Discussion of McKay and Wolf "What Can Time-Series Regressions Tell Us About Policy Counterfactuals"

Daniel J. Lewis

University College London

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Approaches to the Lucas Critique

The Lucas (1976) Critique: We cannot use historical relationships to draw reliable conclusions about the effects of a shock under any other policy rule than that which held historically.

- Cannot draw policy conclusions from semi-structural models, since agents' behaviour would differ under alternative policy

1. **Lucas Program:** Micro-founded structural models that match key moments in the data
2. **Sims & Zha:** Impose counterfactual rules in semi-structural model *ex post* (e.g., "zeroing out")

Main idea

- Agents learn about policy rules and update expectations *ex ante*: need to account for changes in expectations.
- Use not only contemporaneous shocks, but a full menu of news shocks to impose the policy rule **in expectation**.
- Given contemporary shocks $\nu_{0,t}$ and news shocks $\nu_{l,t-l}, \forall l = 1, \ldots \infty$, can impose any policy rule.
- Use the impulse responses to such shocks to infer impulse responses to a non-policy shock under some counterfactual rule.

Simple example: three-equation NK model

$$y_t = y_{t+1} - \frac{1}{\gamma}(i_t - \pi_{t+1})$$

$$\pi_t = \kappa y_t + \beta \pi_{t+1} + (\varepsilon_t + \theta \varepsilon_{t-1})$$

$$i_t = \phi \pi_t + \nu_{0,t} + \sum_{l=1}^{\infty} \nu_{l,t-l}$$

$$i_t = \tilde{\phi} \pi_t \quad \text{(counterfactual rule)}$$

Solve system of 2 eqns (2 horizons) for $\tilde{\nu}_{0,0}, \tilde{\nu}_{1,0}$:

$$\underbrace{\underbrace{\boldsymbol{i}_\phi(\varepsilon_0) + \Theta_{i,\nu 0,\phi}\tilde{\nu}_{0,0} + \Theta_{i,\nu 1,\phi}\tilde{\nu}_{1,0}}_{\text{IR of } i \text{ to } \varepsilon_0, \tilde{\nu}_{0,0}, \tilde{\nu}_{1,0}} = \tilde{\phi} \times \underbrace{[\boldsymbol{\pi}_\phi(\varepsilon_0) + \Theta_{\pi,\nu 0,\phi}\tilde{\nu}_{0,0} + \Theta_{\pi,\nu 1,\phi}\tilde{\nu}_{1,0}]}_{\text{IR of } \pi \text{ to } \varepsilon_0, \tilde{\nu}_{0,0}, \tilde{\nu}_{1,0}}}_{\text{counterfactual rule}}$$

General theory

Two key assumptions on setting:

1. Linear DGP
2. Policy only affects private behaviour through instrument

Main result (Proposition 1): If invertiblity holds historically and under the counterfactual, we can recover the counterfactual IRs of observables and the policy instrument:

$$\tilde{\mathcal{A}}_x \left[\boldsymbol{x}_{\mathcal{A}}(\varepsilon) + \Theta_{x,\nu,\mathcal{A}} \times \tilde{\boldsymbol{\nu}} \right] + \tilde{\mathcal{A}}_z \left[\boldsymbol{z}_{\mathcal{A}}(\varepsilon) + \Theta_{z,\nu,\mathcal{A}} \times \tilde{\boldsymbol{\nu}} \right] = \mathbf{0},$$

$$\text{(counterfactual policy rule)} \quad \tilde{\mathcal{A}}_x \boldsymbol{x} + \tilde{\mathcal{A}}_z \boldsymbol{z} = \mathbf{0}.$$

Best to think about the results as applying to perturbations of policy, not equilibrium/steady state shifts.

Also no asymmetric information (more on this later).

Shocks, not news shocks

The demands of the preceding slides look challenging!

1. Need news shocks for the policy instrument
2. Need news shocks at up to T horizons

Key point: In practice, the shocks do not need to be news shocks, can just be linearly independent measurements of the contemporaneous shock.

In practice, will not have T shock series, but will approximate using n_s shocks.

Reframing as an IRF matching exercise

The paper focuses on a sequence of news shocks for theoretical motivation, and having an adequate menu of shocks to impose (or approximate) a rule.

Completely equivalent to focus instead on impulse responses: find the linear combination of baseline IRs that comes closest to aligning the IRs under the counterfactual rule:

$$\min_{\boldsymbol{s}} \|\tilde{\mathcal{A}}_x(\boldsymbol{x}_{\mathcal{A}}(\varepsilon) + \Omega_{x,\mathcal{A}} \times \boldsymbol{s}) + \tilde{\mathcal{A}}_z(\boldsymbol{z}_{\mathcal{A}}(\varepsilon) + \Omega_{z,\mathcal{A}} \times \boldsymbol{s})\| \quad \text{(OBJ)}$$

Regression in IR space

This problem constitutes a **Regression in Impulse Response Space**, see Barnichon and Mesters (2020), Lewis and Mertens (2022). Treat horizons as "observations", and regress a set of IRs on another.

(OBJ) is equivalent to estimating the OLS regression.

$$\left(\tilde{\mathcal{A}}_x x^h_{\mathcal{A}}(\varepsilon) + \tilde{\mathcal{A}}_z z^h_{\mathcal{A}}(\varepsilon)\right) = -\left(\tilde{\mathcal{A}}_x \Omega^h_{x,\mathcal{A}} + \tilde{\mathcal{A}}_z \Omega^h_{z,\mathcal{A}}\right) \times \boldsymbol{s} + \boldsymbol{u}^h_{IR}, \quad h = 0, \ldots, T-1,$$

where the shock/weight n_s-vector $\boldsymbol{s}$ is the "coefficient" vector.

Problem is similar to Lewis and Mertens (2022).

Implications

Math will be a little different, but

1. **Identification:** Works with either external or internal instruments
2. **Inference:** (robust) frequentist methods available
3. **Weighting:** Does it make sense to equally weight all horizons, or improve efficiency?
4. **Approximation error:** Intuitive units for approximation error available by recasting as "R^2"?
5. **Horizons:** Does it make sense to include all horizons up to $T-1$?

Useful applications?

The most obvious cases for application are monetary and fiscal policy rules (see paper and Valerie Ramey's NBER discussion).

- Potential to be extremely useful, particularly in policy institutions (see paper's MP rules)
- Which applications are accessible depends on how many shock series really needed in practice: many available for MP, fewer for fiscal.
- How linearly dependent are the available shocks?
- A well-scaled measure of error would help interpret and compare applications.
- How much does *historical* variation in policy rules matter?

Which shocks?

- Conceptual challenge: many simultaneously valid MP shock series (Sims-Rudebusch)?
- Should really use internally consistent multi-dimensionsal series like Swanson (2021).
- Central bank information effect presents a big challenge, since it violates key assumption on the effects on policy (e.g., Nakamura and Steinsson (2018) shocks).
- Lewis (2022) and Jarocinski (2022) separate info shocks from the three dimensions Swanson identifies.

Conclusion

- Very helpful answer to Lucas critique without a structural model for policy perturbations!
- Information requirements not as demanding as at first glance - but more work needed to assess approximations.
- The approximation step reduces to an OLS problem, with benefits!
- Need to think carefully about which shocks to use.