag{9}$$

Summing up across regions,

$$P_t g^* + \kappa \overline{B}_{t-1} = \tau_t W_t h_t + (s_N Q_t^N + s_S Q_t^S)\Delta\overline{B}_t + S_t. \tag{10}$$

3.5 The central bank

We allow for a rather rich set of central bank policy instruments, reflecting the expansion of monetary policy tools in recent practice. In particular, the central bank interacts with banks in the "morning,"providing them with collateralized loans, which must be repaid next period. Banks hold these funds as reserves that can be exchanged one-for-one for currency. In particular, banks can use these reserves in the afternoon to repay withdrawing depositors. This morning-afternoon timing ensures that banks must secure needed reserves in advance of potential withdrawals.[11] In detail,

1. The central bank sets the interest rate or, equivalently, the discount price Q_t^F on loans to banks.

2. The central bank demands region-N bonds or region-S bonds as collateral for bank loans and subjects these to haircuts. For each euro's worth of region-γ bonds, the CB provides a

[11]One could consider extending the model to allow for emergency lending by the central bank to banks in the afternoon in case of large withdrawals.

loan of $0 \leq \eta_t^\gamma \leq 1$, $\gamma \in \{N, S\}$, where η_t^S and η_t^N are policy parameters. For tractability, we shall set this haircut choice at a constant throughout, $\eta_t^S = \eta_t^N \equiv \eta$, with $1 - \eta$ being the haircut.[12]

3. The central bank sets the quantity $B_{t,\gamma}^C$ of bonds held outright (rather than as collateral for bank loans) of each region $\gamma \in \{N, S\}$, where the superscript C denotes the central bank's bond holdings.

4. The central bank decides on the supply of money, $\bar{M}_t$, available to the private sector. Throughout, we assume the following money supply rule:

$$\bar{M}_t = \bar{M}_{t-1}\pi_t + Q_t^F \bar{F}_t - R_{t-1}^F Q_{t-1}^F \bar{F}_{t-1} \tag{11}$$

   In other words, absent central bank funding, the money supply is constant in real terms. The last two terms then reflect additional money supply offered via new central bank funding.

Finally, the flow budget constraint of the central bank is given by:

$$\begin{aligned} \overline{M}_t - \overline{M}_{t-1} &= S_t + Q_t^F \overline{F}_t + Q_t^N B_{t,N}^C + Q_t^S B_{t,S}^C \\ &\quad - R_t^N Q_{t-1}^N B_{t-1,N}^C - R_t^S Q_{t-1}^S B_{t-1,S}^C - R_{t-1}^F Q_{t-1}^F \overline{F}_{t-1} \end{aligned} \tag{12}$$

where $R_t^\gamma = \frac{(1-\kappa)Q_t^\gamma + \kappa}{Q_{t-1}^\gamma}$, for $\gamma = N, S$, and $R_t^F = \frac{1}{Q_t^F}$, are the returns from holding government bonds of regions N and S, and from extending loans to banks, respectively.

As a result of these choices, market clearing determines the remaining quantities and prices. Bond market clearing determines the discount price Q_t^N for region-N bonds and the discount price Q_t^S for region-S bonds. Money market clearing determines the currency M_t^h held by households and the reserves M_t held by banks. Banks choose the quantity of loans at face value $\bar{F}_t$, i.e., they receive reserves $Q_t^F \bar{F}_t$ this period for their loans and must repay $\bar{F}_t$ next period to the central bank.

[12] The ECB's haircut schedule features a differentiation according to the credit quality of securities pledged as collateral, resulting in slightly higher haircuts charged on lower-rated sovereigns versus higher-rated sovereigns. The main theme in our analysis, however, is the emphasis on the much larger haircuts imposed by the private sector rather than on the much smaller differentiation among securities in the central bank's haircut schedule.

In many models, central banks target some benchmark interest rate. Likewise, here the central bank could decide to target the bond prices Q_t^N and Q_t^S. It would do so by choosing its instruments above accordingly.[13]

Generally, setting interest rates implies that the quantity will be determined by the market and vice versa. The assets and liabilities of banks are substitutable to a degree, setting limits on how far interest rates can deviate from each other.[14]

Bond holdings are related across time via net bond purchases and bond repayments. Recall that governments repay a fixed fraction κ of the bonds outstanding. Denote the net central bank purchases of bonds by $\Delta B_{t,\gamma}^C$. Thus, $B_{t,\gamma}^C = (1-\kappa)\, B_{t-1,\gamma}^C + \Delta B_{t,\gamma}^C$.

For the episode under consideration and our purpose at hand, we seek to understand, in particular, the impact of the monetary policy choice of allowing banks to borrow from the central bank at favorable haircuts during the crisis. For that reason, we compare the outcomes under two specifications for monetary policy, a benchmark and an alternative. The benchmark is a policy of collateralized lending operations that captures in a stylized manner the main tool used by the ECB during the Great Financial Crisis and the Sovereign Debt Crisis. The alternative is a constant balance-sheet policy that reflects monetary policy before the Great Financial Crisis and provides a counterfactual scenario. The focus on the comparison of these policies furthermore highlights the most novel aspect of our analysis, which is the consideration of collateralized lending to banks at haircuts different from those in the private market.

CO policy For our benchmark *collateralized operations* policy (CO policy for short), we assume that the central bank fixes the haircut η at the favorable pre-crisis private market level $\eta = 0.97$ and fixes the rate on bank loans at $Q^F = 0.997$ close to unity. The amount of bank loans $\bar{F}_t$ is then endogenously chosen by bank demand.

CB policy For our alternative *constant balance-sheet* policy or CB policy, we assume that $\eta = 0$ so that banks never have an incentive to borrow from the central bank, resulting in $\bar{F}_t = 0$.

[13]We have not included interest on reserves in this version of the model in order to avoid the extra notation and issues arising from such an inclusion. It was of no relevance during the euro area sovereign debt crisis and thus the theme at hand, but it could be important to include it in future extensions for other applications.

[14]See the next section for more details.

For both specifications, we assume that purchases of each type of debt are proportional to the region's issuance.[15] Let $B_t^C = B_{t,N}^C + B_{t,S}^C$ and $B_{t,\gamma}^C = s_\gamma B_t^C$. As a result, (12) becomes

$$\begin{aligned}\overline{M}_t - \overline{M}_{t-1} \quad = \quad & S_t + Q_t^F \overline{F}_t + \left(s_N Q_t^N + s_S Q_t^S\right) B_t^C \\ & - \left(R_t^N s_N Q_{t-1}^N + R_t^S s_S Q_{t-1}^S\right) B_{t-1}^C - R_{t-1}^F Q_{t-1}^F \overline{F}_{t-1} \end{aligned} \qquad (13)$$

For both specifications, we assume that the real value of the bonds held outright, $B_{t,\gamma}^C/P_t$, is constant. Bond prices, therefore, arise out of fluctuations in private-sector demand for bonds. Likewise, the amount M_t^h of currency held by households and the amount M_t of reserves held by banks arise from their money demands, given interest rates and market conditions.

3.6 Banks

There is a continuum of banks ("lenders") $i \in [0, 1]$. They are owned by households that are perfectly diversified across banks. Choices in the previous period result in the net worth of a bank at the beginning of the period. Subsequently, their network type and region are drawn. We assume that both are iid across time and banks to ensure the existence of a non-trivial steady state.[16] Banks then make choices regarding assets and liabilities, subject to morning and afternoon constraints, resulting in their net worth at the beginning of the next period.

3.6.1 The morning

In the morning, the date-t type of bank is uncovered as the result of two iid shocks. First, the network type ν is drawn. Bank i is connected ("$\nu = c$") with probability ξ_t and unconnected ("$\nu = u$") with probability $1 - \xi_t$. Unconnected banks will need to pledge bonds in the afternoon in order to borrow in the secured money market, while connected banks can borrow in the afternoon in the unsecured market without posting collateral.

Second, the region γ is chosen. With probability ("size") s_N, a bank is located in $\gamma = N$ ("North"), and with probability $s_S = 1 - s_N$ it is located in $\gamma = S$ ("South"). In each region γ, banks trade and hold only the bond issued by their own government, $B_{t,\gamma}$. Deposit and credit markets are integrated.

[15] The ECB conducted targeted asset purchases during the 2010-2014 period under the so-called "Securities Markets Programme." Those purchases were intended to ensure depth and liquidity in malfunctioning segments of the bond markets. However, such purchases were sterilized and did not affect central bank liquidity outstanding. They are, therefore, not the focus of our study.

[16] If the bank type was fixed forever instead, the type with the highest return would eventually be the only type surviving, except for the creation of new banks.

Knowing their network type and region, banks choose their portfolio of assets and liabilities, and pay dividends. We assume dividends are a constant fraction ϕ of net worth. The ex-dividend net worth is then allocated to capital ($k_{t,i}$), nominal bonds ($B_{t,i}$), and nominal excess reserves ($M_{t,i}$). The bank leverages up using nominal deposits ($D_{t,i}$) and nominal secured loans from the central bank at face value ($F_{t,i}$).

The bank's balance-sheet constraint at the end of the morning is therefore given by

$$P_t Q_t^k k_{t,i} + Q_t^\gamma B_{t,i} + M_{t,i} + \phi N_{t,i} = D_{t,i} + Q_t^F F_{t,i} + N_{t,i}, \tag{14}$$

The collateral constraint at the central bank requires loans not to exceed the value of the bonds pledged, adjusted by the central bank haircut. In equilibrium, banks will pledge just enough collateral to make the collateral constraint bind (even if indifferent between that and pledging more: then, "bind" is a resolution of an indifference). Thus, for all types of banks,

$$F_{t,i} = \eta_t Q_t^\gamma B_{t,i}^F \tag{15}$$

Bonds pledged at the central bank are constrained to be non-negative but also not to exceed the amount of bonds acquired in the morning,

$$0 \leq B_{t,i}^F \leq B_{t,i}. \tag{16}$$

As in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), we assume there is a moral hazard constraint in that bank managers may run away with a fraction λ of their assets at the end of the morning. Let $V_{t,i}$ be the value of a bank at the end of the morning, after the type of the bank is known and assets are purchased, but before dividends are paid. The bank leverage constraint requires the value of the bank $V_{t,i}$ not to fall below the share λ of the value of the assets that the bank manager can run away with:

$$\lambda \left(P_t Q_t^k k_{t,i} + Q_t^\gamma B_{t,i} + M_{t,i} \right) \leq V_{t,i} \tag{17}$$

There are non-negativity constraints for investing in capital, deposits, cash, bonds, and for financing from the central bank, for any bank i:

$$k_{t,i} \geq 0, D_{t,i} \geq 0, M_{t,i} \geq 0, B_{t,i} \geq 0, F_{t,i} \geq 0. \tag{18}$$

3.6.2 The afternoon

In the afternoon, banks face a liquidity management problem, as in Bianchi and Bigio (2022). At the beginning of the afternoon, they experience idiosyncratic liquidity shocks. A bank with end-of-morning deposits $D_{t,i}$ experiences a shock $\omega_i D_{t,i}$. Negative (positive) ω_i denote incoming (outgoing) payments. Here, $\omega_i \in (-\infty, \omega^{\max}]$ is a random variable, which is iid across banks and is distributed according to $F(\omega)$, and $\omega^{\max}$ is a parameter, $0 \leq \omega^{\max} \leq 1$. Payment shocks average out across all banks, so that total deposits remain unchanged. Bank i can settle its payments using reserves $M_{t,i}$ obtained in the morning (outside money) or using interbank loans (IOUs), which are either unsecured or secured by government bond collateral.

We add to the Bianchi and Bigio (2022) structure the distinction between "connected" and "unconnected" bank types. Connected banks can issue unsecured IOUs in the afternoon interbank market. We think of connected banks as operating in a network of banks mutually trusting each other such that collateral is unnecessary.

By contrast, unconnected banks must secure their IOUs with bonds. In the secured (repo) market, we assume that a lending bank imposes a haircut $0 \leq 1 - \widetilde{\eta}^{\gamma} \leq 1$, with $\gamma = S, N$. We allow for haircuts to differ across regions. Haircuts and their regional dependency can be due to different reasons, including the rational assessment of the market on default risks of the underlying bonds. Our focus is on the analysis of the equilibrium in "benign" circumstances. We therefore sidestep modeling those reasons and instead treat the haircuts as exogenous.

The borrowing bank then pledges the amount $\left(B_{t,i} - B^F_{t,i}\right)$ of bonds, as it can only pledge the portion that has not yet been pledged to the central bank. In return, the bank receives the cash amount $\widetilde{\eta}^{\gamma}\left(B_{t,i} - B^F_{t,i}\right)$ in the first leg of the repo, repaying the same amount at the end of the afternoon. The end bond position is therefore the one held in the morning, $B_{t,i}$. Taken literally, there is no risk of bond default here that this haircut could reasonably insure against, but this is just to keep the model simple. Every bank can lend unsecured to connected banks, secured against collateral and within as well as across regions, if they so choose.[17] We impose that the interest rate on interbank IOUs is zero.[18]

[17] Implicitly, we are assuming that the discount window of the central bank is not open in the afternoon, i.e., that banks need to obtain central bank reserves, if any, in the morning as a precaution in anticipation of possible liquidity shocks in the afternoon. This captures the fact that the discount window is rarely used for funding liquidity needs and that these liquidity transactions happen "fast,"compared to central bank liquidity provision.

[18] This can be justified if some banks hold positive reserves $m_{t,i} > 0$: in that case, there is an excess supply of reserves (payment inflows plus morning reserves) compared to the demand for reserves (payment outflows). The market clearing interbank rate then must fall to the price of the alternative storage technology for keeping reserves, i.e., to zero. If no banks wish to hold positive reserves in the morning, supply and demand for interbank

Bankruptcy is not allowed. Hence, if bank i is unconnected, it has to make sure it has enough reserves brought over from the morning and/or enough unpledged collateral to be able to cover the maximum possible afternoon payment outflows, $\omega^{\max} D_{t,i}$. In other words, an unconnected bank i must satisfy

$$\omega^{\max} D_{t,i} \leq M_{t,i} + \widetilde{\eta}_t^{\gamma} Q_t^{\gamma} \left(B_{t,i} - B_{t,i}^F \right), \tag{19}$$

We denote (19) as the unconnected bank's "afternoon constraint."[19]

At the end of the afternoon, the liquidity shocks are reversed, and the interbank loans are repaid. Thus, an initial afternoon liquidity shock creates only a temporary liquidity need that banks must satisfy, in line with the idea of payments circulating in the system.[20] We assume that all within-afternoon interest rates are zero. Therefore, banks will be entirely indifferent between using any of the available sources of liquidity. The balance sheet at the end of the afternoon and before asset returns accrue is the same as the balance sheet at the end of the morning. The only impact of these choices and restrictions is that unconnected banks need to plan ahead of time in the morning to make sure they have enough reserves or collateral in the afternoon.

An overview of the timing of bank decisions is shown in Figure 4.

3.7 Price setting

We assume that prices are determined one period in advance. In particular, we have that:

$$\pi_t = \pi_{t-1}^e \tag{20}$$

In other words, inflation next period is already pre-determined at π_t^e. It endogenously adjusts to ensure that the money market clears. For our perfect foresight experiments, the sole restriction this imposes is that the inflation rate at the time of the shock does not respond.[21]

loans are equal across a range of interest rates. We pick the lowest one, compatible with the storage alternative, implicitly assuming that the borrowing banks have all the bargaining power. Alternatively, one could introduce a minimum reserve requirement so that $m_{t,i} > 0$ always.

[19] We assume that banks will always find defaulting on the payments worse than any precautionary measure they can take against it, and thus rule out payment caps and bank runs by assumption. One might alternatively read the absence of bankruptcy as the result of vigilant regulators.

[20] We follow a long tradition in the banking literature of focusing on the role of interbank money markets in smoothing out idiosyncratic liquidity shocks, as in Bhattacharya and Gale (1987) and Allen and Gale (2000). While analytically convenient, in reality interbank relationships may exhibit more persistent patterns, with some banks being structural borrowers and others as structural lenders in interbank markets (Craig and Ma (2022)).

[21] It turns out that this is important for stability in the evolution of inflation. By ensuring that inflation does not jump at the time of the shock, it prevents unrealistically large shifts in the inflation rate.

3.8 The rest of the world

We assume that a share of the stock of government bonds is held by the rest of the world and that foreigners have an elastic demand for those bonds.[22] Because unconnected banks can buy or sell bonds to foreigners, they can change their bond holdings independently from the government's outstanding stock of debt.

We do not wish to model the foreign sector explicitly. We simply assume that international investors have demands for bonds issued in region γ, $B^w_{t,\gamma}$, that react to movements in the real return,

$$\frac{B^w_{t,\gamma}}{P_t} = \varkappa \left(1 + \frac{1}{\varrho} \log\left(\frac{R^\gamma_{t+1}}{r^\gamma \pi_{t+1}}\right)\right) \tag{21}$$

where $\varrho > 0$, $\varkappa \geq 0$, $\frac{R^\gamma_{t+1}}{\pi_{t+1}}$ is the real one-period holding return on the bond from t to $t+1$, and r^γ is the real rate of return on bond γ in the baseline calibrated steady state. Notice that, if $\varrho = 0$, the bond demand is infinitely elastic. In that case, the real return is fixed and foreign holdings take whatever value is needed to clear the bond market.

The flow budget constraint of the foreign sector is

$$Q^N_t B^w_{t,N} + Q^S_t B^w_{t,S} - R^N_t Q^N_{t-1} B^w_{t-1,N} - R^S_t Q^S_{t-1} B^w_{t-1,S} = -P_t c^w_t \tag{22}$$

where c^w_t is the consumption of the homogeneous good produced in the union. The left-hand side is the net investment of foreign investors in domestic bonds, i.e., the flow in the financial account, while the right-hand side is the corresponding trade balance.

4 UNDERSTANDING BANKS

The decision problem of households and firms is standard. The full optimality conditions for households and firms are reported in Online Appendix A. The key to the model is the decision problem and behavior of the banks. Banks face a number of constraints. In this section, we shed light on their interaction.

[22] We introduce the elastic foreign sector demand for two reasons. First, a large fraction of euro area sovereign debt is held by non-euro area residents, and these bondholders actively re-balance their bond positions. Koijen et al. (2021) document that during the Public Sector Purchase Programme implemented by the ECB in March 2015, for each unit of sovereign bonds purchased by the ECB, the foreign sector sold 0.64 of it. Second, when solving the model we will focus on the parameter space in which connected banks choose not to hold bonds. In a closed economy, unconnected banks would have to absorb whatever amount of bonds is issued by the government (after deducting the fixed amount held by the central bank). The price of the bond would have to adjust to clear the market. Such a direct link between the bond market and the unconnected banks' decisions would become quantitatively implausible.

4.1 Aggregating across banks

First, we establish that the problem of a bank is linear in net worth. Therefore, the distribution of net worth across banks and thus its prior history does not matter for aggregate allocations. Regardless of their size, we assume that all banks behave competitively and take prices as given.

Recall that $V_{t,i}$ is the value of bank i, at the end of the morning, after the type of the bank is known and assets are purchased but before dividends are paid. It is the nominal price a household would be willing to pay for that bank before dividend payments and taking into account the future randomness of net worth due to the future type draws, and is given by

$$V_{t,i} = E\left[\sum_{s=0}^{\infty} \beta^s \frac{u_c\left(c_{t+s}, h_{t+s}\right)}{u_c\left(c_t, h_t\right)} \frac{P_t}{P_{t+s}} \phi N_{t+s,i}\right] \tag{23}$$

This can be rewritten in a recursive fashion. Define $\bar{V}_{t,i}$ as the value of bank i in the morning, before the type draw for t is known. It is given by

$$\bar{V}_{t,i} = E_t[V_{t,i}] \tag{24}$$

where the expectation reflects the type draw.

Proposition 1 (linearity) *The problem of a bank i is linear in net worth and*

$$\bar{V}_{t,i} = \psi_t N_{t,i} \tag{25}$$

for some factor ψ_t which gives the value of a marginal unit of net worth of a bank in the morning, before a bank's type is known. In particular, $\bar{V}_{t,i} = 0$ if $N_{t,i} = 0$.

Proof: Conditional on banks' finding out their network type ν and region γ, the bank problem is linear. Since there are no fixed costs, a bank with twice as much net worth can invest twice as much in the assets. Furthermore, if a portfolio is optimal at some scale for net worth, then doubling every portion of that portfolio is optimal at twice that net worth. Thus, the value of the bank is twice as large, conditional on knowing its type at time t. Taking expectations over the type that will be revealed, which is a linear operator, the linearity is preserved before knowing the type. QED

The proposition above implies that

$$\int_0^1 \bar{V}_{t,i} di = \psi_t \int_0^1 N_{t,i} di \tag{26}$$

which gives the value of a marginal unit of net worth at the beginning of period t, for the aggregate banking sector.

Given the linearity of the bank problem per Proposition 1, it suffices to analyze the problem of a bank with average net worth N_t of network type ν and in region γ. Note that connected banks make the same choices regardless of their region, since the region only matters for unconnected banks in the afternoon. Letting $N_{t+1,\nu,\gamma}$ denote the resulting net worth for each type (ν,γ) of bank when starting out with average net worth N_t, and knowing $N_{t+1,c,N} = N_{t+1,c,S} \equiv N_{t+1,c}$, average net worth in $t+1$ is then

$$N_{t+1} = \xi_t N_{t+1,c} + (1-\xi_t)\left(s_N N_{t+1,u,N} + s_S N_{t+1,u,S}\right) \tag{27}$$

Define $\tilde{V}_{t,\nu,\gamma}$ as the value of a bank with average net worth, network type ν and located in γ at the end of the morning, after the distribution of dividends. We have

$$V_{t,\nu,\gamma} = \phi N_{t,\nu,\gamma} + \tilde{V}_{t,\nu,\gamma} \tag{28}$$

where

$$\tilde{V}_{t,\nu,\gamma} = \beta E_t\left[\frac{u_c\left(c_{t+1},h_{t+1}\right)}{u_c\left(c_t,h_t\right)}\frac{P_t}{P_{t+1}}\psi_{t+1}N_{t+1,\nu,\gamma}\right] \tag{29}$$

with net worth $N_{t+1,\nu,\gamma}$, resulting from the investments of period t, given by

$$N_{t+1,\nu,\gamma} \quad = \quad R^k_{t+1}P_tQ^k_tk_{t,\nu,\gamma} + M_{t,\nu,\gamma} + R^\gamma_{t+1}Q^\gamma_tB_{t,\nu,\gamma} - R^d_tD_{t,\nu,\gamma} - R^F_tQ^F_tF_{t,\nu,\gamma} \tag{30}$$

Equation 29 makes use of Proposition 1 because a bank of type ν,γ at time t is valued at time $t+1$ at $\psi_{t+1}N_{t+1,\nu,\gamma}$. As a result, (28), (29), and (30) deliver a recursive formulation of (23).

In the morning, after the type is known, bank l chooses $k_{t,\nu,\gamma}$, $B_{t,\nu,\gamma}$, $B^F_{t,\nu,\gamma}$, $F_{t,\nu,\gamma}$, $D_{t,\nu,\gamma}$, $M_{t,\nu,\gamma}$ to maximize $V_{t,\nu,\gamma}$, subject to the following constraints:

$$\begin{aligned}
V_{t,\nu,\gamma} &\geq \lambda\left(P_tQ^k_tk_{t,\nu,\gamma} + Q^\gamma_tB_{t,\nu,\gamma} + M_{t,\nu,\gamma}\right)\\
0 &\leq B_{t,\nu,\gamma} - B^F_{t,\nu,\gamma}\\
P_tQ^k_tk_{t,\nu,\gamma} + Q^\gamma_tB_{t,\nu,\gamma} + M_{t,\nu,\gamma} + \phi N_{t,\nu,\gamma} &= D_{t,\nu,\gamma} + Q^F_tF_{t,\nu,\gamma} + N_{t,\nu,\gamma}\\
F_{t,\nu,\gamma} &\leq \eta_tQ^\gamma_tB^F_{t,\nu,\gamma}
\end{aligned}$$

the non-negativity constraints

$$M_{t,\nu,\gamma} \geq 0, B_{t,\nu,\gamma} \geq 0, F_{t,\nu,\gamma} \geq 0, D_{t,\nu,\gamma} \geq 0, k_{t,\nu,\gamma} \geq 0. \tag{31}$$

For the afternoon and unconnected banks

$$\omega^{\max} D_{t,u,\gamma} \leq M_{t,u,\gamma} + \tilde{\eta}_t^{\gamma} Q_t^{\gamma} \left(B_{t,u,\gamma} - B_{t,u,\gamma}^{F} \right). \tag{32}$$

The problems above are linear programming problems, maximizing a linear objective subject to linear constraints. So, the solution is either a corner solution or there will be indifference between certain asset classes, resulting in no-arbitrage conditions. The optimality conditions of the problem of the banks are reported in Online Appendix B. The equilibrium of the model is defined in Online Appendix C. All the equilibrium conditions are also listed there.

4.2 The essential nature of the afternoon and leverage constraints

So far, we have assumed the existence of a leverage constraint for each bank type and an afternoon constraint for the unconnected bank types. We show that both constraints are necessary for changes in the share of unconnected banks ξ or in the market haircuts $\tilde{\eta}^{\gamma}$ to matter for the equilibrium.

4.2.1 A model with no leverage constraint

Suppose that there is no leverage constraint for any bank, but unconnected banks face an afternoon constraint. The following proposition ensures that shocks to $(\xi, \tilde{\eta}^{\gamma})$ do not matter.

Proposition 2 (no leverage constraint) *Consider the model described in Section 3 but without a leverage constraint. Fix all parameters, except to consider two values for the sequence of parameters* $\left(\xi_t, \tilde{\eta}_t^N, \tilde{\eta}_t^S\right)$*, indexed by A and B. If there is an equilibrium for* $\left(\xi_{t;A}, \tilde{\eta}_{t;A}^N, \tilde{\eta}_{t;A}^S\right)$*, then there is an equilibrium for* $\left(\xi_{t;B}, \tilde{\eta}_{t;B}^N, \tilde{\eta}_{t;B}^S\right)$*, where all aggregate quantities are the same.*

The proof is in Online Appendix F. Intuitively, without a leverage constraint, banks are fully unconstrained, and so invest in capital until the risk-adjusted return for shareholders, i.e., households, is the same as the risk-free rate. In other words, banks price assets and liabilities using the household's stochastic discount factor. Consequently, this model is isomorphic to households investing in capital themselves directly without any banks, and so shocks to characteristics of the interbank market are irrelevant.

4.2.2 A model with no afternoon constraint

Suppose banks in region γ face a leverage constraint but there is no afternoon constraint for unconnected banks. The following proposition ensures that shocks to $(\xi, \tilde{\eta}^{\gamma})$ do not matter.

Proposition 3 (no afternoon constraint) *Consider the model described in Section 3 but without an afternoon constraint for unconnected banks. Then the equilibrium is independent of any shocks to* $(\xi, \tilde{\eta}^{\gamma})$*.*

Intuitively, without any afternoon constraint, the two bank types are identical to each other. Therefore, any movement in ξ does not affect the equilibrium because adjusting shares across banks that are identical has no impact. In addition, $\tilde{\eta}^{\gamma}$ disappears from the optimization problem of banks. There is no need for banks to post collateral as they all have access to unsecured markets. As a result, as we show in Online Appendix E, with no afternoon constraint, there is no collateral premium attached to bonds by banks.

4.3 Analyzing the bank problem

We are now ready to analyze the decision of a bank of region-type (μ, γ). We focus on the portfolio choice for banks in a given period. In order to neatly convey the key trade-offs that are crucial for understanding the mechanisms in the full model, in this section we make a simplifying assumption: no aggregate shocks. In this setting, the returns on assets and liabilities (that banks take as given) are known with certainty. The bank then simply chooses a portfolio that maximizes the return on net worth subject to the afternoon and leverage constraints.

Of course, allowing for aggregate shocks requires a dynamic model. The bank's full problem and the associated optimality conditions are reported in Online Appendix B. The definition of the equilibrium and the full system of the equilibrium conditions are reported in Online Appendix C and D, respectively. For now, though, we stick to this stylized setup.

We drop the time subscript and make two assumptions that turn out to always be true in equilibrium in the full model: (i) $R^k > R^{\gamma}$ and (ii) $R_d < R_F$. Banks will then only borrow from the central bank if it offers additional advantages. For connected banks, there are no such advantages as they never need collateralized funding when they have access to unsecured markets. Therefore, their portfolio choice in the morning is simply

$$D_{c,\gamma} = \frac{V_{c,\gamma}}{\lambda} - (1-\phi)\,N \quad \text{and} \quad Q^k k_{c,\gamma} = \frac{V_{c,\gamma}}{\lambda}$$

Connected banks invest entirely in capital as bond returns are lower. In the afternoon, they use unsecured funding to cover temporary deposit withdrawals.

Understanding the portfolio choice of unconnected banks in the morning, however, requires some careful analysis and is key for our dynamic results. It starts by deciphering how banks optimally plan to cover deposit withdrawals in the afternoon, given their decision on $D_{u,\gamma}$ and $F_{u,\gamma}$ in the morning. As dictated by the afternoon constraint, banks can do so, using money ($M_{u,\gamma}$) or bonds ($B_{u,\gamma}$). Which of the two the bank holds depends on the "collateral premium" for bonds, Λ^γ, equaling [23]

$$\Lambda^\gamma = \frac{R^k - R^\gamma}{\widetilde{\eta}^\gamma}. \tag{33}$$

Intuitively, to issue $(1/\omega^{\max})$ of deposits, a bank must hold $(1/\tilde{\eta}^\gamma)$ units of bonds. The cost of doing so is the returns forgone. Λ^γ is precisely this: the returns forgone from having to invest $(1/\tilde{\eta}^\gamma)$ units in bonds instead of capital. By the same logic, if a bank uses money to back $(1/\omega^{\max})$ deposit units, the returns forgone are $R^k - 1$.

$$M_{u,\gamma} = \begin{cases} 0 & \text{if} \quad \Lambda^\gamma < R_k - 1 \\ \in [0, \omega^{\max} D_{u,\gamma}] & \text{if} \quad \Lambda^\gamma = R_k - 1 \\ \omega^{\max} D_{u,\gamma} & \text{if} \quad \Lambda^\gamma > R_k - 1 \end{cases} ; \quad B_{u,\gamma} = \frac{F_{u,\gamma}}{Q^\gamma \eta} + \frac{1}{Q^\gamma \tilde{\eta}^\gamma}\left(\omega^{max} D_{u,\gamma} - M_{u,\gamma}\right) \tag{34}$$

There are three cases, as shown in equation (34). If $\Lambda^\gamma > R^k - 1$, money is a cheaper source of collateral than bonds. Thus, money is used exclusively to satisfy the afternoon constraint. Conversely, if $\Lambda^\gamma < R^k - 1$, only bonds are used, while if $\Lambda^\gamma = R^k - 1$, then any bond-money mix is optimal. Note also that extra bonds must be held as collateral for any central bank funding, at a haircut η, as bonds are the only acceptable collateral for the central bank.

Armed with this analysis, we can now understand how the bank chooses between the two funding sources: deposits ($D_{u,\gamma}$) and central bank funding ($F_{u,\gamma}$). An additional unit of deposits earns X_d:

[23] In Online Appendix E, we show that the collateral premium is strictly positive if the afternoon constraint is binding or unconnected banks are collateral-constrained in their borrowing from the central bank.

$$X_d = R_k - R_d - \omega^{\max} \min\{R_k - 1, \Lambda^\gamma\} \tag{35}$$

The first term, $R_k - R_d$, is the return earned if the bank invests the deposit unit in capital. The final term reflects the fact that banks cannot invest all in capital but must hold collateral to back the deposit unit. This is costly because of the forgone returns, as explained above. The min operator is a result of choosing the least-cost type of collateral: bonds or money.

Conversely, an additional unit of central bank funding earns X_f:

$$X_f = R_k - R_f - \frac{\widetilde{\eta}^\gamma}{\eta}\Lambda^\gamma \tag{36}$$

Here, the collateral cost is without a min operator because bonds must be posted to the central bank. The term $\tilde{\eta}^\gamma/\eta$ is because the central bank applies its own haircut, $1 - \eta$, not the private haircut $(1 - \tilde{\eta}^\gamma)$, to the bond collateral.

Now, suppose that $\max\{X_d, X_f\} > 0$. Then the bank will access funding until the leverage constraint binds because more funding enhances portfolio returns. In other words, $(D_{u,\gamma}, F_{u,\gamma})$ is such that

$$D_{u,\gamma} + F_{u,\gamma} = \frac{V_{u,\gamma}}{\lambda} - (1 - \phi)N \tag{37}$$

To determine the allocation across deposits and central bank funding, the bank compares X_d and X_f. If $X_d > X_f$, then $F_{u,\gamma} = 0$, while if $X_d < X_f$, then $D_{u,\gamma} = 0$. If $X_d = X_f$ any non-negative $(D_{u,\gamma}, F_{u,\gamma})$ combination satisfying (37) is optimal.[24],[25]

This scenario always prevails in the dynamics. Here, we can already make two key observations. First, a lower $\tilde{\eta}$ reduces the returns to deposit issuance because it requires banks to hold more costly collateral instead of capital. Second, central bank policy matters: lowering R_f and/or raising η makes central bank funding more attractive and can make $X_f \geq X_d$, especially if banks are facing high private haircuts $(1 - \tilde{\eta}^\gamma)$.

This section makes clear the trade-offs banks face in their portfolio choice and how non-negativity constraints on portfolio holdings can turn from slack to binding and vice versa. To fully understand the effects of central bank policy, private haircuts, and other shocks, however,

[24]For completeness, if $\max\{X_d, X_f\} \leq 0$, an optimal choice is simply for the bank to only invest net worth into capital and not utilize any other funding sources.

[25]In the full dynamic model, it is always the case that $X_d \geq X_f$ as central bank funding is never made sufficiently attractive such that $X_d < X_f$. That is why we can assume cases where $D_{\nu,\gamma} > 0$.

we must turn to the general equilibrium setup where asset returns, bank value, and asset supply all become endogenous.

5 CALIBRATION

5.1 Parameters

This section describes the calibration of the model parameters to euro area data. A period is one quarter. Table 2 provides the values for all parameters. Here, we focus on key choices.

We focus on cases where banks choose to raise deposits and to extend loans, i.e., we focus on equilibria, where $D_{t,\nu,\gamma} > 0, k_{t,\nu,\gamma} > 0$ for all ν and γ. However, we explicitly allow for corner solutions for $M_{t,\nu,\gamma}$, $B_{t,\nu,\gamma}$ and $F_{t,\nu,\gamma}$.

One central parameter capturing the liquidity management constraints faced by unconnected banks is $\omega^{\max}$. We calibrate it using the information embedded in the liquidity coverage ratio (LCR) - a prudential instrument that requires banks to hold high-quality liquid assets (HQLA) in an amount that allows them to meet 30 days of liquidity outflows under stress. We implicitly assume here that regulators can estimate with high precision the 30 day outflows in a period of stress. We therefore calibrate $\omega^{\max}$ so that the maximum amount of liquidity demand in the model, $\omega^{\max} D_{t,u}$, equates to the observed holdings of HQLA.[26] More specifically, we use the European Banking Authority report from December 2013, which provides LCR data for 2012Q4 and covers 357 EU banks from 21 EU regions. We take $\omega^{\max}$ to be the ratio of aggregate HQLA over total assets, implying that $\omega^{\max} = 0.1$.

We set the fraction of government bonds repaid each period, κ, to 0.042, corresponding to an average maturity of the outstanding stock of euro area sovereign bonds of around 6 years.[27]

We choose the parameter $\varkappa$ of the foreign demand for bonds to ensure that if foreign bond holdings take a value consistent with their observed share in total debt, then Q^{γ} and π also take their average value at that steady state. We take data reported by Koijen et al. (2021) on

[26] In our model, whenever the afternoon constraint binds, banks hold liquid assets in the amount of $M_u + \tilde{\eta} Q \left(B_u - B_u^F\right)$ to cover afternoon withdrawals $\omega^{\max} D$. Since $F = 0$ in our calibrated steady state, and net worth is a small fraction of total liabilities, we approximate D with total assets. Alternatively, we can approximate $\omega^{\max}$ using the run-off rates on deposits, as specified in the LCR regulation (e.g., a run-off rate of 10% means that 10% of the deposits are assumed to possibly leave the bank in 30 days). Run-off rates for deposits range from 5% for the most stable, fully insured deposits to 15% for less stable deposit funding. Our calibration of $\omega^{\max}$ at 0.1 is consistent with these rates.

[27] Average maturity is computed as a weighted average of all maturities of euro area government bonds, with weights given by outstanding amounts in 2011. Source: Bloomberg, ECB and authors' calculations. Bond-level data used in Andrade et al. (2016) give a similar average maturity in 2015, pointing to a stable maturity structure of euro area debt over time.

average foreign holdings of euro area government bonds over the periods 2013Q4-2014Q4 and 2015Q2 to 2015Q4. We compute the%age change in foreign holdings between the two periods to be -3.3%. We then calculate the%age change between the same periods in the average real return on euro area government bonds to be -38%. We then set ϱ to replicate the observed elasticity of foreign bond holdings with respect to changes in the real return on bonds, i.e., $\varrho = 1.76$, thus interpreting it as a demand elasticity for the purpose of our analysis. We check the robustness to alternative values (not reported) and find little impact on our quantitative analysis.

We parameterize central bank policies as follows. Under a CO policy, there are two key parameters to be set: the interest rate on central bank loans, $1/Q^F$, and the haircut on collateral charged by the central bank, $1 - \eta$. We set the former equal to 1.0025 and the latter equal to 0.03, corresponding to the haircuts charged by the ECB on high-quality government bonds. Under these conditions, banks do not take up central bank funding in the baseline steady state. Under the constant balance-sheet policy, we ensure that banks do not borrow from the central bank, by setting $\eta = 0$, and we hold the real value of central bank bond holdings b^C constant.

We treat the monetary union as composed of symmetric regions and assume that $Q^N = Q^S = Q$ and $B_N^C = B_S^C = B^C$. We calibrate the remaining six parameters to match the model-based predictions of some key variables to their empirical counterparts over the pre-crisis period 1999-2006 (see Table 3).[28] The parameters are the share of net worth distributed by banks as dividends, ϕ, the share of assets bankers can run away with, λ, the coefficient determining the utility from money holdings for households, χ, the expenditure on public goods, g^*, the real stock of government bonds purchased by the central bank, b^C, and the stock of real debt in the economy, $\bar{b}^*$. The targeted variables are: i) average ratio of government expenditure to GDP; ii) bank leverage; iii) government bond spread (annual); iv) share of banks' bond holdings in

[28]The average government expenditure to GDP ratio is computed using data for euro area (EU12) governments from Eurostat. The value of bank leverage is taken from Andrade et al. (2016). The share of banks' bond holdings in total debt is set at the value reported in Koijen et al. (2021) for 2015, 23%. To compute the share of the foreign sector's bond holdings, we first use data from the ECB's Statistical Data Warehouse to calculate the share the of central bank's holdings in total government debt. We include in this item not only outright purchases of government bonds but also collateralized loans extended in refinancing operations (the main instrument through which the ECB injects liquidity in normal times). The ratio to total sovereign debt is 10%. Koijen et al. (2021) report that households hold 3% of government bonds. We then impute to the foreign sector the remaining share, which amounts to 64%. The government bond spread is computed using data from SDW. We build average government bond yields by weighting the yields of all euro area government bonds, for all maturities, with the respective amounts in 2011. We then build the spread relative to the overnight rate, the EONIA. Average inflation is computed using quarterly changes in the HICP index taken from SDW.

total debt; v) share of foreign sector's bond holdings in total debt; and vi) average inflation (annual). As Table 3 reports, the model perfectly replicates the six targeted moments and provides a good fit for two non-targeted moments: the ratios of central bank bond holdings to GDP and government debt to GDP.

5.2 Shock processes

The dynamics of the model are driven by the dynamics of the share ξ_t of "connected" banks and the private market haircut, $1 - \widetilde{\eta}_t^{\gamma}$.

For the baseline pre-crisis steady state, we use data from the Euro Money Market Survey and sum up the turnover in the secured and in the unsecured segments (where 2003 is the first available observation in the survey and 2007 is the last year before the Global Financial Crisis). We set $\xi = 0.42$, corresponding to the 2003-2007 average share of cumulative quarterly total turnover executed in the secured market.

We set the haircuts on government bonds in private markets and at the central bank equal to each other, at $1 - \widetilde{\eta}^S = 1 - \widetilde{\eta}^N = 1 - \eta = 0.03$ (corresponding to a 3% haircut). The private haircuts are consistent with those observed in 2010, while the central bank haircuts are consistent with those imposed by the ECB on sovereign bonds with credit quality 1 and 2 (corresponding to a rating of AAA to A-) in 2010. This calibration ensures that, in the baseline, private banks do not take up any central bank funding in the steady state.

For the dynamic analysis in the model, we assume that ξ_t and $\tilde{\eta}_t$ are constant at their benchmark steady-state levels for $t < 1$. At $t = 1$, agents learn that ξ_t follows a new and deterministic path, but assume that $\tilde{\eta}_t^{\gamma}$ remains constant for $\gamma \in \{N, S\}$. [29] At $t = 13$, agents learn that $\tilde{\eta}_t^S$ also follows a new and deterministic path. We formulate these paths such that the model matches the observed evolution of the secured shares and private market haircuts, with $t = 1$ corresponding to the first quarter of 2009.

In our model, there is a tight relationship between ξ_t and the secured shares within the North.[30] Therefore, we consider a process for ξ_t that matches as closely as possible the data

[29] We treat the evolution of the share of unconnected banks, ξ_t as exogenous. One may think that the observed decline in unsecured money market lending reflects the introduction of ECB policies that injected large amount of central bank reserves, and that those reserves made money market lending redundant. Two facts documented in figure 1 support our view that the observed developments in money markets were exogenous to monetary policy. First, the trend decline in unsecured funding in both the North and South of Europe preceded the GFC and the ECB policies that increased reserves. Second, money markets did not become redundant; rather, the decline in unsecured funding induced a corresponding increase in secured interbank lending.

[30] This is for two reasons: 1) no banks in the North take up central bank funding, and 2) connected and unconnected banks issue very similar amounts of deposits.

on secured shares in the North. We assume that $\log \xi_t$ converges to the new steady state ξ_∞ at a constant rate,

$$\log\left(\xi_t\right) = \left(1-\rho_\xi\right)\log\left(\xi_\infty\right) + \rho_\xi \log\left(\xi_{t-1}\right) \tag{38}$$

where $\xi_{-1} = 0.42$ and $\xi_\infty = 0.1$. We choose ρ_ξ to minimize the squared distance between the model and the data, and find a value of $\rho_\xi = 0.95$. The left panel of Figure 5 compares the process (38) used in the model to the data evolution of the unsecured share in interbank markets in Northern European regions, also shown in Figure 1, Panel B.

We assume that $\tilde{\eta}_t^N$ remains constant for all $t \geq 1$ and that $\tilde{\eta}_t^S$ remains constant for $t \leq 12$. From $t = 13$ onward, we assume that $\log\left(\tilde{\eta}_t^S\right)$ follows an AR(2) process in order to replicate the hump-shaped process observed in the data,

$$\log\left(\tilde{\eta}_t^S\right) = \left(1-\rho_{\tilde{\eta},1}-\rho_{\tilde{\eta},2}\right)\log\left(\tilde{\eta}_\infty\right) + \rho_{\tilde{\eta},1}\log\left(\tilde{\eta}_{t-1}^S\right) + \rho_{\tilde{\eta},2}\log\left(\tilde{\eta}_{t-2}^S\right) + \epsilon_{\tilde{\eta}_t^S}, \tag{39}$$

where the shocks are zero, $\epsilon_{\tilde{\eta}_t^S} = 0$, except for $t = 13$. We search for $\rho_{\tilde{\eta},1}$, $\rho_{\tilde{\eta},2}$ as well as the shock $\epsilon_{\tilde{\eta}^S,13}$ in order to fit the data closely, and pick

$$\rho_{\tilde{\eta},1} = 1.65; \rho_{\tilde{\eta},2} = -0.7; \epsilon_{\tilde{\eta},13} = -0.11$$

The right panel of Figure 5 compares the AR(2) of equation (39) to the data, represented as a splined interpolation through the available data points.

6 NUMERICAL ANALYSIS

Our goal is to quantitatively understand and analyze the evolution of the banking system during the euro area crisis and its impact on the economy, resulting from the increased share of collateralized interbank lending and private-sector haircuts on government bonds in the South.

In order to build up an intuition for the mechanisms that will play an important role in the dynamics, we first run a comparative statics exercise where we permanently change the share of unconnected banks in the monetary union $(1-\xi)$ or the haircut on bonds issued in the South $(1-\tilde{\eta}^S)$.

The comparative statics exercise highlights the complex interactions between various occasionally binding constraints of the banks described and analyzed in Section 4, which is a novel feature of our model. As we will see below, some constraints turn from slack to binding, and vice versa, as parameters are changed. Therefore, the subsequent numerical calculation of the dynamics needs to allow for such changes over time as well, requiring a non-linear solution method.

6.1 Comparative statics

Changing share of unconnected banks. We consider first the macroeconomic impact of equally shrinking the unsecured money market segment in both regions, under a CO policy. We do so by permanently shifting the steady-state share of unconnected banks, $1-\xi$, between 0.58 and 0.9. Figure 6 shows the results.

The solid red line denotes the share of unconnected banks under our benchmark calibration ($1-\xi = 0.58$). The dashed orange lines indicate the level of $1-\xi$ at which unconnected banks start holding money so that the multiplier μ_u^M becomes zero.

In the calibrated steady state (at the solid red line), the collateral premium on bonds is positive and the afternoon constraint binds for unconnected banks. Indeed, as we prove in Online Appendix E, the collateral premium on bonds is always positive as long as the afternoon constraint binds. The amount of deposits raised by connected and unconnected banks is of a broadly comparable magnitude. Unconnected banks, however, invest less in capital than connected banks, as they need to invest part of the funds in bonds to be pledged in the secured market in the afternoon. At this point, the return on bonds is higher than the return on money (not shown), and unconnected banks choose not to hold money to satisfy their afternoon liquidity needs.

If more banks in the economy are unconnected (moving rightward on the x-axis), a larger number of banks face an afternoon liquidity management constraint, which raises the aggregate demand for bonds and the bond price. In the region where $1-\xi < 0.61$, the amount of bonds held by each unconnected bank, b_u, declines, as more banks need to hold bonds as collateral, and the supply of bonds is fixed. When the share of unconnected banks increases further, i.e., when $1-\xi$ exceeds 0.61, the high price of bonds lowers the return on bonds to the point where it is equalized with the return on money. From this point onward (indicated by the dashed orange lines), unconnected banks also use money to self-insure against afternoon withdrawals.

Although the central bank stands ready to provide collateralized loans, central bank funding is not used because deposit funding is less expensive. Higher demand for money by unconnected banks is accommodated by an increase in the deposit rate, which induces households to reduce their money holdings. Scarce money balances are therefore reallocated from households to unconnected banks. Inflation also rises,[31] which increases the opportunity cost of holding money for unconnected banks and further tightens their afternoon constraint. Unconnected banks respond by reducing their deposit intake and, therefore, investment in capital, exerting upward pressure on the return on capital. As the net worth of unconnected banks declines, and there is an increasing share of those, the aggregate net worth - which is equally distributed to all banks in the morning - declines. This results in a tightening of the run-away constraint of connected banks, which induces them to also reduce their investment in capital and their deposit intake. Therefore, aggregate deposits and capital fall and so does output. The overall decline in output between a steady state with 0.58 share of unconnected banks and that with 0.89 share (pre- to post-2008 average share of secured turnover in total) is around 1.8%.

In sum, reduced access to the unsecured market can reduce investment and output via two channels. First, since unconnected banks need to satisfy liquidity shocks by holding bonds and/or by holding money, it crowds out capital investment. Second, as more banks become unconnected, bonds and money become more scarce, tightening the banks' afternoon constraint. As a consequence, banks raise fewer deposits, invest less in capital, and aggregate output falls.

Changing haircuts on Southern bonds. Next, we analyze the macroeconomic effects of changing collateral value by permanently increasing the private haircut in the South from the benchmark pre-crisis value of 3%, under a CO policy. The results are shown in Figure 7. The solid red line denotes the secured market haircut under our benchmark calibration ($1 - \widetilde{\eta}^S = 0.03$). The dashed orange lines indicate the level of $1 - \widetilde{\eta}^S$ at which unconnected banks in the South start holding money so that the multiplier μ_u^M becomes zero. The dashed purple lines indicate the level of $1-\widetilde{\eta}^S$ at which unconnected banks in the South start borrowing from the central bank so that the multiplier μ_u^F becomes zero. The dashed green lines indicate the level of $1 - \widetilde{\eta}^S$ at which unconnected banks in the South pledge their entire bond holdings at the central bank and no longer use the secured market.

In the calibrated steady state (at the solid red line), the collateral premium on bonds is positive and the afternoon constraint binds for unconnected banks. At higher haircut levels

[31] This is an artefact of our steady-state analysis in which the Fisher equation holds. An alternative way to think about the adjustment in response to a higher demand for real money balances when the nominal money supply is fixed is that the price level must decrease so that the real money supply increases. That is, increased demand for scarce money balances necessitates deflation. In our setting, however, real money balances remain constant across steady states by assumption, and so demand must be weakened to clear the market, hence the rise in inflation.

(moving rightward in the figure), it becomes more difficult for unconnected banks in the South to satisfy their liquidity needs in the secured market because the pledgeable collateral supply has fallen. This drives up the price for bond collateral, the collateral premium, in the South. When haircuts reach 0.06, it becomes optimal for unconnected banks in the South to start holding money as collateral, which helps to ease the shortage. As shown in Figure 7, as haircuts rise and the collateral shortage intensifies, money holdings rise. Moreover, under CO policy, banks can access central bank funding as an alternative to deposit funding. The advantage of this source of funding is that haircuts are more favorable, but at the expense of having to use more collateral.[32] When the private haircut rises above 0.27, it becomes advantageous for unconnected banks in the South to substitute deposits for central bank funding.

As the central bank provides funding to banks, its balance sheet expands and so does the money supply. Therefore, unconnected banks in the South can further increase their money holdings, without the need for a reallocation of money from households. As there is no need for an increase in the deposit rate, the increase in the collateral premium is very contained, and in fact falls because of the money injection. When the private haircut increases above 0.32 (indicated by the dashed green lines), unconnected banks in the South pledge all their bond collateral at the central bank and stop using the secured market to manage their afternoon liquidity needs, relying solely on money holdings instead. From this point onward, the economy is insulated from further increases in the secured market haircut.

The impact on the real economy is very limited: output falls by 0.15% when private haircuts, $1-\tilde{\eta}^S$, rise to 0.4. Crucially, this is because there are minimal spillover effects on unconnected banks in the North and on connected banks. In fact, they mainly counteract the reduction in capital holdings of unconnected banks in the South and take advantage of capital risk premia. Unconnected banks in the North have their own collateral holdings whose haircuts $((1-\tilde{\eta}^N))$ remain unchanged. This segmented collateral market ensures that they don't face the same collateral shortage as the South. Indeed, as seen clearly in Figure 7, collateral premia in the North are lower than in the South as $1-\tilde{\eta}^S$ rises. As a result, unconnected banks in

[32] Consider the pledgeable value of collateral post-haircut at the central bank and in the private market, respectively. To access 1 unit of central bank funding, a bank needs to post 1 pledgeable unit of collateral, whereas covering deposit funding requires the bank to hold only ω^{max} pledgeable units of collateral, in accordance with the afternoon constraint.

the North remain in a strong position to increase their balance sheets and keep the effects on the real economy muted.[33]

6.2 Dynamic analysis

We now turn to the numerical calculation of the dynamics resulting from a progressive but permanent decline in the share of transactions in the unsecured vs the secured market segment and a temporary but persistent increase in haircuts set on sovereign bonds issued by some EU member states. For the monetary policy benchmark, we assume that the central bank follows a policy of collateralized lending to banks (CO policy). We then compare the outcomes to a counterfactual scenario of the central bank maintaining a constant balance sheet (CB policy). We solve the model non-linearly under perfect foresight.[34]

We assume that the model is at its benchmark pre-crisis steady state for $t < 1$. At $t = 1$ and corresponding to the first quarter of 2009, agents learn that ξ_t follows a new path, while at $t = 13$, they learn that $\tilde{\eta}_t^S$ follows a new path described in Subsection 5.2 and shown in the top quadrants of Figure 8. The scenario in blue is one where only the unanticipated shock to the unconnected share, $(1 - \xi)$, is realized at time $t = 1$. In this case, at $t = 1$, agents understand fully and with certainty that the share of connected banks, ξ, will transition to a permanently lower level in accordance with the top right quadrant of Figure 8. The scenario in red is where we also have the unanticipated persistent shock to $\tilde{\eta}^S$, realized at time $t = 13$. Agents in the model, from $t = 13$ onward, understand that, with certainty, $\tilde{\eta}^S$ will evolve according to the top left quadrant.[35] Therefore, the difference between the red and blue lines conveys the effect caused by the $\tilde{\eta}^S$ shock. Note the key difference in the characteristics of the two shocks. The shock to ξ represents a gradual transition to a permanently lower level, whereas the $\tilde{\eta}^S$ shock is a persistent but non-permanent change in haircuts.

[33] This conclusion would be very different if collateral markets were not segmented. This does not require us to relax the assumption of segmented bond markets. Suppose another type of collateral, in this case money, is held by banks in the North and the South. If collateral in the South experiences higher haircuts, banks in the South experience a scarcity of collateral. This drives up the price for collateral that banks in the South are willing to pay. In equilibrium, this drives up the collateral premium for money, which is the price also faced by banks in the North. In other words, there is a perfect spillover of collateral premia to the North, causing banks to also divest. This significantly increases the negative effects on output.

[34] We use Dynare to calculate the numerical solution. We chose the Levenberg-Marquardt mixed complementarity problem solver. This solver is more suited to models with occasionally binding constraints than the more commonly used Newton algorithm. The path to final equilibrium is imposed in finite time T, but we set the end point large enough (T = 400) to ensure that the economy converges to the steady state.

[35] It arrives at time $t = 13$ to reflect the fact that, in the data, the increase in haircuts experienced by sovereign bonds in the South occurred approximately three years after the beginning of the notable rise in the secured share.

The bottom two quadrants of Figure 8 tell us the implied evolutions in (i) secured shares and (ii) the take-up of central bank funding as a percent of assets held by unconnected banks in the South, assuming that the central bank follows the benchmark policy of collateralized lending to banks.[36] Item (i) connects with Observation 2, while (ii) connects with Observation 3. Overall, the dynamic model does well in replicating the empirical observations we documented: a rise in secured shares and a take-up of central bank funding at the peak of private haircuts.

Figure 9 conveys the impulse responses of key variables of interest in our two scenarios and under the benchmark CO policy: in blue when just the shock to ξ is realized, and in red when we also have the shock to $\tilde{\eta}^S$ realized at time $t = 13$. The scenario in red is consistent with the empirical observations documented in Section 2. The vertical orange lines represent the time at which the $\tilde{\eta}^S$ shock is realized. The intersection with the y-axis denotes the point where the economy is in time $t = 0$, either in levels or as a percent change from the baseline steady state.

As the shift in ξ is permanent, the economy ultimately transitions to a new steady state, regardless of whether or not the $\tilde{\eta}^S$ shock occurs. The new steady state is represented by the horizontal dashed lines, which indicate the new long-run level (or percent change from the baseline steady state).

6.2.1 Impact of changes in ξ only

We can gain the main intuition for the impulse responses by understanding the shift in the steady state induced by the long-run shift in the unconnected share, $1 - \xi$. The key outcome across steady states in the model is the decline in the capital stock, falling 10% on aggregate. This is due to the "crowding-out" effect on capital, for two reasons. First, as $(1 - \xi)$ rises, connected banks get replaced by unconnected banks. In the baseline steady state, their balance sheets are very similar in size but differ crucially in the composition of assets held: unconnected banks need to hold collateral to satisfy their afternoon constraint. This mechanically crowds out capital investment on aggregate.

Second, each individual bank invests less in capital as a result of lower deposits. This is especially the case for unconnected banks. As the number of unconnected banks increases, the demand for collateral also increases to satisfy the afternoon constraint. This drives up the

[36] See Online Appendix G for details on how we evaluate the secured share. We assume that unconnected banks cover deposit withdrawals with reserves before accessing the secured market.

collateral premium banks pay to hold bonds. When this premium is high enough, unconnected banks prefer bringing money over to the afternoon, which they can hold because an increase in the deposit rate induces households to hold less of it.[37] The higher cost of deposits, however, leads unconnected banks to downsize their balance sheets and reduce capital investment funded by deposits. The increase in the cost of deposits spills over also to connected banks in the form of a higher cost of investing in capital and reduced net worth.

On aggregate, the capital stock declines at the new steady state, and investment declines by the same proportion. This depresses output as a consequence. Moreover, there are no stabilizing interventions undertaken by the central bank in equilibrium. As we know from the steady-state analysis, no CB funding is taken up by banks when ξ changes across steady states. As private haircuts, $1-\tilde{\eta}^S$ remained unchanged, and there is no advantage in tapping more collateral-intensive central bank funding as there is no difference between central bank and private market haircuts.

However, the transition toward the steady state is far from smooth. Indeed, output and investment initially undershoot the value they reach in the new steady state, despite the share of connected banks being still far above the new steady-state level. The coming gradual decline of ξ - and thereby the gradual need to forgo returns in favor of holding bonds or money for liquidity reasons - lowers the value of operating a bank. This happens immediately, resulting in an immediate, endogenous reduction of leverage and assets held. Moreover, as banks understand that capital investment will be crowded out in the future, the current price of capital falls. This drives down net worth and, combined with lower leverage, leads to persistently lower capital investment. Investment then gradually recovers to the new lower steady-state level as the net worth of banks is replenished.[38]

Finally, the bottom left quadrant of Figure 8 shows that a permanent fall in ξ produces a permanent rise in the secured shares. Intuitively, both bank types use only deposits as a liability source. The increasing share of unconnected banks creates a larger need to use the secured market to manage short-term deposit withdrawals. Therefore, there is a very tight relationship between ξ and the secured share. As we will see, this relationship does not

[37]The size of the central bank balance sheet remains constant but the composition of liabilities switches from the issuance of currency to households toward issuance of reserves to banks.

[38]Note that connected banks partially counteract the decline in investment on impact as they do not suffer from higher collateral premia. However, this is not enough to prevent aggregate investment from falling.

necessarily hold if unconnected banks had an incentive to use an alternative liability source, such as central bank funding.

6.2.2 Impact of changes in ξ and $\tilde{\eta}$

We now investigate what happens if the economy additionally experiences the unanticipated persistent drop in $\tilde{\eta}^S$ at time $t = 13$, comparing the red impulse responses in Figure 9 to the ξ-shock-only responses in blue.

Bonds available to unconnected banks in the South are now worse collateral. This drives up collateral prices and makes deposit issuance more expensive, inducing banks to downsize their liabilities. Moreover, Q^S falls because these bonds are now less effective as collateral, driving net worth lower. As a result, capital investment by unconnected banks in the South falls.

However, unconnected banks in the South rebalance their portfolios to mitigate the effects of collateral scarcity. First, they increase the money they bring over to the afternoon. Second, at the peak of the rise in private haircuts, banks substitute expensive deposits with central bank funding because of the preferential haircuts central banks charge on bonds relative to the private market. Indeed, this take-up is significant, equalling 2.5% of total bank assets, as seen in the bottom right panel of Figure 8. This is much in accordance with Observation 2, both in direction and in magnitude. These two effects mitigate the collateral scarcity faced by unconnected banks in the South, helping them downsize investment by less. Although unconnected banks rebalance toward central bank funding in the morning, they continue tapping into the private secured market to manage their afternoon liquidity needs, albeit to a lesser extent than before the shock. This is in line with Figure 1, Panel A, which shows continued use of the private secured market in the South throughout the crisis.

Turning to the aggregate effect, output and investment fall because of aggregate disinvestment, mainly driven by unconnected banks in the South. This is partially counteracted by connected banks' rise in value as capital returns increase. This endogenously allows them to increase their leverage, thus both investing more in capital and raising more deposits.

Quantitatively, however, a key determinant of the negative effect is the response of unconnected banks in the North. They do not face any shock to $\tilde{\eta}^N$ but still get impacted indirectly via higher collateral premia in the economy. Intuitively, unconnected banks in the South experience a collapse in the pledgeable value of their collateral. As a result, they move toward holding money to satisfy the afternoon constraint, thereby driving up its cost. But crucially, unconnected banks in the North already hold increased amounts of money as collateral following the ξ shock. Hence, they also pay a higher price for this collateral. This tightens the afternoon constraint for them as well. Therefore, despite the increase in returns to capital and despite not being directly exposed to the private haircut shocks, they hardly change their capital investment. In other words, unconnected banks in the North face a pecuniary externality through higher collateral premia because of the shock to collateral quality for those in the South.

This is in stark contrast to the steady-state analysis in Section 6.1, where the spillover effects of higher haircuts, $1 - \tilde{\eta}^S$, were small. Importantly, this is because the collateral markets were entirely segmented in the baseline steady state: unconnected banks in the North only used bonds issued in the North as collateral. But in this experiment, as all unconnected banks endogenously turn to money as collateral, collateral markets endogenously become integrated. As a result, all banks now face the same collateral premium.

As the shock subsides and the haircuts revert back toward the pre-shock level, investment recovers and, in fact, overshoots the path absent the $\tilde{\eta}^S$ shock. The intuition is as follows. As the haircuts recover somewhat and returns to capital remain elevated, the bank now has access to favorable investment opportunities while at the same time being less constrained by collateral needs. As a result, banks can increase leverage, which causes the overshooting effect to bring capital back to its new steady state.

6.2.3 Comparison of impulse responses: CO policy vs CB policy

We now compare the dynamics under the benchmark policy of collateralized lending to banks (CO policy) to a counterfactual scenario of the central bank maintaining a constant balance sheet (CB policy). In the ξ-only-shock scenario, there is no difference: since central bank haircuts are the same as private-sector haircuts under both policies, banks strictly prefer not to borrow from the central bank. Therefore, Figure 10 compares the dynamics resulting from the CO policy in blue to the dynamics resulting from CB policy, subtracting out the dynamics resulting from the ξ-only-shock scenario. Put differently, Figure 10 shows the additional dynamics under these two policies, arising from the additional $\tilde{\eta}$-shock.

With Figure 9, we have shown that a shock to $\tilde{\eta}^S$ has an adverse effect on output and investment and that the capital stock declines under the benchmark CO policy. As Figure 10 demonstrates, the impact on the real economy is stronger under the CB policy. In particular, output and investment fall around twice as much on impact. The key difference here is of course the access to central bank funding under the benchmark CO policy. For unconnected banks in the South and when $\tilde{\eta}^S$ falls, the afternoon constraint is tighter and collateral premia rise. If this shock is large enough, unconnected banks divert their liability sources from expensive deposits (as the afternoon constraint is tighter) to central bank funding under that policy. This mitigates collateral scarcity as fewer deposits need to be covered with collateral, thereby dampening the rise in the collateral premium.

By mitigating the rise in the collateral premium, the CO policy especially benefits unconnected banks in the North. Looking at the bottom left, capital investment $k_{U,N}$ rises under the CO policy, while it falls under the CB policy. If unconnected banks in the South access central bank funding, banks in the North also benefit indirectly by having to pay less for collateral in the form of reserves. The spillover from South to North and to the unconnected banks in the North, therefore, plays a critical role in the overall dynamics, even though the latter are not directly affected by the rise of the private-sector haircuts in the South. The larger fall on impact in investment and output under the alternative CB policy increases returns to investing in capital and thereby raises the value of operating a bank with some delay. The resulting endogenous rise in leverage then leads to a larger recovery under the alternative CB policy than the benchmark CO policy.

7 CONCLUSIONS

We have documented four observations concerning the euro area developments around the financial and sovereign debt crises. The share of secured interbank lending increased throughout. Private-sector haircuts as well as borrowing from the central bank increased in the South. Depositor funding remained stable. In order to understand these developments and the role played by central bank policies, we have developed a general equilibrium model featuring banks in two regions and interbank markets for both secured and unsecured credit, and a common central bank that can conduct open market operations as well as lend to banks against collateral. The model features a number of occasionally binding constraints. The interactions between these constraints are key in determining trade-offs in the bank's decision problem and macroeconomic outcomes. We characterize the bank's decision problem, weighing the various trade-offs.

We show how both secured and unsecured money market frictions force banks to either divert resources into unproductive but liquid assets (bonds or money) or to de-lever (raise fewer deposits as it is deposit funding that exposes banks to liquidity shocks). This leads to less lending and output in the economy. In our numerical analysis calibrated to the euro area developments, we demonstrate that the gradual decline in unsecured interbank lending leads to an immediate decline in bank lending. We show that the impact of the rise in private market haircuts is more dramatic when it interacts with changes in the proportion of unsecured interbank lending. We compare the impact of the benchmark central bank policy of lending against collateral at haircuts more favorable than those of the private market to an alternative central policy that does not allow such additional lending. We calculate that the initial fall in output would have been twice as large under the alternative policy than it was under the benchmark policy, but that the rebound would be stronger. We show how pecuniary externalities between the Southern banks and the Northern banks are crucial to understanding the comparative statics and dynamic impacts, even though the Northern banks are not directly affected by the rise of the private-sector haircuts in the South. There is a rich set of interactions, which we have highlighted and illuminated in our analysis.

The model, therefore, provides not only an explanation and examination of the four key observations we highlighted but also a rich laboratory in which to explore the interaction between liquidity management and leverage, interbank market frictions, and the role of central bank lending. This should prove useful in many applications beyond the particular episode examined here.

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TABLES

Table 1: Weighted average market haircuts in the "South," in %

2010	2011	2012	2013	2014	2015	2017	2019
4.03	6.77	14.49	25.16	25.16	15.30	11.48	11.08

Notes: The table presents the time series evolution of private market haircuts (here: LCH.Clearnet) on Southern government bonds. Region-level haircuts for Italy, Spain, and Portugal are obtained as simple averages across maturities. Region-group-level haircuts are constructed as weighted averages across the Southern regions, with weights given by the shares of the respective banking sector assets in total in 2010 (49%, 44%, and 7% for IT, ES, and PT, respectively). Source: Data on banking sector assets are from the ECB, while data on haircuts are from LCH.Clearnet (snapshots available as of 2010).

Table 2: Parameter values

Parameter	Description	Value
θ	Capital share in income	0.330
δ	Capital depreciation rate	0.020
β	Discount rate households	0.994
ϵ	Inverse Frisch elasticity	0.400
χ^{-1}	Coefficient in households' utility	0.006
g	Government spending	0.566
κ^{-1}	Average maturity bonds (years)	5.952
ϕ	Fraction net worth paid as dividends	0.025
ξ	Fraction banks with access to unsecured market	0.420
$\widetilde{\eta}$	Haircut on bonds set by banks	0.970
η	Haircut on bonds set by central bank	0.970
λ	Share of assets bankers can run away with	0.701
$\omega^{\max}$	Max possible liquidity demand as share of deposits	0.100
$\varkappa$	Intercept foreign demand function	10.120
B_C	Bonds held by central bank	0.968
B^*	Stock of debt	7.443
ϱ	Parameter foreign bond demand	1.757
Q^F	Price central bank loans	0.997

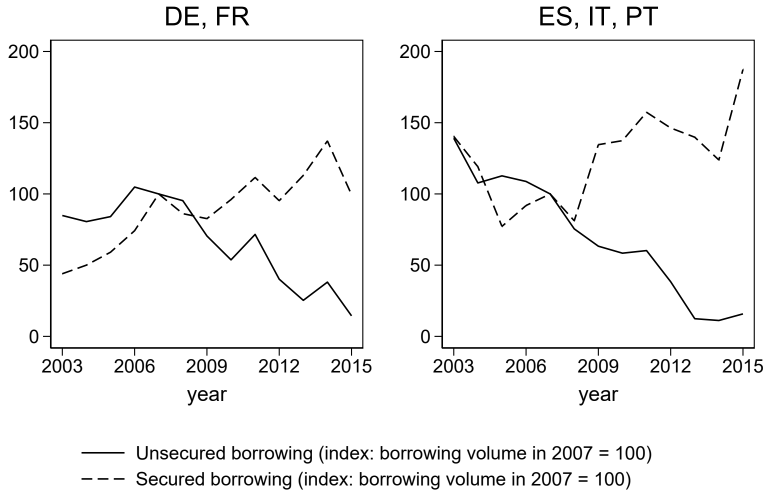
Table 3: Calibration

Targeted variables	**Data**	**Model**
Govt expenditure/GDP	0.20	0.20
Bank leverage	6.00	6.00
Govt bond spread (annual)	0.002	0.002
Share bonds held by banks	0.23	0.23
Share bonds foreign sector	0.64	0.64
Inflation (annual)	0.02	0.02
Non-targeted variables	**Data**	**Model**
CB bond holdings/GDP	0.06	0.08
Govt debt/GDP	0.69	0.66

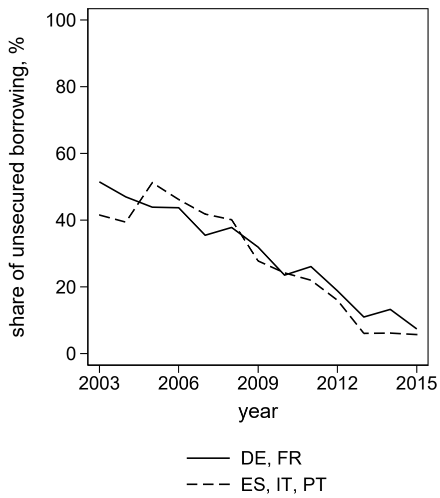
FIGURES

Figure 1: Unsecured and secured interbank borrowing

Panel A

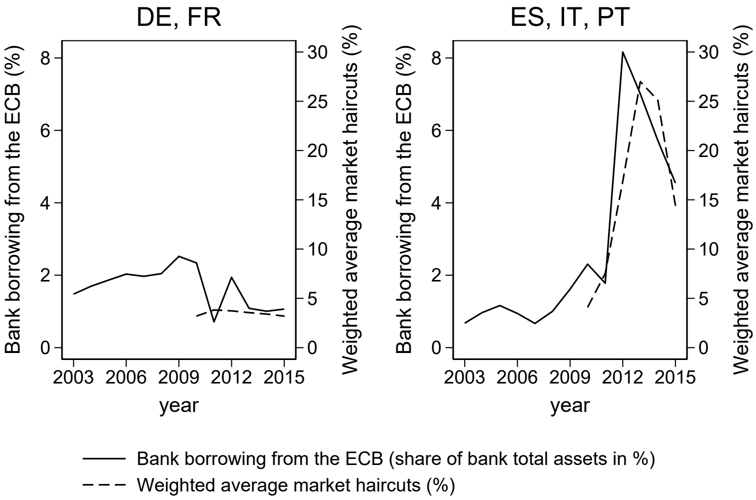


Panel B



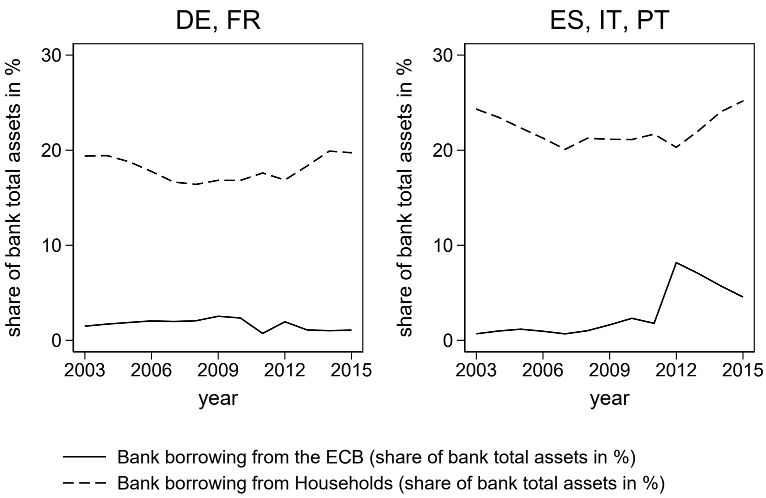
Notes: Panel A presents cumulative quarterly turnover in the unsecured and secured interbank borrowing over 2003-2015 (the time span of the data collection), distinguishing between Northern regions (left-hand-side panel) and Southern regions (right-hand-side panel). Turnover is defined as the sum of all borrowing transactions over the second quarter of each year (the quarter in which the data were collected) reported by banks participating in the ECB's Money Market Survey (MMS) and normalized to 100 in 2007. The MMS panel comprised 98 euro area banks. Panel B presents the relative share of unsecured borrowing in total, distinguishing between Northern regions (solid line) and Southern regions (dashed line). Source: Euro Area Money Market Survey.

Figure 2: Bank borrowing from the ECB and market haircuts, % of total assets



Notes: The figure presents bank borrowing from the ECB as % of bank total assets (solid line, left scale in each figure) and weighted average market haircuts in % (dotted line, right scale in each figure) over 2003-2015. The left-hand-side panel displays the evolution for Northern regions, while the right-hand-side panel displays the evolution for Southern regions. Region-level haircuts are obtained as simple averages across maturities. Region-group-level haircuts are constructed as weighted averages across the Northern (Southern) regions, with weights given by the shares of the respective banking sector assets in total in 2010 (the weights are 48% and 52% for DE and FR, and 49%, 44% and 7% for IT, ES and PT, respectively). Source: ECB and LCH.Clearnet.

Figure 3: Bank borrowing from households and the ECB, % of total assets



Notes: The figure presents bank borrowing from the ECB (solid line) and from households (dotted line) as % of bank total assets over 2003-2015. The left-hand-side panel displays the evolution for Northern regions, while the right-hand-side panel displays the evolution for Southern regions. Source: ECB.

Figure 4: Timeline of the model

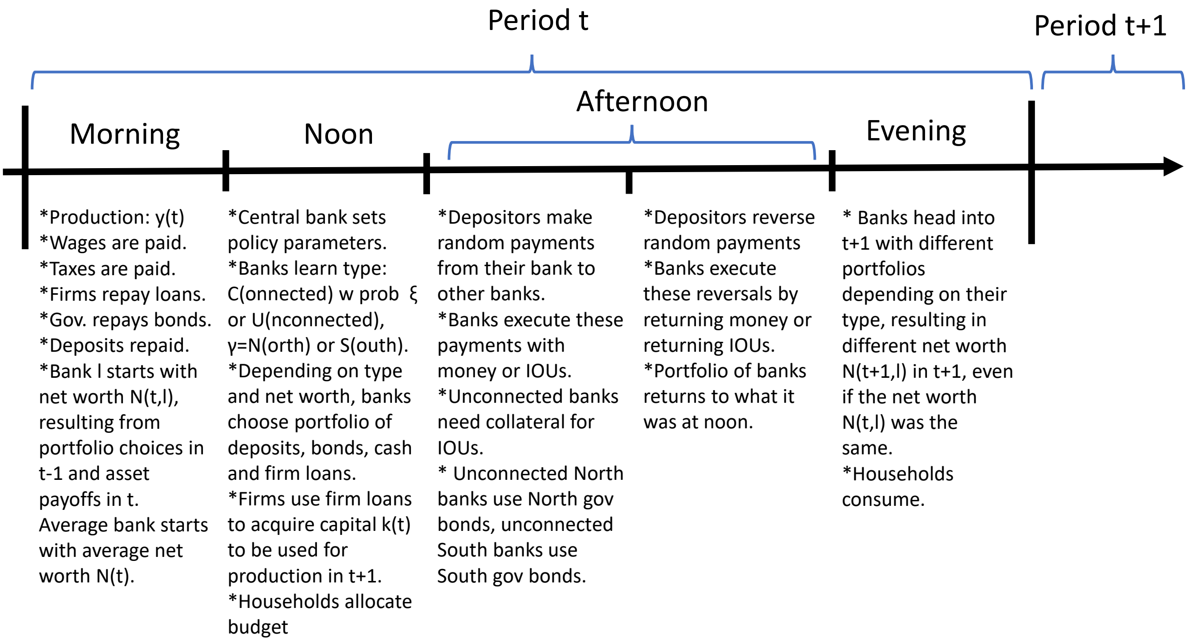
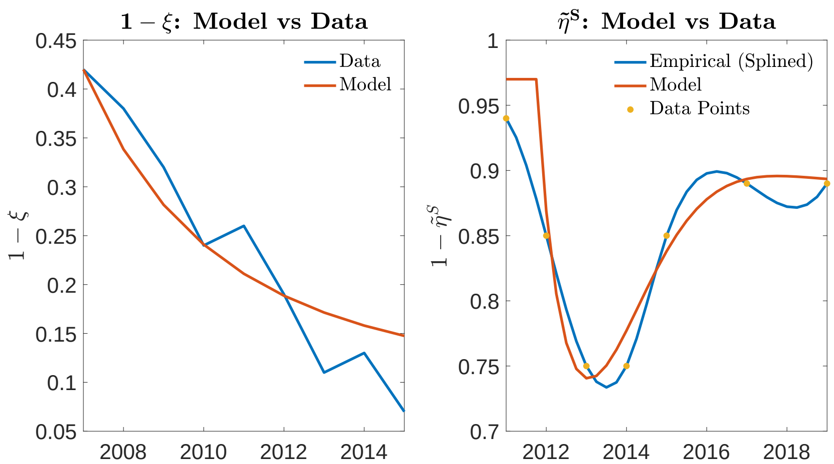
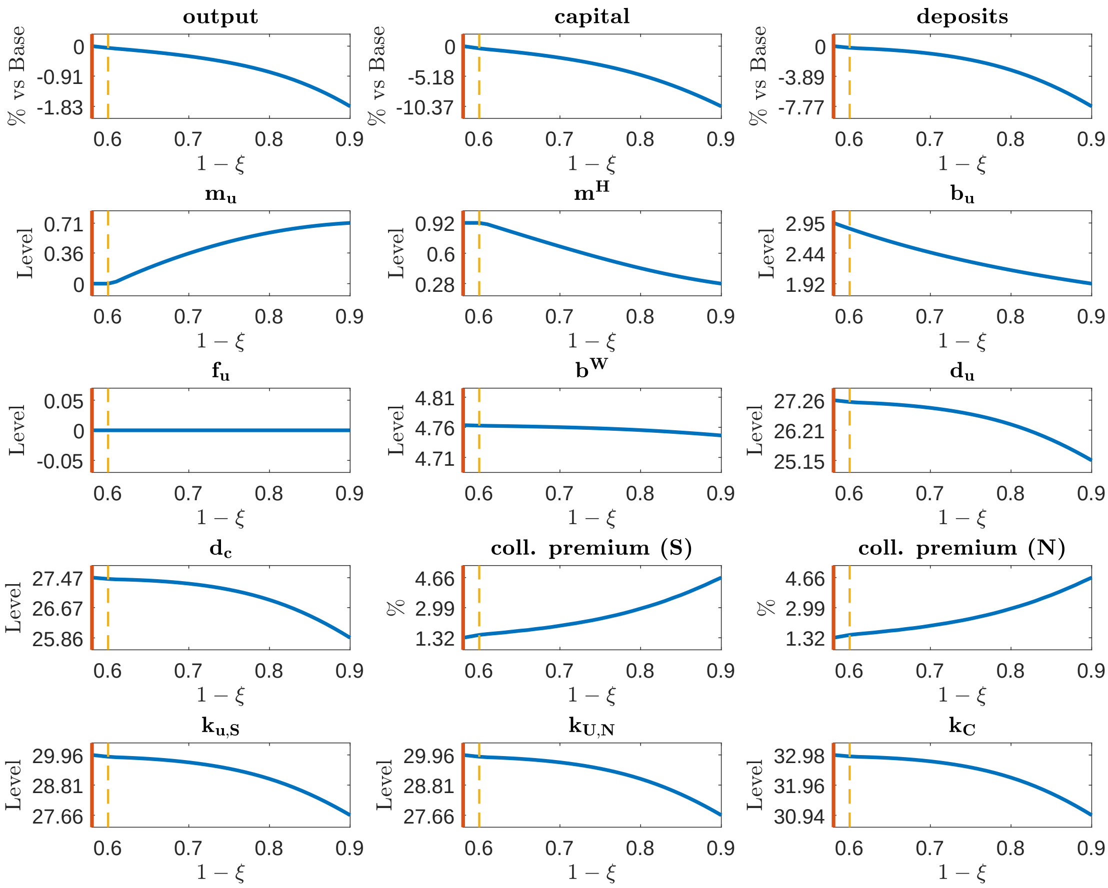


Figure 5: Evolution of $(1 - \xi)$ and $\tilde{\eta}^S$: Model vs. data



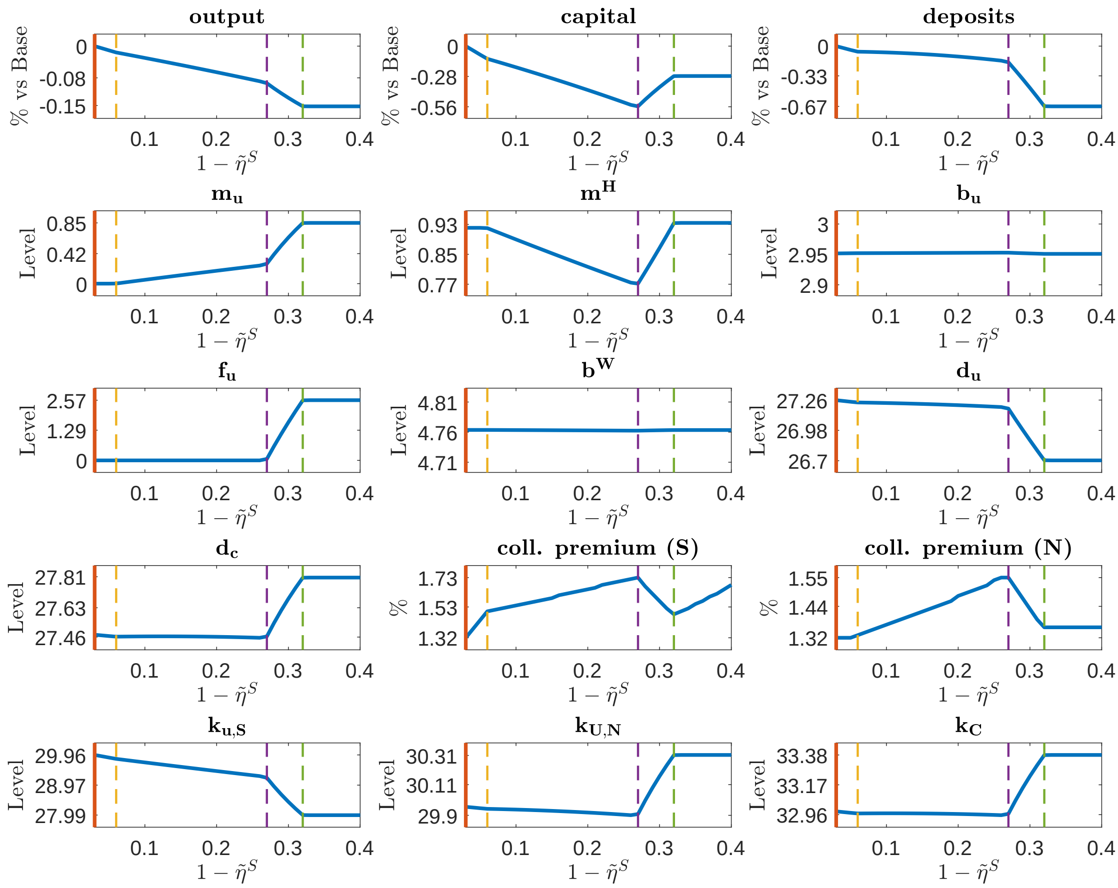
Notes: Conveys the evolution of a) the unsecured share in the secured market and b) the level of haircuts applied to government collateral in the South. The blue line is the data taken from Section 2, while the red line is the fitted model-implied evolution of these variables described in Section 5.2. The Data Points for $\tilde{\eta}^S$ come from Table 1, while the Model line is a spline interpolation through these points that we attempt to fit with an AR(2) process that is fit into the model.

Figure 6: steady state Comparative Statics: CO policy; Vary $1-\xi$



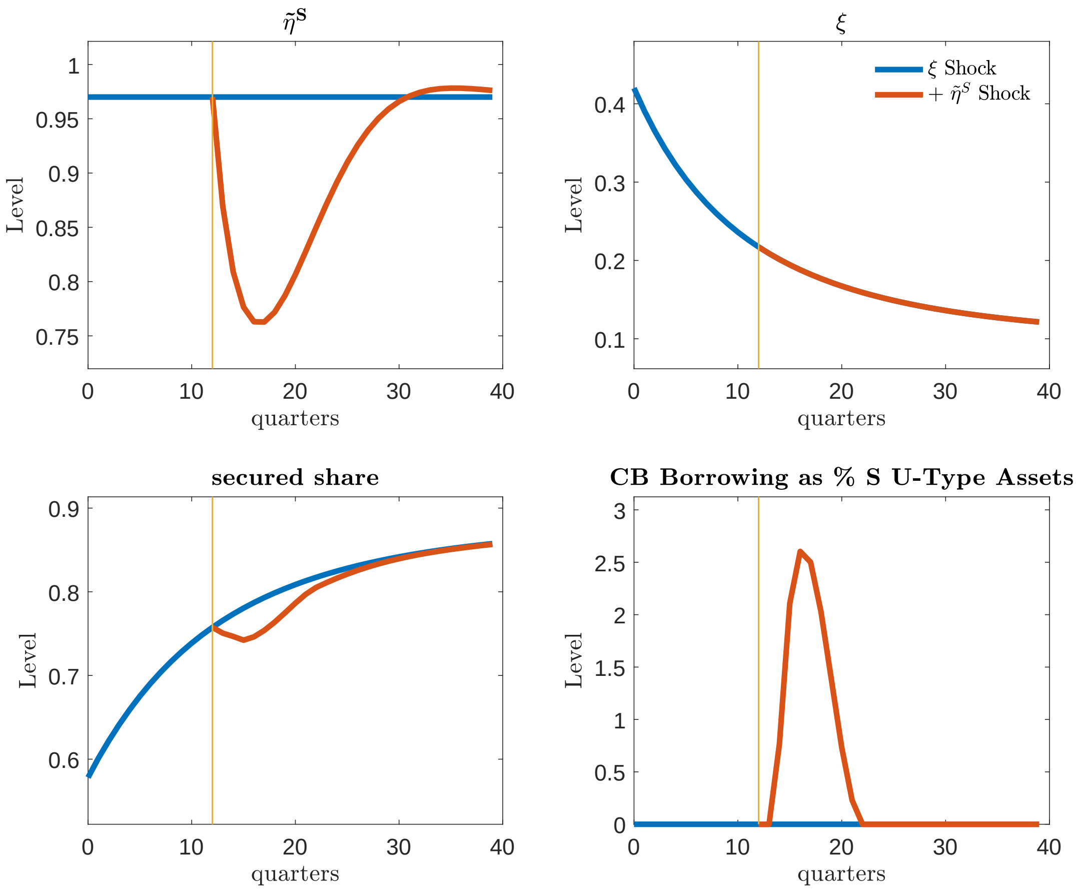
Notes: Red solid lines denote the calibrated steady state. Orange dashed lines denote the share of unconnected banks at which the non-negativity conditions on their cash holdings become slack. First row: output, capital, deposits. Second row: money holdings of unconnected banks, money holdings of households, and bonds held by unconnected banks. Third row: central bank funding taken up by unconnected banks, world bond demand, and deposits issued by unconnected banks. Fourth row: deposits issued by connected banks, and the collateral premium in the North and South (as defined in Section 4.3). Fifth row: capital held by unconnected banks in the South and the North, and connected banks. The y-axis label "% vs Base" means the % difference in the value of the variable relative to the baseline steady state.

Figure 7: Steady-state comparative statics: CO policy; Vary $1-\tilde{\eta}$



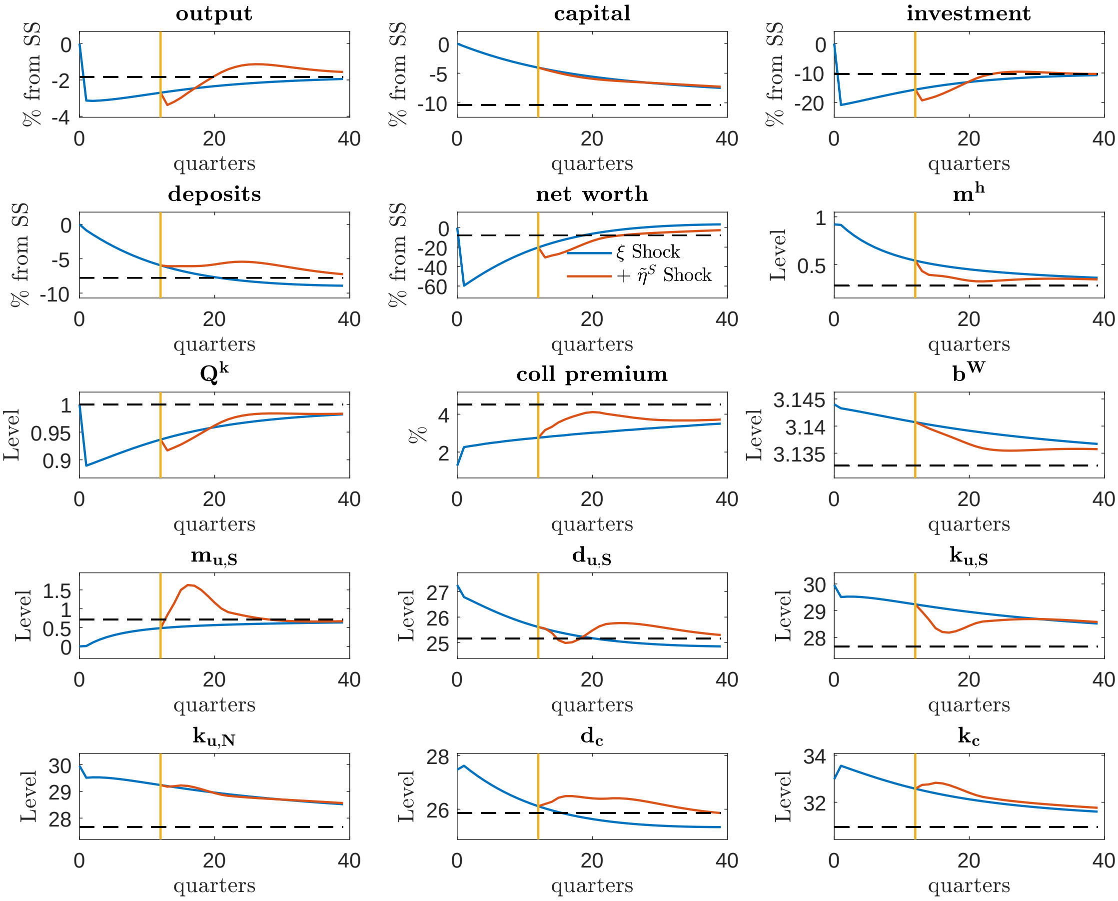
Notes: Solid red lines denote the calibrated steady state. Dashed orange lines denote the share of unconnected banks at which the non-negativity conditions on their cash holdings become slack. Dashed purple lines denote the share of unconnected banks at which banks draw funding from the central bank. The dashed green lines denote the share of unconnected banks at which unconnected banks no longer secure funding in the private market. First row: output, capital, deposits. Second row: money holdings of unconnected banks, money holdings of households, and bonds held by unconnected banks. Third row: central bank funding taken up by unconnected banks, world bond demand, and deposits issued by unconnected banks. Fourth row: deposits issued by connected banks, and the collateral premium in the North and South (as defined in Section 4.3). Fifth row: capital held by unconnected banks in the South and the North, and connected banks. The y-axis label "% vs Base" means the % difference in the value of the variable relative to the baseline steady state.

Figure 8: Impulse responses: Full experiment (ξ + $\tilde{\eta}$ shock) vs. ξ shock only



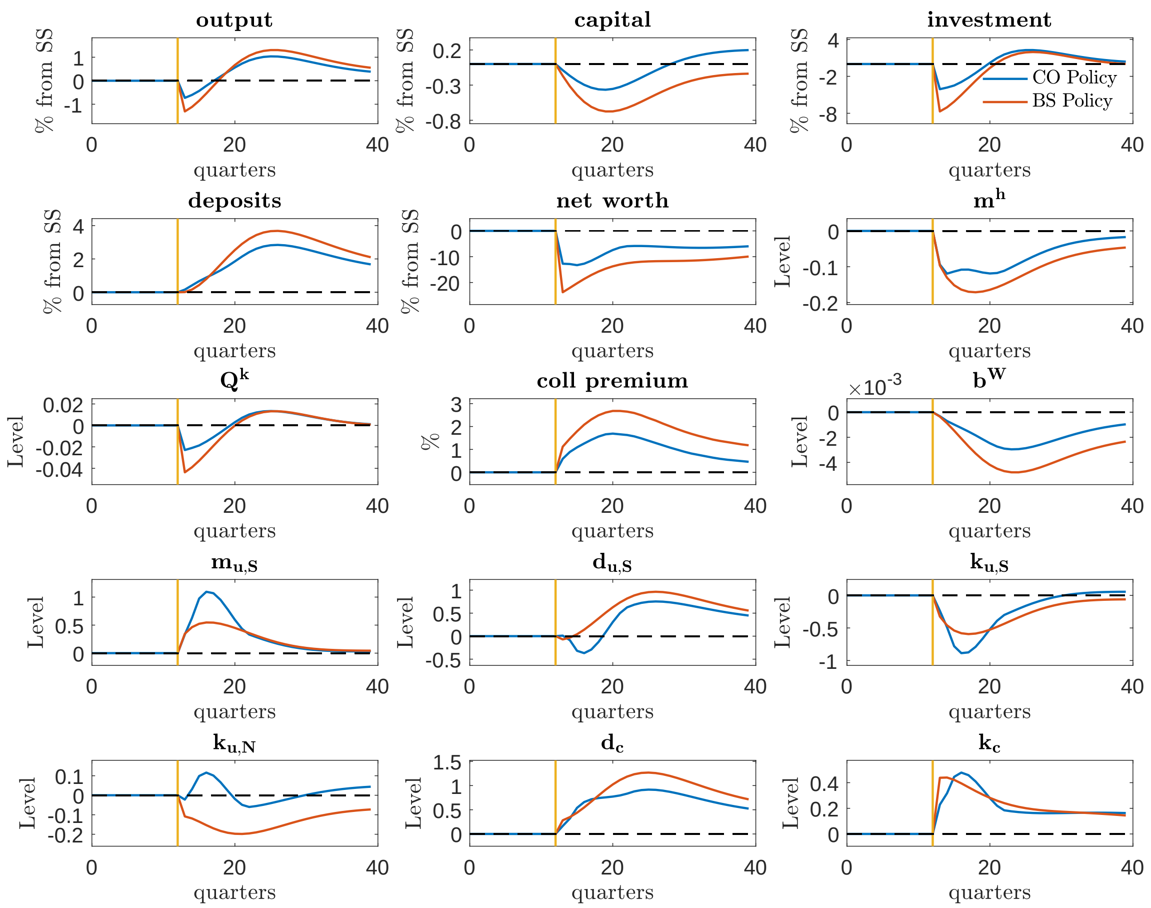
Notes: Conveys the impulse responses of key variables to exogenous shocks under CO Policy. The timing and process of the shocks in the full experiment are explained in Section 5.2. The solid blue lines denote the response to the permanent shock to ξ only that occurs at time 0. The solid red lines denote the response of variables when there is an additional shock to $\tilde{\eta}^S$ that occurs at time $t = 13$. The timing of the shock to $\tilde{\eta}^S$ is conveyed by the thin orange line. First row: process for $\tilde{\eta}^S$ and ξ. Second row: the secured share in interbank markets and borrowing from central bank as a % of total assets held by unconnected banks in the South.

Figure 9: Impulse responses: $\xi + \tilde{\eta}$ shock (red) vs. ξ shock only (blue)



Notes: Conveys the impulse responses of key variables to exogenous shocks under CO Policy. The timing and process of the shocks in the full experiment are explained in Section 5.2. The solid blue lines denote the response to the permanent shock to ξ only that occurs at time 0. The solid red lines denote the response of variables when there is an additional shock to $\tilde{\eta}^S$ that occurs at time $t = 13$. The timing of the shock to $\tilde{\eta}^S$ is conveyed by the thin orange line. The dashed black lines indicate the permanent % change / new level of the variable in response to the permanent shock to ξ. First row: % change in aggregate output, capital, and investment. Second row: % change in aggregate deposits, bank net worth and money held by households. Third row: price of capital, bond collateral premium (as defined in Section 4.3), and world bond demand. Fourth row: money, deposits and capital held by U-types in the South. Fifth row: capital held by U-types in the North, and deposits issued and capital held by connected banks.

Figure 10: Impulse responses: CO policy and constant balance-sheet policy



Notes: Conveys the response of variables in the full experiment in comparison to a counterfactual where only the ξ shock occurs. In other words, it illustrates the additional effect induced by the shock to $\tilde{\eta}^S$. The timing and process of the shocks in the full experiment are explained in Section 5.2. The solid blue lines denote the response under CO policy, while the red solid lines denote the response under a constant balance-sheet policy. The timing of the shock to $\tilde{\eta}^S$ is conveyed by the thin orange line at $t = 13$. First row: % change in aggregate output, capital and investment. Second row: % change in aggregate deposits, bank net worth and money held by households. Third row: price of capital, bond collateral premium (as defined in Section 4.3), and world bond demand. Fourth row: money, deposits and capital held by U-types in the South. Fifth row: capital held by U-types in the North, and deposits issued and capital held by connected banks.

ONLINE APPENDIX

A Optimality conditions of households and firms

We assume the following functional form of the households' utility function:

$$u\left(c_t, h_t\right)+v\left(\frac{M_t^h}{P_t}\right)=log(c_t)+\frac{1}{\chi}log(\frac{M_t^h}{P_t})-\frac{h_t^{1+\epsilon}}{1+\epsilon}.$$

The household maximizes his preferences, equation (2), subject to the budget constraints

$$D_t^h+M_t^h\leq R_{t-1}^d D_{t-1}^h+M_{t-1}^h+\left(1-\tau_t\right)W_t h_t+\phi N_t+\Pi_t^{CP}-P_t c_t \tag{A.1}$$

Note that there are further restrictions on the choice variables, i .e. $c_t>0, h_t>0$, $M_t^h>0$ and $D_t^h\geq 0$. We do not list these constraints separately for the following reasons. For $c_t>0$, $h_t>0$, and $M_t^h>0$, we can assure non-negativity with appropriate choice for preferences and per the imposition of Inada conditions. We constrain the analysis a priori to $D_t^h>0$, (see section 5 below) despite the possibility that it could be negative when allowing for more generality. [39]

We define a real variable as the corresponding nominal (capital letter) variable, divided by the contemporaneous price level, i.e. $x_t=\frac{X_t}{P_t}$, and

$$\pi_t=\frac{P_t}{P_{t-1}}$$

as the inflation from period $t-1$ to t.

For households, the first order conditions over bonds, labour and money are:

$$1 = \beta E_t\left[\frac{c_t}{c_{t+1}}\frac{R_t^d}{\pi_{t+1}}\right] \tag{A.2}$$

$$c_t h_t^{\zeta} = \left(1-\tau_t\right)w_t \tag{A.3}$$

$$\frac{c_t}{\chi m_t^h} = \frac{R_t^d-1}{R_t^d} \tag{A.4}$$

Final goods firms choose labor $h_t>0$, and capital $k_t>0$ to maximize their profits. The first order conditions over labor and capital are:

$$\frac{W_t}{P_t}h_t = \left(1-\theta\right)y_t \tag{A.5}$$

$$r_t k_t = \theta y_t \tag{A.6}$$

Finally, the capital-producing firm solves the problem in (4), resulting in the following price for capital:

[39] We have not analyzed this matter for the dynamic evolution of the economy. It may well be that net worth of banks temporarily exceeds the funding needed for financing the capital stock, and that therefore deposits ought to be negative, rather than positive. For now, the attention is on the steady state analysis, however, and on returns to capital exceeding the returns on deposits.

$$Q_t^k = \left(\frac{i_t}{\delta k_{t-1}}\right)^{\zeta} \tag{A.7}$$

Intuitively, if the price of capital Q_t^k is 1, investment replaces depreciated capital, exceeds replacement if below 1 and vice versa.

B The optimality conditions of the banks

We characterize the dynamic problem of an average connected bank and an average unconnected banks at region S and N. Banks are indexed by their type $\nu \in \{c, u\}$ and location $\gamma \in \{S, N\}$

A generic bank $\{\nu, \gamma\}$ chooses $k_{t,\nu,\gamma}$, $b_{t,\nu,\gamma}$, $b^F_{t,\nu,\gamma}$, $f_{t,\nu,\gamma}$, $d_{t,\nu,\gamma}$, $m_{t,\nu,\gamma}$ to maximize

$$v_{t,\nu,\gamma} = \phi n_t + E_t\left[\tilde{\psi}_{t+1} n_{t+1,\nu,\gamma}\right] \tag{B.1}$$

where $\tilde{\psi}_t = \beta \frac{c_{t-1}}{c_t}\psi_t$, subject to the evolution of net worth[40]

$$n_{t+1,\nu,\gamma} = R_{k,t+1} Q_t^k k_t + \frac{m_{t,\nu,\gamma}}{\pi_{t+1}} + \frac{R^{\gamma}_{b,t+1}}{\pi_{t+1}} Q_t^{\gamma} b_{t,\nu,\gamma} - \frac{R_t^d}{\pi_{t+1}} d_{t,\nu,\gamma} - \frac{R_t^F}{\pi_{t+1}} Q_t^F f_{t,\nu,\gamma} \tag{B.2}$$

and the constraints

$$\begin{array}{llrr}
\left(\mu^{BC}_{t,\nu,\gamma}\right): & d_{t,\nu,\gamma} + Q_t^F f_{t,\nu,\gamma} + n_t - \left(Q_t^k k_{t,\nu,\gamma} + Q_t^{\gamma} b_{t,\nu,\gamma} + m_{t,\nu,\gamma} + \phi n_t\right) & \geq 0 & \text{(B.3)} \\
\left(\mu^{RA}_{t,\nu,\gamma}\right): & v_{t,\nu,\gamma} - \lambda\left(Q_t^k k_{t,\nu,\gamma} + Q_t^{\gamma} b_{t,\nu,\gamma} + m_{t,\nu,\gamma}\right) & \geq 0 & \text{(B.4)} \\
\left(\mu^{F}_{t,\nu,\gamma}\right): & f_{t,\nu,\gamma} & \geq 0 & \text{(B.5)} \\
\left(\mu^{M}_{t,\nu,\gamma}\right): & m_{t,\nu,\gamma} & \geq 0 & \text{(B.6)} \\
\left(\mu^{C}_{t,\nu,\gamma}\right): & Q_t^{\gamma}\left(b_{t,\nu,\gamma} - b^F_{t,\nu,\gamma}\right) & \geq 0 & \text{(B.7)} \\
\left(\mu^{CC}_{t,\nu,\gamma}\right): & \eta_t Q_t^{\gamma} b^F_{t,\nu,\gamma} - f_{t,\nu,\gamma} & \geq 0 & \text{(B.8)}
\end{array}$$

where the variable included in brackets on the left of each constraint denotes the lagrangean multiplier associated to it.

Unconnected banks located in either region γ face in addition the afternoon constraint:

$$(\mu_{t,u,\gamma}): \quad m_{t,u,\gamma} + \tilde{\eta}_t^{\gamma} Q_t\left(b_t - b^F_{t,u,\gamma}\right) - \omega^{max} d_{t,u,\gamma} \quad \geq 0 \tag{B.9}$$

Note that we no longer feature the constraint

$$\left(\mu^B_{t,u,\gamma}\right): \quad Q_t^{\gamma} b_{t,u,\gamma} \quad \geq 0 \tag{B.10}$$

Equation (B.5) and (B.8) imply that $b^F_{t,u,\gamma} \geq 0$. But then, with (B.7), we also get $b_{t,u,\gamma} \geq 0$.

[40]Notice that the pricing kernel, $\tilde{\psi}_t$, is the same for all banks. This is because they value a unit of net worth the same way because all banks face the same distribution for their type at $t+1$.

The FOCs for unconnected banks where $\gamma \in \{S, N\}$ are:

$$\frac{\partial}{\partial k_{t,u,\gamma}}: \quad \left(1+\mu^{RA}_{t,u,\gamma}\right) E_t\left[\tilde{\psi}_{t+1} R_{k,t+1}\right] = \mu^{BC}_{t,u,\gamma} + \lambda \mu^{RA}_{t,u,\gamma} \tag{B.11}$$

$$\frac{1}{Q^{\gamma}_t}\frac{\partial}{\partial b_{t,u,\gamma}}: \quad \left(1+\mu^{RA}_{t,u,\gamma}\right) E_t\left[\tilde{\psi}_{t+1} R^{\gamma}_{b,t+1}\right] = \mu^{BC}_{t,u,\gamma} + \lambda \mu^{RA}_{t,u,\gamma} - \mu^{C}_{t,u,\gamma} - \tilde{\eta}^{\gamma}_t \mu_{t,u,\gamma} \tag{B.12}$$

$$\frac{\partial}{\partial m_{t,u,\gamma}}: \quad \left(1+\mu^{RA}_{t,u,\gamma}\right) E_t\left[\tilde{\psi}_{t+1}\right] = \mu^{BC}_{t,u,\gamma} + \lambda \mu^{RA}_{t,u,\gamma} - \mu_{t,u,\gamma} - \mu^{M}_{t,u,\gamma} \tag{B.13}$$

$$(-1) * \frac{\partial}{\partial d_{t,u,\gamma}}: \quad \left(1+\mu^{RA}_{t,u,\gamma}\right) E_t\left[\tilde{\psi}_{t+1} R^{d}_{t}\right] = \mu^{BC}_{t,u,\gamma} - \omega^{max} \mu_{t,u,\gamma} \tag{B.14}$$

$$(-1) * \frac{1}{Q^{F}_t}\frac{\partial}{\partial f_{t,u,\gamma}}: \quad \left(1+\mu^{RA}_{t,u,\gamma}\right) E_t\left[\tilde{\psi}_{t+1} R^{F}_{t}\right] = \mu^{BC}_{t,u,\gamma} - \frac{\mu^{CC}_{t,u,\gamma}}{Q^F_t} + \frac{\mu^{F}_{t,u,\gamma}}{Q^F_t} \tag{B.15}$$

$$\frac{1}{Q^{\gamma}_t}\frac{\partial}{\partial b^F_{t,u,\gamma}}: \quad \mu^{CC}_{t,u,\gamma}\eta = \mu^{C}_{t,u,\gamma} + \tilde{\eta}^{\gamma}_t \mu_{t,u,\gamma} \tag{B.16}$$

The FOCs for connected banks are:

$$\frac{\partial}{\partial k_{t,c,\gamma}}: \quad \left(1+\mu^{RA}_{t,c,\gamma}\right) E_t\left[\tilde{\psi}_{t+1} R_{k,t+1}\right] = \mu^{BC}_{t,c,\gamma} + \lambda \mu^{RA}_{t,c,\gamma} \tag{B.17}$$

$$\frac{1}{Q^{\gamma}_t}\frac{\partial}{\partial b_{t,c,\gamma}}: \quad \left(1+\mu^{RA}_{t,c,\gamma}\right) E_t\left[\tilde{\psi}_{t+1} R^{\gamma}_{b,t+1}\right] = \mu^{BC}_{t,c,\gamma} + \lambda \mu^{RA}_{t,c,\gamma} - \mu^{C}_{t,c,\gamma} \tag{B.18}$$

$$\frac{\partial}{\partial m_{t,c,\gamma}}: \quad \left(1+\mu^{RA}_{t,c,\gamma}\right) E_t\left[\tilde{\psi}_{t+1}\right] = \mu^{BC}_{t,c,\gamma} + \lambda \mu^{RA}_{t,c,\gamma} - \mu^{M}_{t,c,\gamma} \tag{B.19}$$

$$(-1) * \frac{\partial}{\partial d_{t,c,\gamma}}: \quad \left(1+\mu^{RA}_{t,c,\gamma}\right) E_t\left[\tilde{\psi}_{t+1} R^{d}_{t}\right] = \mu^{BC}_{t,c,\gamma} \tag{B.20}$$

$$(-1) * \frac{1}{Q^{F}_t}\frac{\partial}{\partial f_{t,c,\gamma}}: \quad \left(1+\mu^{RA}_{t,c,\gamma}\right) E_t\left[\tilde{\psi}_{t+1} R^{F}_{t}\right] = \mu^{BC}_{t,c,\gamma} - \frac{\mu^{CC}_{t,c,\gamma}}{Q^F_t} + \frac{\mu^{F}_{t,c,\gamma}}{Q^F_t} \tag{B.21}$$

$$\frac{1}{Q^{\gamma}_t}\frac{\partial}{\partial b^F_{t,c,\gamma}}: \quad \mu^{CC}_{t,c,\gamma}\eta = \mu^{C}_{t,c,\gamma} \tag{B.22}$$

The complementary slackness conditions for each (ν, γ) combination are

$$\mu^{F}_{t,\nu,\gamma} f_{t,\nu,\gamma} = 0 \tag{B.23}$$

$$\mu^{M}_{t,\nu,\gamma} m_{t,\nu,\gamma} = 0 \tag{B.24}$$

$$\mu^{C}_{t,\nu,\gamma}\left(b_{t,\nu,\gamma} - b^F_{t,\nu,\gamma}\right) = 0 \tag{B.25}$$

$$\mu^{RA}_{t,\nu,\gamma}\left[\phi n_t + \tilde{v}_{t,\nu,\gamma} - \lambda\left(Q^k_t k_{t,\nu,\gamma} + Q^{\gamma}_t b_{t,\nu,\gamma} + m_{t,\nu,\gamma}\right)\right] = 0 \tag{B.26}$$

and for each location γ, for unconnected banks

$$\mu_{t,u,\gamma}\left[\omega^{\max} D_{t,u,\gamma} - M_{t,u,\gamma} - \widetilde{\eta}^{\gamma}_t Q^{\gamma}_t\left(B_{t,u,\gamma} - B^F_{t,u,\gamma}\right)\right] = 0.$$

C Equilibrium

Defining the equilibrium requires us to firstly define some aggregate variables. As argued in section 4.1, it suffices for us to look at the decision problem of the average region-type bank. Then, to move from these averages to economy-wide aggregates, for capital:

$$k_t = \xi_{t,c} k_{t,c} + \xi_{t,u}(1 - s_N) k_{t,u,S} + \xi_{t,u} s_N k_{t,u,N} \tag{C.1}$$

where to get aggregate capital we scale each region-type average bank by their mass relative to the mass of banks on aggregate. We then do the same for $b_t, b_{t,\gamma}, f_t, d_t$ and m_t. This allows us to proceed to the equilibrium.

An equilibrium is a vector of sequences such that when:

1. Given $P_t, \tau_t, W_t, R^d_{t-1}, N_t$, the representative household chooses $c_t > 0, h_t > 0, D^h_t \geq 0, M^h_t \geq 0$ to maximise lifetime utility subject to their budget constraint (A.1)

2. Final good firms choose capital and labor to maximize their expected profits from production, which makes use of the technology (3)

3. Given capital prices Q^k_t, capital-producing firms decide on how much investment i_t to make by maximising profits (4)

4. Given the paths for the endogenous variables c_t, h_t, r_t, P_t, Q^γ_t, Q^F_t, η_t, and exogenous sequence for $\widetilde{\eta}^\gamma_t$, the average region-type bank choose $k_{t,\nu,\gamma}$, $b_{t,\nu,\gamma}$, $b^F_{t,\nu,\gamma}$, $f_{t,\nu,\gamma}$, $d_{t,\nu,\gamma}$, $m_{t,\nu,\gamma}$ to maximize its value function, with the full problem described in appendix B

5. The central bank chooses money supply $\overline{M}_t$ from (11), the haircut parameter η_t, the discount factor on central bank funds Q^F_t, constant real bond holdings b^C, as well as the seignorage payments S_t to satisfy the budget constraint (13)

6. The government in each region chooses a rule for bond issuances following (D.19), a constant government spending level $g^\star$ and chooses the proportional tax rate τ_t to satisfy the budget constraint (D.23) given lump-sum transfers

7. The foreign sector demands bonds in each region from demand function (21), resulting in a claim on domestic goods c^w_t to satisfy the budget constraint in (22)

8. The goods, capital and asset markets clear

$$c_t + g_t + i_t + c^w_t = y_t \tag{C.2}$$

$$k_t = (1 - \delta) k_{t-1} + \Phi\left(\frac{i_t}{k_{t-1}}\right) k_{t-1} \tag{C.3}$$

$$\overline{b}_{t,N} = b_{t,N} + b^C_{t,N} + b^w_{t,N} \tag{C.4}$$

$$\overline{b}_{t,S} = b_{t,S} + b^C_{t,S} + b^w_{t,S} \tag{C.5}$$

$$\overline{f}_t = f_t \tag{C.6}$$

$$d^h_t = d_t \tag{C.7}$$

$$\overline{M}_t = M_t + M^h_t \tag{C.8}$$

D Dynamic System of Equations

Here we describe the full system of equations that characterizes the model dynamics. There are 90 variables and 91 equations. One equation is redundant by the Walras law, e.g. the goods market clearing condition (D.30).

The variables are (without time subscript)

$$\left\{ \begin{array}{c} y, k, c, c^w, l, d, n, m^h, b^w_S, b^w_N, f, m, \bar{v}, \bar{b}, \tau, T_N, T_S \\ \psi, \tilde{\psi}, w, r, Q^S, Q^N, Q^F, r^S, R^N, R^d, R^k, \pi, I, s, \overline{f}, \overline{m}, \pi^e, Q^k \end{array} \right\}$$

plus

$$\left\{k_{u,S}, m_{u,S}, f_{u,S}, b_{u,S}, b^F_{u,S}, d_{u,S}, n_{u,S}, v_{u,S}, \widetilde{v}_{u,S}, \mu^F_{u,S}, \mu^M_{u,S}, \mu^C_{u,S}, \mu^{RA}_{u,S}, \mu^{BC}_{u,S}, \mu^{CC}_{u,S}, \mu_{u,S}\right\},$$
$$\left\{k_{u,N}, m_{u,N}, f_{u,N}, b_{u,N}, b^F_{u,N}, d_{u,N}, n_{u,N}, v_{u,N}, \widetilde{v}_{u,N}, \mu^F_{u,N}, \mu^M_{u,N}, \mu^C_{u,N}, \mu^{RA}_{u,N}, \mu^{BC}_{u,N}, \mu^{CC}_{u,N}, \mu_{u,N}\right\},$$
$$\{k_u, m_u, f_u, d_u, v_u, n_u\}$$
$$\left\{k_c, m_c, d_c, n_c, v_c, \widetilde{v}_c, \mu^M_c, \mu^{RA}_c, \mu^{BC}_c\right\}$$

plus the value of the three monetary policy instruments

$$R^F, b^C, \eta$$

and the rule for government spending and debt increase,

$$\overline{b}^{\Delta}, g$$

and the exogenous variables

$$\xi, \tilde{\eta}^S, \tilde{\eta}^N.$$

The dynamics is characterized by the equilibrium conditions listed below.

D.1 Households and firms

For households, the first order conditions over bonds, labour and money are:

$$1 = \beta E_t \left[\frac{c_t}{c_{t+1}} \frac{R^d_t}{\pi_{t+1}} \right] \tag{D.1}$$

$$c_t h_t^{\zeta} = (1 - \tau_t)\, w_t \tag{D.2}$$

$$\frac{c_t}{\chi m^h_t} = \frac{R^d_t - 1}{R^d_t} \tag{D.3}$$

The budget constraint is

$$c_t + m^h_t + d_t = \frac{m^h_{t-1}}{\pi_t} + R^d_{t-1}\frac{d_{t-1}}{\pi_t} + (1 - \tau_t) w_t h_t + \phi n_t \tag{D.4}$$

First-order conditions arising from the problem of the final good firms are

$$\begin{aligned} y_t &= k_{t-1}^{\theta} h_t^{1-\theta}, && \text{(D.5)} \\ w_t h_t &= (1-\theta)\ y_t, && \text{(D.6)} \\ r_t k_{t-1} &= \theta y_t, && \text{(D.7)} \\ k_t &= (1-\delta)\, k_{t-1} + I_t && \text{(D.8)} \\ R_{k,t} &= \frac{Q_t^k}{Q_{t-1}^k}(r_t + 1 - \delta) && \text{(D.9)} \end{aligned}$$

The first-order condition for the capital-producing firms equals:

$$Q_t^k = \left(\frac{i_t}{\delta k_{t-1}}\right)^{\zeta} \tag{D.10}$$

D.2 Central Bank

The money supply rule is given by

$$\bar{m}_t = \bar{m}_{t-1} + Q_{f,t}\bar{f}_t - \frac{R_{t-1}^F}{\pi_t} Q_{f,t-1}\bar{f}_{t-1} \tag{D.11}$$

We define a rule over the interest rate charged on central bank funding,

$$R_t^F = \left(R^F\right)^{\star} \tag{D.12}$$

The rule on bond purchases is:

$$b_t^c = (b^c)^* \tag{D.13}$$

The haircut applies to central bank funding is:

$$\eta_t = \eta^* \tag{D.14}$$

The value of η^* differs depending on the policy regime. Under CB Policy, $\eta^* = 0$, whereas under CO Policy $\eta^* = 0.97$.

The central bank budget constraint is

$$\begin{aligned} s_t = &\left(\frac{R_{t-1}^F}{\pi_t}\right) Q_{t-1}^F \bar{f}_{t-1} + \left[s_N \left(\frac{R_t^N}{\pi_t}\right) Q_{t-1}^N + (1-s_N)\left(\frac{R_t^S}{\pi_t}\right) Q_{t-1}^S\right] b_{t-1}^C \\ &- \left(\frac{1}{\pi_t}\right) \bar{m}_{t-1} + \bar{m}_t - Q_t^F \bar{f}_t - \left[s_N Q_t^N + (1-s_N) Q_t^S\right] b_t^C \end{aligned} \tag{D.15}$$

with returns defined as

$$R_t^N = \frac{\kappa + (1-\kappa)Q_t^N}{Q_{t-1}^N} \tag{D.16}$$

$$R_t^S = \frac{\kappa + (1-\kappa)Q_t^S}{Q_{t-1}^S} \tag{D.17}$$

$$R_t^F = 1/Q_t^F \tag{D.18}$$

D.3 Government

Each region γ is of size s_γ. Bond issuance is the same in each region, adjusted for size, so that $\bar{B}_{t,\gamma} = s_\gamma \bar{B}_t$. Government expenditure is also the same adjusted for size i.e. $g_{t,\gamma} = s_\gamma g_t$. On aggregate, we have rules

$$\bar{b}_t^\Delta = \alpha\left(b^* - (1-\kappa)\frac{\bar{b}_{t-1}}{\pi_t}\right) \tag{D.19}$$

$$g_t = g^* \tag{D.20}$$

where, as a result, we have the following evolution of bond holdings:

$$\bar{b}_t = (1-\kappa)\frac{\bar{b}_{t-1}}{\pi_t} + \bar{b}_t^\Delta \tag{D.21}$$

The budget constraint for the government in region N is

$$P_t s_N g^* + \kappa \overline{B}_{t-1,N} = s_\gamma \tau_t W_t h_t + Q_t^N \Delta \overline{B}_{t,N} + s_N S_t + T_{t,N}, \tag{D.22}$$

while for region S it is

$$P_t s_S g^* + \kappa \overline{B}_{t-1,N} = s_\gamma \tau_t W_t h_t + Q_t^S \Delta \overline{B}_{t,S} + s_S S_t + T_{t,S}, \tag{D.23}$$

where the cross-region transfers net out to zero,

$$T_{t,N} + T_{t,S} = 0 \tag{D.24}$$

This implies the following aggregate government budget constraint

$$g_t + \kappa\frac{\bar{b}_{t-1}}{\pi_t} = \tau_t w_t h_t + \left[s_N Q_t^N + (1-s_N)Q_t^S\right]\bar{b}_t^\Delta + s_t$$

D.4 Price Setting

Prices are determined one period in advance:

$$\pi_t = \pi_{t-1}^e \tag{D.25}$$

D.5 Market Clearance

$$
\begin{aligned}
\overline{f}_t &= f_t && \text{(D.26)}\\
\overline{m}_t &= m_t + m_t^h && \text{(D.27)}\\
s_N \bar{b}_t &= (1-\xi_t) s_N b_{t,u,N} + b_{t,u,N}^w + s_N b_t^C && \text{(D.28)}\\
(1-s_N)\bar{b}_t &= (1-\xi_t)(1-s_N) b_{t,u,S} + b_{t,u,S}^w + (1-s_N) b_t^C && \text{(D.29)}\\
y_t &= c_t + c_t^w + g_t + I_t && \text{(D.30)}
\end{aligned}
$$

where, as remarked at the beginning of this section, the last equation can be dropped per Walras' law.

D.6 Banks

Common conditions to c, $\{u, N\}$ and $\{u, S\}$ banks:

$$
\begin{aligned}
\bar{v}_t &= \psi_t n_t && \text{(D.31)}\\
\tilde{\psi}_t &= \beta \frac{c_{t-1}}{c_t} \psi_t && \text{(D.32)}
\end{aligned}
$$

Equations for the problem of bank $\{u, N\}$:
The value of the bank before dividend distribution is equal to:

$$
v_{t,u,N} = \phi n_t + \tilde{v}_{t,u,N} \quad \text{(D.33)}
$$

where the value of the bank after dividend distribution is:

$$
\tilde{v}_{t,u,N} = E_t \left[\tilde{\psi}_{t+1} n_{t+1,u,N} \right] \quad \text{(D.34)}
$$

Net worth evolves according to:

$$
n_{t+1,u,N} = R_{k,t} Q_t^k k_{t,u,N} + \frac{m_{t,u,N}}{\pi_{t+1}} + \frac{R_{t+1}^N}{\pi_{t+1}} Q_t^S b_{t,u,N} - \frac{R_t^d}{\pi_{t+1}} d_{t,u,N} - \frac{R_t^F}{\pi_{t+1}} Q_t^F f_{t,u,N} \quad \text{(D.35)}
$$

The following are the corresponding constraints and Kuhn-Tucker conditions arising from solving the problem for the bank of type $\{u, S\}$: the budget constraint (D.36), the leverage constraint (D.37), the afternoon constraint (D.38), the CB funding constraint (D.39), the non-negativity constraints on money, (D.40), and CB funding (D.41), and the constraint on collateral posted to the central bank (D.42):

$$\begin{aligned}
Q_t^k k_{t,u,N} + Q_t^N b_{t,u,N} + m_{t,u,N} - (1-\phi)n_t - Q_t^F f_{t,u,N} &= d_{t,u,N} && \text{(D.36)}\\
\min\left\{\mu_{t,u,N}^{RA}, v_{t,u,N} - \phi n_t - \lambda\left(Q_t^k k_{t,u,N} + Q_t^N b_{t,u,N} + m_{t,u,N}\right)\right\} &= 0 && \text{(D.37)}\\
\min\left\{\mu_{t,u,N}, m_{t,u,N} + \widetilde{\eta}_t^N Q_t^N \left(b_{t,u,N} - b_{t,u,N}^F\right) - \omega^{\max} d_{t,u,N}\right\} &= 0 && \text{(D.38)}\\
f_{t,u,N} &= \eta_t Q_t^N b_{t,u,N}^F && \text{(D.39)}\\
\min\left\{\mu_{t,u,N}^F, f_{t,u,N}\right\} &= 0 && \text{(D.40)}\\
\min\left\{\mu_{t,u,N}^M, m_{t,u,N}\right\} &= 0 && \text{(D.41)}\\
\min\left\{\mu_{t,u,N}^C, b_{t,u,N} - b_{t,u,N}^F\right\} &= 0 && \text{(D.42)}
\end{aligned}$$

Equations for the problem of bank $\{u, S\}$:

The value of the bank before dividend distribution is equal to:

$$v_{t,u,S} = \phi n_t + \tilde{v}_{t,u,S} \tag{D.43}$$

where the value of the bank after dividend distribution is:

$$\tilde{v}_{t,u,S} = E_t\left[\tilde{\psi}_{t+1} n_{t+1,u,S}\right] \tag{D.44}$$

and net worth evolves according to:

$$n_{t+1,u,S} = R_{k,t} Q_t^k k_{t,u,S} + \frac{m_{t,u,S}}{\pi_{t+1}} + \frac{R_{t+1}^S}{\pi_{t+1}} Q_t^S b_{t,u,S} - \frac{R_t^d}{\pi_{t+1}} d_{t,u,S} - \frac{R_t^F}{\pi_{t+1}} Q_t^F f_{t,u,S} \tag{D.45}$$

The following are the corresponding constraints and Kuhn-Tucker conditions arising from solving the problem for the bank of type $\{u, S\}$: the budget constraint (D.46), the leverage constraint (D.47), the afternoon constraint (D.48), the CB funding constraint (D.49), the non-negativity constraints on money, (D.50), and CB funding (D.51), and the constraint on collateral posted to the central bank (D.52):

$$\begin{aligned}
Q_t^k k_{t,u,S} + Q_t^S b_{t,u,S} + m_{t,u,S} - (1-\phi)n_t - Q_t^F f_{t,u,S} &= d_{t,u,S} && \text{(D.46)}\\
\min\left\{\mu_{t,u_G}^{RA}, v_{t,u,S} - \phi n_t - \lambda\left(Q_t^k k_{t,u,S} + Q_t^S b_{t,u,S} + m_{t,u,S}\right)\right\} &= 0 && \text{(D.47)}\\
\min\left\{\mu_{t,u,S}, m_{t,u,S} + \widetilde{\eta}_t^S Q_t^S \left(b_{t,u,S} - b_{t,u,S}^F\right) - \omega^{\max} d_{t,u,S}\right\} &= 0 && \text{(D.48)}\\
f_{t,u,S} &= \eta_t Q_t^S b_{t,u,S}^F && \text{(D.49)}\\
\min\left\{\mu_{t,u,S}^F, f_{t,u,S}\right\} &= 0 && \text{(D.50)}\\
\min\left\{\mu_{t,u,S}^M, m_{t,u,S}\right\} &= 0 && \text{(D.51)}\\
\min\left\{\mu_{t,u,S}^C, b_{t,u,S} - b_{t,u,S}^F\right\} &= 0 && \text{(D.52)}
\end{aligned}$$

Equations for the problem of bank c:

This bank type never has an incentive to hold bonds. The following exposition is thus simplified. The value of the bank before dividend distribution is equal to:

$$v_{t,c} \;=\; \phi n_t + \tilde{v}_{t,c} \tag{D.53}$$

where the value of the bank after dividend distribution is:

$$\tilde{v}_{t,c} \;=\; E_t\left[\tilde{\psi}_{t+1} n_{t+1,c}\right] \tag{D.54}$$

Net worth evolves according to:

$$n_{t,c} \;=\; R_{k,t} Q^k_{t-1} k_{t-1,c} + \frac{m_{t-1,c}}{\pi_t} - \frac{R^d_{t-1}}{\pi_t} d_{t-1,c} \tag{D.55}$$

The following are the corresponding constraints and Kuhn-Tucker conditions arising from solving the problem for the bank of type c: the budget constraint (D.56), the leverage constraint (D.57), and the non-negativity constraints on money, (D.58):

$$Q^k_t k_{t,c} + m_{t,c} - (1-\phi) n_t \;=\; d_{t,c} \tag{D.56}$$

$$\min\left\{\mu^{RA}_{t,c}, v_{t,c} - \phi n_t - \lambda\left(Q^k_t k_{t,c} + m_{t,c}\right)\right\} \;=\; 0 \tag{D.57}$$

$$\min\left\{\mu^{M}_{t,c}, m_{t,c}\right\} \;=\; 0 \tag{D.58}$$

First Order Conditions for Banks $\{u, N\}$:

Here we have the first order conditions for unconnected banks in capital (D.59), bonds (D.60), money (D.61), deposits (D.62), CB funding (D.63) and CB collateral-posting (D.64):

$$(1+\mu^{RA}_{t,u,N}) E_t[\tilde{\psi}_{t+1} R_{k,t+1}] \;=\; \mu^{BC}_{t,u,N} + \lambda \mu^{RA}_{t,u,N} \tag{D.59}$$

$$(1+\mu^{RA}_{t,u,N}) E_t\left[\tilde{\psi}_{t+1} \frac{R^N_{t+1}}{\pi_{t+1}}\right] \;=\; \mu^{BC}_{t,u,N} + \lambda \mu^{RA}_{t,u,N} - \mu^{C}_{t,u,N} - \tilde{\eta}^N_t \mu_{t,u,N} \tag{D.60}$$

$$(1+\mu^{RA}_{t,u,N}) E_t\left[\tilde{\psi}_{t+1} \frac{1}{\pi_{t+1}}\right] \;=\; \mu^{BC}_{t,u,N} + \lambda \mu^{RA}_{t,u,N} - \mu_{t,u,N} - \mu^{M}_{t,u,N} \tag{D.61}$$

$$(1+\mu^{RA}_{t,u,N}) E_t\left[\tilde{\psi}_{t+1} \frac{1}{\pi_{t+1}}\right] R^d_t \;=\; \mu^{BC}_{t,u,N} - \omega^{\max} \mu_{t,u,N} \tag{D.62}$$

$$(1+\mu^{RA}_{t,u,N}) E_t\left[\tilde{\psi}_{t+1} \frac{1}{\pi_{t+1}}\right] R^F_t \;=\; \mu^{BC}_{t,u,N} - \frac{\mu^{CC}_{t,u,N}}{Q^F_t} + \frac{\mu^{F}_{t,u,N}}{Q^F_t} \tag{D.63}$$

$$\mu^{CC}_{t,u,N} \eta \;=\; \mu^{C}_{t,u,N} + \tilde{\eta}^N_t \mu_{t,u,N} \tag{D.64}$$

First Order Conditions for Banks $\{u, S\}$:

Here we have the first order conditions for unconnected banks in capital (D.65), bonds (D.66), money (D.67), deposits (D.68), CB funding (D.69) and CB collateral-posting (D.70):

$$\begin{aligned}
(1+\mu^{RA}_{t,u,S})E_t[\tilde{\psi}_{t+1}R_{k,t+1}] &= \mu^{BC}_{t,u_G} + \lambda\mu^{RA}_{t,u,S} && \text{(D.65)}\\
(1+\mu^{RA}_{t,u,S})E_t\left[\tilde{\psi}_{t+1}\frac{R^S_{t+1}}{\pi_{t+1}}\right] &= \mu^{BC}_{t,u,S} + \lambda\mu^{RA}_{t,u,S} - \mu^{C}_{t,u,S} - \tilde{\eta}^S_t\mu_{t,u,S} && \text{(D.66)}\\
(1+\mu^{RA}_{t,u,S})E_t\left[\tilde{\psi}_{t+1}\frac{1}{\pi_{t+1}}\right] &= \mu^{BC}_{t,u,S} + \lambda\mu^{RA}_{t,u,S} - \mu_{t,u,S} - \mu^{M}_{t,u,S} && \text{(D.67)}\\
(1+\mu^{RA}_{t,u,S})E_t\left[\tilde{\psi}_{t+1}\frac{1}{\pi_{t+1}}\right]R^d_t &= \mu^{BC}_{t,u,S} - \omega^{\max}\mu_{t,u,S} && \text{(D.68)}\\
(1+\mu^{RA}_{t,u,S})E_t\left[\tilde{\psi}_{t+1}\frac{1}{\pi_{t+1}}\right]R^F_t &= \mu^{BC}_{t,u,S} - \frac{\mu^{CC}_{t,u,S}}{Q^F_t} + \frac{\mu^{F}_{t,u,S}}{Q^F_t} && \text{(D.69)}\\
\mu^{CC}_{t,u,S}\eta &= \mu^{C}_{t,u,S} + \tilde{\eta}^S_t\mu_{t,u,S} && \text{(D.70)}
\end{aligned}$$

First-Order Conditions for Banks c:

Here we have the first order conditions for connected banks in capital (D.71) money (D.72) and deposits (D.73):

$$\begin{aligned}
(1+\mu^{RA}_{t,c})E_t[\tilde{\psi}_{t+1}R_{k,t+1}] &= \mu^{BC}_{t,c} + \lambda\mu^{RA}_{t,c} && \text{(D.71)}\\
(1+\mu^{RA}_{t,c})E_t\left[\tilde{\psi}_{t+1}\frac{1}{\pi_{t+1}}\right] &= \mu^{BC}_{t,c} + \lambda\mu^{RA}_{t,c} - \mu^{M}_{t,c} && \text{(D.72)}\\
(1+\mu^{RA}_{t,c})E_t\left[\tilde{\psi}_{t+1}\frac{1}{\pi_{t+1}}\right]R^d_t &= \mu^{BC}_{t,c} && \text{(D.73)}
\end{aligned}$$

Unconnected bank aggregation equations:

$$\begin{aligned}
k_{t,U} &= s_N k_{t,u,N} + (1-s_N)k_{t,u,S} && \text{(D.74)}\\
d_{t,U} &= s_N d_{t,u,N} + (1-s_N)d_{t,u,S} && \text{(D.75)}\\
f_{t,U} &= s_N f_{t,u,N} + (1-s_N)f_{t,u,S} && \text{(D.76)}\\
m_{t,U} &= s_N m_{t,u,N} + (1-s_N)m_{t,u,S} && \text{(D.77)}\\
v_{t,U} &= s_N v_{t,u,N} + (1-s_N)v_{t,u,S} && \text{(D.78)}\\
n_{t,U} &= s_N n_{t,u,N} + (1-s_N)n_{t,u,S} && \text{(D.79)}
\end{aligned}$$

Bank aggregation equations:

$$\begin{aligned}
k_t &= \xi_t k_{t,c} + (1-\xi_t)k_{t,u} && \text{(D.80)}\\
d_t &= \xi_t d_{t,c} + (1-\xi_t)d_{t,u} && \text{(D.81)}\\
f_t &= (1-\xi_t)f_{t,u} && \text{(D.82)}\\
m_t &= \xi_t m_{t,c} + (1-\xi_t)m_{t,u} && \text{(D.83)}\\
\bar{v}_t &= \xi_t v_{t,c} + (1-\xi_t)v_{t,u} && \text{(D.84)}\\
n_t &= \xi_{t-1} n_{t,c} + (1-\xi_{t-1})n_{t,u} && \text{(D.85)}
\end{aligned}$$

Exogenous processes:

The parameters that measure connectedness and private bank haircuts follow the processes:

$$\frac{\xi_t}{\xi^\star} = \left(\frac{\xi_{t-1}}{\xi^\star}\right)^{(1-\rho_\xi)} \exp(\epsilon_{\xi,t}) \tag{D.86}$$

$$\frac{\tilde{\eta}_t^S}{\tilde{\eta}^\star} = \left(\frac{\tilde{\eta}_{t-1}^S}{\tilde{\eta}^\star}\right)^{(1-\rho_{\tilde{\eta},1})} \left(\frac{\tilde{\eta}_{t-2}^S}{\tilde{\eta}^\star}\right)^{(1-\rho_{\tilde{\eta},2})} \exp(\epsilon_{\tilde{\eta}_t^S}) \tag{D.87}$$

$$\tilde{\eta}_t^N = \tilde{\eta}^\star \tag{D.88}$$

D.7 Rest of the world

Foreign bond demand functions are

$$b_{t,N}^w = s_N \left(\varkappa + \frac{1}{\varrho} \log\left(\frac{E\left[R_{t+1}^N/\pi_{t+1}\right]}{r^N}\right)\right) \tag{D.89}$$

$$b_{t,S}^w = (1-s_N) \left(\varkappa + \frac{1}{\varrho} \log\left(\frac{E\left[R_{t+1}^S/\pi_{t+1}\right]}{r^S}\right)\right) \tag{D.90}$$

where r^N, r^S are the real rates of return on bonds in the baseline calibrated steady state.[41]

The foreign sector satisfies the budget constraint

$$Q_t^N b_{t,N}^w + Q_t^S b_{t,S}^w + c_t^w = \frac{R_t^N}{\pi_t} Q_{t-1}^N b_{t-1,N}^w + \frac{R_t^S}{\pi_t} Q_{t-1}^S b_{t-1,S}^w. \tag{D.91}$$

E Collateral premium

Combine the first-order conditions of an unconnected bank $\nu = u$ located in γ with respect to capital and bonds to get

$$\left(1+\mu_{t,\nu,\gamma}^{RA}\right) \tilde{\psi}_{t+1} \left(R_{t+1}^k - R_{b,t+1}^\gamma\right) = \mu_{t,\nu,\gamma}^C + \mu_{t,\nu,\gamma} \widetilde{\eta}_t^\gamma \tag{E.1}$$

Since $\mu_{t,\nu,\gamma}^C \geq 0$, $\mu_{t,\nu,\gamma}^{RA} \geq 0$, $\mu_{t,\nu,\gamma} \geq 0$, and $\tilde{\psi}_{t+1} > 0$, it must be that $\left(R_{t+1}^k - R_{b,t+1}^\gamma\right) \geq 0$.

We show that the collateral premium is strictly positive if the afternoon constraint is binding or unconnected banks are collateral-constrained in their borrowing from the central bank. First, if unconnected banks are constrained in the afternoon, $\mu_{t,\nu,\gamma} > 0$. It follows that $\left(R_{t+1}^k - R_{b,t+1}^\gamma\right) > 0$. Second, if unconnected banks are collateral-constrained at the central bank, $\mu_{t,\nu,\gamma}^C > 0$ and the same conclusion follows.

F Proof of Proposition 2

Consider the equilibrium for the parameters indexed by A. Consider an individual bank and recall its budget constraint (dropping subscript i)

$$n_t = \phi n_t + k_t + Q_t^{\gamma_t} b_t + m_t - d_t \tag{F.1}$$

[41] In the Baseline calibration, $r^N = r^S$.

where the portfolio choices and returns result in the net worth n_{t+1} for the next period,

$$n_{t+1} = R_{k,t+1}k_t + \frac{m_t}{\pi_{t+1}} + \frac{R_{t+1}^{\gamma_t}}{\pi_{t+1}}Q_t^{\gamma_t}b_t - \frac{R_t^d}{\pi_{t+1}}d_t \tag{F.2}$$

Given no aggregate uncertainty, banks simply want to maximize next period's net worth, n_{t+1}, subject to the constraints that they face. For unconnected banks, if they issue an additional unit of deposits next period, they add the following to net worth

$$R_{k,t+1} - \frac{R_{d,t}}{\pi_{t+1}} + \omega^{\max}\max\left\{\frac{R_{k,t+1} - R_{t+1}^{\gamma}/\pi_{t+1}}{\tilde{\eta}_t}, R_{k,t+1} - \frac{1}{\pi_{t+1}}\right\} \leq R_{k,t+1} - \frac{R_{d,t}}{\pi_{t+1}} \tag{F.3}$$

where the third term on the left reflects the collateral premium banks pay on bonds or money to back one deposit unit and is non-negative. Connected banks face no afternoon constraint and obtain the net return on the right of the equation (F.3).

Therefore, there must be two cases: either unconnected banks are priced out of issuing deposits, or unconnected banks issue deposits, but the constraint does not bind, and returns on any assets held by unconnected banks are the same as the return on capital. Consider the latter case.

Since there is no leverage constraint, there is, therefore, also an equilibrium, where the deposits together with their corresponding assets are reallocated from the unconnected banks to the connected banks, i.e., where deposits are only issued by connected banks.[42]. This equilibrium has the same aggregate quantities. With zero deposits, the afternoon constraint no longer matters for the unconnected banks, and thus, the parameters $\left(\xi_t, \tilde{\eta}_t^N, \tilde{\eta}_t^S\right)$ no longer matter for the net worth evolution of the unconnected banks. It is, therefore, also a feasible allocation for the economy with parameters B. It remains to show that it is optimal for unconnected banks in economy B not to issue deposits: it then follows that this allocation is an equilibrium. Hold market prices fixed and suppose there is a solution to the problem of unconnected banks, where they issue positive deposits. By the same argument as above, it then is similarly optimal for connected banks to do so, i.e., the allocation, where unconnected banks do not issue deposits, is also optimal.

G Derivation of Secured Share

This section derives the secured share of interbank markets. For any given bank, when they issue deposits d, they face realized withdrawals of:

$$\hat{\epsilon}\omega^{max}d \qquad \text{s.t.}\hat{\epsilon} \sim U\left[-1, 1\right]$$

where $\hat{\epsilon} > 0$ implies withdrawals, and $\hat{\epsilon} < 0$ inflows. $\hat{\epsilon}$ is bounded above by 1 as ω^{max} is the max fraction of withdrawals one receives. We assume $\hat{\epsilon}$ follows a uniform distribution between -1 and 1 i.e. $\hat{\epsilon} \sim U[-1, 1]$.

[42]Suppose, for example, that unconnected banks hold money, with $m_t \geq d_t$. Then, increase the deposits issued by connected banks by this amount d_t, increase their money holdings by $m_t - d_t$, and decrease the money holdings of unconnected banks by $m_t - d_t$. The budget constraints (F.1) for both types of banks continue to hold. The reshuffling is a solution to their optimization problem, per the remarks made.

We assume that, for any region-type bank (ν, γ), they can cover any deposit withdrawals firstly with reserve holdings, and then subsequently draw from the interbank market The expected volume of interbank funding for a bank of each region-type combination, $f^P_{\nu,\gamma}$, is:

$$\begin{aligned} f^P_{\nu,\gamma} &= E\left[\hat{\epsilon}\omega^{max} d_{\nu,\gamma} - m_{\nu,\gamma} | \hat{\epsilon}\omega^{max} d_{\nu,\gamma} - m_{\nu,\gamma} > 0\right] Pr\left(\hat{\epsilon}\omega^{max} d_{\nu,\gamma} - m_{\nu,\gamma} > 0\right) \\ &= E\left[\hat{\epsilon}\omega^{max} d_{\nu,\gamma} - m_{\nu,\gamma} | \hat{\epsilon} > \frac{m_{\nu,\gamma}}{\omega^{max} d_{\nu,\gamma}}\right] Pr\left(\hat{\epsilon} > \frac{m_{\nu,\gamma}}{\omega^{max} d_{\nu,\gamma}}\right) \\ &= \left(\frac{\omega^{max} d_{\nu,\gamma} - m_{\nu,\gamma}}{2}\right)\left(1 - \frac{m_{\nu,\gamma}}{\omega^{max} d_{\nu,\gamma}}\right)^2 \end{aligned} \tag{G.1}$$

We know that connected banks draw from unsecured markets while unconnected banks draw from secured markets. The secured share s_{SEC} is therefore:

$$s_{SEC} = \frac{(1-\xi)\left(s_N f^P_{u,N} + (1-s_N) f^P_{u,S}\right)}{\xi\left(s_N f^P_{c,N} + (1-s_N) f^P_{c,S}\right) + (1-\xi)\left(s_N f^P_{u,N} + (1-s_N) f^P_{u,S}\right)} \tag{G.2}$$

where $(f^P_{c,N}, f^P_{c,S}, f^P_{u,N}, f^P_{u,S})$ are defined in equation (G.2) and depend on the portfolio choice made by the respective bank type.

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