

Fiscal Rules and Sovereign Default*

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Abstract

Recurrent concerns over debt sustainability in emerging and developed nations have prompted renewed debate on the role of fiscal rules. Their optimality, however, remains unclear. We provide a quantitative analysis of fiscal rules in a standard model of sovereign debt accumulation and default modified to incorporate quasi-hyperbolic preferences. For reasons of political economy or aggregation of citizens' preferences, government preferences are present biased, resulting in over-accumulation of debt. Calibrating this parameter with values in the literature, the model can reproduce debt levels and frequency of default typical of emerging markets even if the household impatience parameter is calibrated to local interest rates. A quantitative exercise finds welfare gains of the optimal fiscal policy to be economically substantial, and the optimal rule to not entail a countercyclical fiscal policy. A simple debt rule that limits the maximum amount of debt is analyzed and compared to a simple deficit rule that limits the maximum amount of deficit per period. Whereas the deficit rule does not perform well, the debt rule yields welfare gains virtually equal to the optimal rule.

JEL classification: F34, H63.

Key words: sovereign debt, hyperbolic discounting, fiscal rules.

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1 Introduction

Recurrent concerns over debt sustainability in emerging markets and the ongoing European debt crisis have prompted renewed debate in academic and policy circles on the role of fiscal rules. A data set compiled by the Fiscal Affairs Department of the IMF identifies countries' adoption of fiscal policy restrictions.¹ Only five countries had fiscal rules in place in 1990, more than 80 by 2014. National fiscal rules are most commonly responses to pressure on public finances. Adoption in emerging economies was typically motivated by debt excesses resulting from the debt crises of the 1980s and banking and economic crises in the 1990s. Rules enacted in response to the more recent global financial crisis attempt to provide credible commitment to long-term fiscal discipline.

Fiscal rules also play a role in business cycle frequencies. Standard economic theory holds that fiscal policy should be countercyclical (Barro, 1979), yet most emerging countries, possibly owing to limited access to credit markets (Bauducco and Caprioli, 2014) or distorted political incentives (Alesina and Tabellini, 2008), follow pro-cyclical fiscal policies, which tend to exacerbate already pronounced cycles (Kaminsky, Reinhart and Vegh, 2005; Vegh and Vuletin, 2012). Governments may adopt fiscal rules that constrain their behavior in order to correct distorted incentives to overspend, particularly in good times. This, in turn, would alleviate distress on rainy days.

In this paper, we examine the welfare implications of fiscal rules in the context of sovereign debt and default. The prevalent and increased use of fiscal rules is suggestive of a desire to constrain a sovereign's incentives to overspend. Despite widespread use, less research has been devoted to understanding the optimality and role of such rules in preventing sovereign default. Are the potential welfare gains of fiscal rules significant? Should such rules take into account the economic cycle? How do the welfare implications compare between simple and sophisticated rules? These questions relate to the broader rules-versus-discretion debate, that is, to whether a

¹ See www.imf.org/external/datamapper/FiscalRules.

commitment should be required, and more generally to why governments hold positive amounts of debt.

To address these questions, we transform the traditional model of sovereign debt and default by assuming governments' preferences to be time inconsistent. Specifically, our formulation of government preferences corresponds to the quasi-hyperbolic consumption model (Laibson, 1997). The consequent conflict between today's government and tomorrow's generates an incentive to pre-commit to a particular fiscal rule. This argument is the primary motivator of our use of time-inconsistent government preferences. We reconcile the impatience of a government with its citizens by recalling Jackson and Yariv (2014, 2015), who propose that aggregating citizens' time-consistent preferences naturally results in time-inconsistent preferences. Even if benevolent ex-ante, the sovereign thus ends up with preferences that display an extra discount parameter that captures the ex-post present bias. Alternatively, a deficit bias may also emerge as an outcome of political game as in [Aguiar and Amador \(2011\)](#).

A second, more technical motivation relates to the calibration of models of sovereign debt. As documented in the literature (Reinhart and Rogoff, 2009), emerging countries are able, despite repeatedly defaulting, to accumulate debt levels close to 60% of GDP. To obtain the observed levels of debt and default in an artificial economy, the intertemporal discount parameter must be calibrated to extremely low numbers. [Aguiar and Gopinath \(2006\)](#), [Alfaro and Kanczuk \(2005, 2009\)](#), and [Arellano \(2008\)](#) use annual data and beta values between 0.40 and 0.80, values much lower than what would be obtained if calibration were to local interest rates.² Notwithstanding this unintuitive calibration, the government is assumed to be benevolent and to maximize household preferences. The use of inconsistent time preferences by the government removes this calibration restriction allowing the household impatience parameter to be calibrated to the interest rate.

² The literature has used other features to match higher debt levels. The use of asymmetric default costs, for example, allows matching higher debt levels, but as noted by [Mendoza and Yue \(2012\)](#) and [Chatterjee and Eyigungor \(2016b\)](#), the calibrated debt levels are still much lower than in the data.

Calibrating the model to the Brazilian economy, a typical emerging economy, yields the following results.³ First, if the government is assumed to have hyperbolic preferences, and calibrating this parameter with values in the literature (Angeletos et al., 2001), the model can reproduce the Brazilian level of debt and frequency of default even if the household impatience parameter is calibrated to local interest rates. Second, adoption of the optimal fiscal rule implies substantive welfare gains relative to the absence of a rule. Third, the optimal fiscal rule does not entail a countercyclical fiscal policy, as is usually believed. Fourth, under the optimal fiscal rule, the country would never opt to default on its debt. Fifth, a simple deficit rule that sets the maximum amount of deficit per period incurs welfare losses relative to no rule. Because a deficit rule reduces the potential benefits of defaulting by limiting consumption after default, the government consequently resists default for large amounts of debt but achieves lower welfare levels. Sixth, welfare gains are similar between a simple debt rule that sets the maximum amount of debt and the optimal fiscal rule. Given its simplicity and ease of contractibility, the simpler fiscal rule seems to be the best option for emerging countries.

We analyze the robustness of the quantitative implications of our results to different assumptions and parameter values. In terms of the role of debt, the hypothesis that governments benefit from borrowing and thus have a motive to hold positive amounts of debt is essential to our results. This is the case of emerging countries, which will eventually catch up to developed ones. Given that our analysis shows the benefits from frontloading household consumption to be quantitatively more important than those from consumption smoothing, fiscal rules should be designed to allow consumption frontloading and avoid the cost of default. We analyze the robustness to changes in risk aversion. When the benefits from consumption smoothing become larger, fiscal policy turns countercyclical. However, the estimates and results contradict the result

³ Two additional features make Brazil particularly interesting. First, President Dilma Rousseff's impeachment in 2016 was due to disobedience of the existing fiscal rule; second, Congress passed additional fiscal restrictions in December 2016.

contradicts evidence regarding consumer preferences in emerging markets. Finally, since there are many political economy motivations for excessive indebtedness including common pool problem-externalities that lead to a deficit bias, and interest groups, we also contrast the results to the case of a non-benevolent sovereign.⁴

This paper relates to the vast literature on sovereign debt and default (see Aguiar and Amador (2014) and Aguiar et al. (2016) for recent surveys of the literature). In a related paper, Hatchondo et al. (2015), study the role of sovereign default and fiscal rules limiting the maximum sovereign premium due to the time-inconsistency of debt dilution. Differently from their work, in our model government preferences display a present bias, which creates a natural alternative role for fiscal rules.⁵ Alternatively, in Dovis and Kirpalani (2017), fiscal rules arise because governments want to signal their type.⁶ More generally, this paper is related to a recent literature on rules versus discretion in self-control settings (Amador, Werning and Angeletos, 2006; Halac and Yared, 2014, 2015). If fiscal rules cannot define policy instructions for every possible shock or eventuality, there is a cost from lack of flexibility and some discretion can be optimal. Differently from this literature, we explicitly consider the possibility of default and its effect on debt accumulation. We also assume the private sector to know as much as the government about the state of the economy.

The rest of the paper is organized as follows. The model is developed in Section 2 and calibrated in Section 3. Results are reported in section 4, discussion and robustness exercises are presented in Section 5. Section 6 concludes.

⁴ See Alesina and Drazen (1991), Persson and Svensson (1989), and Alesina and Passalacqua (2015) for a recent review of the literature.

⁵ See also discussion in Chatterjee and Eyigungor (2016 a,b). Amador (2003, 2016) also advances a sovereign debt model with present bias preferences. Gomtsyan (2017) explores related models with less attention to the first-best benchmark. Chatterjee and Eyigungor (2016b) advance a political economy setup.

⁶ Bauducco and Caprioli (2014) analyze limited commitment in an open economy optimal fiscal policy model. Balke and Ravn (2016) analysis allows for a richer set of fiscal instruments and studies how commitment affects optimal policies.

2 Model

Our economy is populated by a benevolent government (the sovereign) that borrows from a continuum of risk-neutral investors. Endowment being risky, the government may optimally choose to default on its commitments to smooth consumption. As in Aguiar and Gopinath (2006), Alfaro and Kanczuk (2005), and Arellano (2008), default is assumed to temporarily exclude the government from borrowing and the sovereign incurs additional output costs. Our environment is thus quite similar to standard sovereign debt models incorporating the assumption of quasi-hyperbolic preferences.

In more precise terms, we assume the sovereign's preferences to be given by:

$$U_t = E_t \left[u(g_t) + \beta \sum_{\tau=1}^{\infty} \delta^\tau u(g_{t+\tau}) \right] \quad (1)$$

with
$$u(g) = \frac{g^{1-\sigma} - 1}{(1-\sigma)}, \quad (2)$$

where E is the expectation operator, g denotes government consumption (or public spending), $\sigma > 0$ measures the curvature of the utility, and $\delta \in (0, 1)$ and $\beta \in (0, 1)$ are the traditional discount factor and an additional discount parameter, respectively.

Note that, the discount function being a discrete time function with values $\{1, \beta\delta, \beta\delta^2, \beta\delta^3, \dots\}$, these preferences are dynamically inconsistent in the sense that preferences at date t are inconsistent with preferences at date $(t + 1)$. This function, as Laibson (1997) explains, has the advantage of mimicking the qualitative property of the hyperbolic discount function while maintaining analytical tractability, and has been proposed, moreover, by economists and psychologists as a characterization of human behavior.

One motivation for using such preferences in our set up is that government preferences are an aggregation of citizens' preferences. As Jackson and Yariv (2014, 2015) show, with any heterogeneity in preferences, every non-dictatorial aggregation method that respects unanimity

must be time inconsistent even if citizens' preferences are time consistent. Moreover, any such method that is time separable must entail a present bias. We can thus interpret the $\beta < 1$ parameter as the bias resulting from this aggregation.

If the government chooses to repay its debt, its budget constraint is given by:

$$g_t = \tau \exp(z_t) - d_t + q_t d_{t+1}, \quad (3)$$

where d_t denotes the government debt level in period t , τ is the exogenous and constant tax rate, and z_t is the technology state that determines the output level, $\exp(z_t)$, in the present period. The debt price functions, $q(s_t, d_{t+1})$, are endogenously determined in the model and potentially depend on all the states of the economy, s_t , as well as the government's decisions.

We assume the technology state z_t can take a finite number of values and evolves over time according to a Markov transition matrix with elements $\pi(z_i, z_j)$. That is, the probability that $z_{t+1} = z_j$ given that $z_t = z_i$ is given by the matrix π element of row i and column j .

When the government chooses to default, the economy's constraint is:

$$g_t = \tau(1 - \phi) \exp(z_t) \quad (4)$$

where the parameter ϕ governs the additional loss of output in autarky, a common feature in sovereign debt models.⁷ After defaulting, the government is temporarily excluded from issuing debt. We assume θ to be the probability that it regains full access to credit markets.

Investors are risk neutral, their opportunity cost of funds given by ρ , denotes the risk-free rate. The investor action is to choose the debt price q_t , which depends on the perceived likelihood of default. For investors to be indifferent between the riskless asset and lending, it must be the case that:

$$q_t = \frac{(1 - \psi_t)}{(1 + \rho)} \quad (5)$$

⁷ To keep matters simple, we assume these output costs are a constant fraction of output. Alternative costs specifications as well the addition of elements of political uncertainty, debt maturity and renegotiation have been shown to be useful ways to increase the amount of debt in equilibrium, see discussion in Mendoza and Yue (2012).

where ψ_t is the probability of default, endogenously determined and dependent on the government incentive to repay debt.

We intentionally do not specify whether investors are international or domestic. Therefore, d_t stands for the total, domestic and international, government debt level. The assumption that investors are risk neutral implies that investors compete away their profits (and thus do not affect the government's utility). Another implicit assumption is that households' private consumption does not affect the utility they derive from public expenditures, which occurs, for example, when the two types of consumption are separable in households' preferences.

The timing of decisions is as follows. The government begins each period with debt level d_t and receives tax revenue endowment $\tau \exp(z_t)$. Taking the bond price schedule $q(s_t, d_{t+1})$ as given, the government faces two decisions, (i) whether to default, and if it decides not to default, (ii) the next level of debt, d_{t+1} .

The model described is a stochastic dynamic game played by a large agent (the government) against many small agents (the continuum of investors). We focus exclusively on the Markov perfect equilibria, in which the sovereign (government) is not committed and players act sequentially and rationally. This definition of equilibrium is identical to that of Arellano (2008) and Alfaro and Kanczuk (2009), among many others, the only difference being that it was adapted to deal with the time inconsistency problem that results from the sovereign's preferences. The quasi-hyperbolic assumption implies that the solution can nevertheless be written as a recursive problem.

Note that investors are passive and their actions can be completely described by equation (5). To write the government problem recursively, let w^G denote the value function if the government decides to maintain a good credit history (G stands for good credit history), and let w^B denote the value function if the government defaults (B stands for bad credit history) in the present period. The value of a good credit standing at the start of a period can then be defined as:

$$w = \text{Max}\{w^G, w^B\} \quad (6)$$

This indicates that the government defaults if $w^G < w^B$. The “good credit” value function w^G and policy function D^G can be written as,

$$w^G(s_t) = \text{Max}\{u(g_t) + \beta\delta Ev(s_{t+1})\} \quad (7a)$$

$$D^G(s_t) = \text{ArgMax}\{u(g_t) + \beta\delta Ev(s_{t+1})\} \quad (7b)$$

subject to (3). The “bad credit” value function w^B can be written as

$$w^B(s_t) = u(g_t) + \beta\delta[\theta Ev(s_{t+1}) + (1 - \theta)Ev^B(s_{t+1})] \quad (8)$$

subject to (4). In its turn, the “good credit” continuation value v^G is written as,

$$v^G(s_t) = u(g_t) + \delta Ev^G(s_{t+1}) \quad (9)$$

subject to (3) and to

$$d_{t+1} = D^G(s_t). \quad (10)$$

That is, it is evaluated using the policy function obtained in the good credit optimization.

The “bad credit” continuation value v^B is written as,

$$v^B(s_t) = u(g_t) + \delta[\theta Ev^G(s_{t+1}) + (1 - \theta)Ev^B(s_{t+1})] \quad (11)$$

subject to (4). And the continuation value at the start of the period (with good credit standing) is:

$$v = \text{Max}\{v^G, v^B\} \quad (12)$$

To compute the equilibrium, it is useful to define a default set of states of the economy in which the government chooses to default. The default set, in turn, determines the price q_t through expression (5). With these prices, one can solve the government problem (7) to (12). The solution for (6) determines the default set, which can be used in the next iteration.

The recursive equilibrium is defined by the set of policy functions for government asset holdings and default choice such that (i) taking the price functions as given, the government policy functions satisfy the government optimization problem, and (ii) the price of bonds is consistent with the government decisions.

To grasp some intuition, recall the government has two instruments to affect the time path of consumption: default and borrowing. In choosing default, the government is opting for a higher level of immediate consumption in exchange for being excluded from capital markets and incurring output costs. Default may thus make sense as a means of escape from a situation in which high indebtedness and low technology would result in extremely low consumption levels. Debt, the other instrument, can potentially be used (i) to smooth income fluctuations relative to the mean level of income, in the same manner as default, and (ii) to tilt the consumption profile towards the present, given that the impatience level is higher for the country than for investors.

Because, in this model, available financial contracts are state dependent, frontloading consumption is easier during high-income shocks when debt is cheaper and borrowing limits looser. The two objectives of the debt instrument thus tend to conflict. When the technology shock is good, it is cheaper to frontload consumption but also makes sense to save for rainy days. The converse is true when the technology shock is bad. The policy rule obtained by solving the model reflects which objective, to smooth consumption or frontload its profile, is quantitatively more important. Note also, that in our specification, as in models with no hyperbolic preferences, more impatience (lower β) leads to a lower equilibrium debt level and more frequent defaults.

3 Calibration

We calibrate the model to Brazilian annual data from 1955. Brazil is a relatively large and typical emerging economy. Its sovereign debt level, frequency of default and business cycle characteristics are similar to those of Mexico, for example.⁸

⁸ Aguiar et al. (2016) document that typical emerging market countries, such as Mexico and Brazil, are less volatile than Argentina which demonstrates particular outlier behavior.

We set the risk-free (international) interest rate $\rho = 0.04$ and inter-temporal substitution parameter $\sigma = 2$, as is usual in real-business-cycle research in which each period corresponds to one year. We set tax rate $\tau = 30\%$, which is the average tax burden over the period.

We calibrate the technology state z^T by considering the (logarithm of) GDP to follow an $AR(1)$ process, that is, $z_{t+1}^T = \alpha z_t^T + \varepsilon_t$, where $\varepsilon_t \approx N(0, \sigma_\varepsilon^2)$. We obtain $\alpha = 0.85$ and $\sigma_\varepsilon = 0.044$. We discretize this technology state and use the quadrature method to calculate transition probabilities. We also discretize the space state of debt sufficiently to avoid affecting the decision rules.

We set the probability of redemption at $\theta = 0.2$, which implies an average stay in autarky of five years, which is in line with estimates by Reinhart and Rogoff (2009) for Brazil. Direct output costs, modeled from default episodes, equal $\phi = 10\%$.

We calibrate $\delta = 0.90$ using the Brazilian average real interest rate. The fact of impatience being higher for the country than for investors is motivated by the fact of growth being higher in emerging than in developed markets. That poorer countries should catch up with richer ones provides the incentive to frontload consumption.

Debt maturity in Brazil changed a lot over the last decades. During the inflation years, debt maturity was under 3 months. Recently it is more than 3 years. In the model, each period corresponds to one year, which is the maturity assumed. This is probably not very different from the average maturity over the calibration period (complete data to calculate it is not available).

Finally, to obtain reasonable levels of debt in equilibrium we set the additional inter-temporal factor related to government present bias at $\beta = 0.70$ as in Angeletos et al., (2001).

Table 1 summarizes the parameter values.

4 Simulation Results

As our analysis is normative, we proceed by studying welfare under several policy rules. We start with a no rule benchmark case and compare against the optimal fiscal rule. We compare these cases against the rules typically applied: fiscal rules and debt rules.

4.1 No Rule

We first consider the case in which the government is not subject to any fiscal rules. Figures 1a and 1b represent the policy functions obtained by solving the model and depict, respectively, the default decision and choice of next period debt level contingent on not defaulting. The path denoted “high technology” refers to a good shock, or “boom,” while that denoted “low technology”, refers to a bad shock, or “recession.”

Consider a sovereign’s choice to default, and compare expressions (3) and (4). On one hand, default implies an instantaneous reduction in the cost of rolling debt. This means higher consumption, especially when the debt service is large. On the other hand, default implies lower output, and a reduction in the possibility of smoothing consumption in future periods. This is the case because the sovereign loses access to credit markets, and is forced to have zero debt. Default becomes attractive then when debt is high and the technology shock is low, to avoid extremely low consumption as debt becomes more expensive.

Figure 1a shows when the technology shock is high, the government defaults when the debt level exceeds 70% of GDP. The maximum amount of sustainable debt when the technology shock is low is 58% of GDP. Default, if used as a means to smooth consumption, is thus more likely the lower the output levels.

Figure 1b shows the next period debt level to be higher the higher the technology state and current period debt. The positive relationship between consecutive debt levels was anticipated

because, for a given technology shock, the government would attempt to avoid sharp changes in the debt level that would imply higher consumption volatility.

The relationship between technology and debt, a fairly surprising but previously obtained result (Arellano, 2008; Kanczuk and Alfaro, 2009), implies that the government does not use debt to smooth consumption, a departure from the “pure” Eaton and Gersovitz (1981) framework, but rather primarily to frontload consumption given that the discount factor δ is lower than the inverse of the risk-free interest rate. Consumption smoothing is mostly achieved, as noted earlier, through default, as in contingent debt service models such as that of Grossman and Van Huyck (1988).

Calculating the invariant distribution of the states, we determine the government to be excluded from the market 3.2% of the time and average debt to be 60.1% of output, as reported in the first line of Table 3. These results are broadly consistent with the stylized facts. Notice that by assuming hyperbolic preferences on the part of the government, the model can reproduce the level of debt and frequency of default typical of emerging markets, even if the household impatience parameter is calibrated to local interest rates.

In Table 2 we report the business cycle statistics of the model and compare them with the Brazilian data. The model matches the data in that it simultaneously delivers a higher volatility of government consumption relative to income, countercyclical interest spreads, and countercyclical debt changes. These results suggest the model is a good measurement instrument to perform the normative exercises we are interested in.

4.2 Optimal Fiscal Rule

We next consider the case of the optimal fiscal rule. This benchmark corresponds to what Amador, Werning and Angeletos (2006) denominate the “first-best allocation”, and Halac and Yared (2014) denominate “ex-ante optimal rule”. In our setup, this corresponds to the case of $\beta = 1$, if the rule was implemented last period, just after the government expenditure. Our analysis

contrasts with these papers in that there is no private information about the technology shocks. In the latter, the optimal rule has to balance discretion with commitment, and is shown to be history dependent, as it provides dynamic incentives. In our simpler case, given the absence of private information, the first best policy can be implemented with full commitment, that is, by giving the government a pre-determined contingent path of efficient consumption.

The invariant distribution properties of the economy under the optimal fiscal rule are reported in the second row of Table 3. Under the optimal rule, the invariant distribution displays no default, and thus no spreads, and the average debt level drops to 50.2% of GDP. In other words, the government present bias is responsible for debt over-accumulation of about 10% of GDP and the occurrence of default episodes.

Table 3 also compares, in terms of consumption, the welfare level of the economy under the optimal rule and no rule. Considering, for both economies, the transition path from a starting point with no debt to their respective invariant distribution, we obtain that adoption of the optimal rule results in welfare gains of 0.277% of GDP, substantial compared to typical business cycle welfare gains.

Figures 2a and 2b depict, respectively, the government's default decision and choice of next period debt level contingent on not defaulting. Figure 2a looks like Figure 1a, but has different threshold axis values. As with no rule, default is more likely the lower the output levels. The maximum amount of sustainable debt is 63% of GDP under a high, and 50% of GDP under a low, technology shock. Although there is no default in equilibrium, the threshold values for default are lower under the optimal than under no rule.

Comparing Figure 2b with Figure 1b reveals the optimal policy to also have similar qualitative properties to the solution with no fiscal rule. Again, contrary to the usual intuition, debt accumulation does not increase when the economy is hit by a bad shock. In other words, whereas default is (potentially) used to smooth consumption, debt is used to tilt the consumption profile,

and this holds regardless of the fact that default does not occur in equilibrium, at least after the economy has converged to its invariant distribution.

Figure 2b shows that when the debt level is relatively low, the economy saves the same regardless of the output shock. When the debt level is high, however, the economy borrows more in booms than in recessions because of the countercyclical interest rate schedules. In other words, when debt (and the incentive to default) is greater, the borrower would like to borrow heavily during bad shocks, but cannot because such financial contracts are too expensive. Consequently, when debt is large the optimal fiscal policy is pro-cyclical.

4.3 Debt Rules

We now consider the case of simple debt rules under the hypothesis that the government is present biased ($\beta = 0.70$). These rules prohibit the government from choosing debt levels above a previously set threshold. We solve for many thresholds and report the corresponding invariant distribution properties in Table 3.

Note that when the debt threshold is set to 65% or 60% of GDP, the average debt level is lower than under the no rule case but smaller than the threshold. Thus, the threshold is constraining debt accumulation, but the government still has some margin. Additionally, for these threshold levels the frequency of default is as high as under no rule.

When the debt threshold is close to 50% of GDP or lower, the invariant distribution debt level is equal to the threshold. Debt accumulation becomes, in fact, binding all the time and there is no longer default in equilibrium (after converging to the invariant distribution).

Figure 3 depicts, for the case in which output is neither high nor low (i.e., $z = 0$), the policy function for the cases with no rule, with the optimal fiscal rule, and with a simple rule with threshold $d \leq 50\%$ of GDP. Note that the simple rule implies an average debt level similar to the

optimal rule, but the constraint affects consumption smoothing, making the government accumulate the same level of debt regardless of its previous indebtedness.

Welfare gains vary widely depending on the debt threshold. When the threshold is too high or too low, the simple rule incurs substantial welfare losses even relative to no rule. In the case of very high threshold levels, the rule does not prevent default but limits consumption smoothing. When the threshold levels are too low, the costs of preventing the sovereign to frontload consumption outweigh any gain from preventing defaults. More interesting, when the threshold is set to 50%, welfare gain is virtually equal to that of the optimal rule. That is, a very simple rule can yield gains comparable to a fairly complex one.

We believe this finding may have relevant policy implications. Threshold debt rules, being easily contractible and yielding welfare gains virtually equal to those afforded by the optimal fiscal rule, would seem to be the better alternative for implementation in practice.

4.4 Deficit Rules

Lastly, we consider the case of simple deficit rules, again under the hypothesis that the government is present biased ($\beta = 0.70$). These rules prohibit the government from choosing deficit levels (or changes in debt, $\Delta d \equiv d_{t+1} - d_t$) above a previously set threshold.⁹ We solve for many thresholds and report the corresponding invariant distribution properties in Table 4.

Note that as the maximum amount of deficit is reduced, and the constraint becomes binding, we observe fewer defaults and higher amounts of debt. It is unexpected, but promising, that the deficit constraint can generate high amounts of debt, and thus more frontloading, without incurring the costs of default. But examination of the welfare implications shows the arrangement to not be working as expected.

⁹ In the case of Brazil, the fiscal rule approved in December 2016 is actually a transition rule to reign in fiscal expenditures. It fixes government expenditure in real terms for twenty years. As GDP grows, and revenues increase, this fiscal rule will generate an increasingly large fiscal surplus and assets, and the equilibrium will be ill defined.

Figure 4 presents the debt policy (d_{t+1} as a function of d_t) for the deficit policy ($\Delta d < 2\%$) compared with the optimal policy (always when technology is neutral, i.e., $z = 0$). As expected, in the relevant region the deficit rule results in a policy very close to the 45-degree line, that is, debt is restricted to very slow change.

To illustrate the practical implication of the deficit rule, Figure 5 displays the consumption profile for an economy that begins with zero debt in the beginning of time and always faces neutral technology shocks ($z = 0$). Recall that zero debt is the case after a sovereign defaults, and is also the initial condition adopted in our welfare calculations.

Under the optimal policy, the household frontloads consumption, which starts at a much higher level and converges to its steady state in about six years. Under the deficit rule, the government, restricted to increasing debt by small amounts, cannot increase initial consumption and convergence to the steady state is much slower. In other words, the government foregoes the benefits from frontloading consumption.

An indirect effect of the deficit rule is that the potential benefits of default are greatly reduced. Default enables a country to shed debt and thus increase consumption. But under a deficit rule, under which consumption can increase only slowly, little utility is derived from default. The government consequently resists default even for large amounts of debt, thus creating the apparent contradiction of being able to hold large amounts of debt without defaulting but, achieving lower welfare levels.

5 Discussion and Robustness Exercises

5.1 Risk Aversion, Distortionary Taxes, and Fiscal Pro-Cyclicality

A surprising result of our simulations is that optimal fiscal policy is not countercyclical. We are used to the idea that because tax distortion costs are convex, debt should fluctuate in order to

keep tax rates constant (see Barro, 1979). However, as discussed, this logic does not hold in our calibrated model.

In principle, our simple endowment economy already has the ingredients that should make countercyclical fiscal policy optimal. That is, modeling production, investment, and further tax distortions would not imply more tax smoothing than our simpler model does.

On the revenue side, our model assumes output is stochastically endowed and considers the extreme case in which tax rates are constant. Suppose instead the government could choose the tax rate (to reduce distortions in output production). Then, the maximum counter-cyclicality would be obtained when tax rates are set constant, which is already the case in our model, by assumption.

On the expenditure side, rather than assumed to be constant, consumption is chosen to maximize utility. Because preferences are concave, the government also has incentives to use debt to smooth consumption. In other words, the government is motivated to hold expenditures constant and thus implement a countercyclical fiscal policy. Our calibrated results indicate, however, that this motive is dominated by other motivations, in particular, under the optimal fiscal rule, to use debt to frontload rather than smooth consumption.

To investigate how quantitatively robust this result is we modify the model to increase the gains from consumption smoothing. A simple way to do so is to modify the calibration by increasing the risk aversion parameter σ . Because this parameter also plays the role of inter-temporal elasticity, it controls the benefits of smoothing consumption.

Figure 6 displays the correlation between (the logarithm of) output and government savings (the change in debt) for various risk aversion parameters. Note that for σ less than or equal to 4, the model displays a pro-cyclical fiscal policy. When $\sigma = 5$, fiscal policy is completely a-cyclical. For higher values of σ , when the benefits from consumption smoothing become larger, fiscal policy turns countercyclical. These calibrations, however, contradict much evidence about consumer preferences.

We take this result as an indication that, from a quantitative perspective, the gains from frontloading consumption seem to be more important than those from tax smoothing for a wide range of economies.

5.2 Dynamics and Initial Debt

Note that we obtained the quantitative results under the assumption that the initial debt level is zero, which is the natural benchmark. But the qualitative result is a general one. For all initial debt levels lower than the maximum sustainable debt, the sovereign would gain from frontloading consumption and quickly increasing its debt holdings. If initial debt level is higher than its maximum sustainable level, the sovereign would default in the first period, debt would be equal to zero in the second period, and the same analysis would apply.

Our case, Brazil, is in the process of discussing a national fiscal rule, but does not currently have one operating. The best known fiscal rule is the Euro area Stability and Growth Pact. A common criticism is that there was no adherence to the rule, because the prescribed punishments were not credible. However, even though many countries' debt holdings are above the 60% of GDP limit, few have actually defaulted. The most emblematic example is Germany, which held a debt level equal to 81 % of GDP in 2010, and faced no apparent risk of default. In our framework, this is an indication that the maximum sustainable debt for Germany must be well above 80%. (In fact, the 60% limit was based on the median public debt levels at the time of the framing of the Maastricht Treaty. It was not considered the optimal or the maximum sustainable debt level.)¹⁰ Thus, according to our framework, the Euro area debt limit was set too low. Many countries would benefit from having more debt and frontloading consumption without incurring defaults.

¹⁰ Countries such as Greece and Italy joined the Euro with initial debt levels well beyond the limit.

5.3 Private Information

As noted above, our framework has not incorporated private information about technology shocks. In other words, we assumed households to know as much as the government about the state of the economy (and contracts thus can be written contingent on shocks). Although this is the natural hypothesis to entertain, there are possible rationales for considering the case of private information whereby the government is better informed than households. One is that private information is an indirect way to capture the fact that some fiscal contingencies are not easily contractible and a value for flexibility.

How would our results change if only the government observed the technology shock? The optimal policy would be much more complex, as shown by Halac and Yared (2014), but would yield welfare gains (to households) less than or equal to those obtained under the optimal rule with complete information. More precisely, the welfare gains of the private information optimal rule lie somewhere between the welfare gains under the public information optimal rule and simple debt threshold rules.

That welfare gains under the simple debt rule are almost identical to those under the optimal rule, as highlighted in our analysis, is a good reason for using the simple threshold rule, given that it is easily contractible. One can conclude that the same policy implication holds when there is private information.

5.4 Self-Interested (Non-Benevolent) Government

The analysis in the previous sections hypothesized a benevolent government with quasi-hyperbolic preferences that motivated adoption of fiscal rules. A vast literature in political economy has analyzed the over-accumulation of debt by non-benevolent, or self-interested, governments.

A simple way to capture this possibility in our framework is to set $\beta = 1$ and calibrate δ^{gov} such that our artificial economy displays observed debt levels consistently. That is, by supposing that government preferences are not time inconsistent, but differ from those of the country's citizens, who are less impatient.

Setting $\delta^{gov} = 0.80$, the invariant distribution displays debt equal to 62% of GDP and projects default to occur 3.2% of the time, which are broadly consistent with both the stylized facts and our basic calibration. As expected, the policy functions are also quite similar.

If the government discount factor is $\delta^{gov} = 0.80$ and the citizens' discount factor is $\delta = 0.90$, the same policy implications as before would apply. The optimal rule and results of simple threshold rules would also be the same as before.

An important omission from the analysis is why the optimal rule (or threshold rules) would be applied. The self-interested government would certainly oppose adoption of such rules. Although we do not analyze how, citizens need to somehow, perhaps by means of elections, discipline politicians and force them to adopt such rules.

6 Conclusion

Emerging countries, as they catch up to developed ones, can borrow in order to frontload their consumption profile. In practice, however, they tend to over borrow and often resort to defaulting on their debt. Governments' preferences, for reasons of political economy or aggregation of heterogeneous citizens' preferences, may display a present bias.

Fiscal rules are a potentially useful commitment technology to solve this problem. The continued approval of fiscal rules, as documented by the IMF, whether debt or deficit related, is suggestive of a desire to constrain a sovereign's incentives to overspend. In the context of a traditional model of sovereign debt and default, we analyze the welfare gains from alternative fiscal rules. We find the gains from the optimal fiscal rule to be economically relevant, and observe

that the optimal fiscal rule does not entail pro-cyclical fiscal policy. Although beyond the scope of the paper, the question remains whether fiscal or debt rules are easier to enforce. However, we find that a simple, easily contractible threshold rule can generate gains virtually as high as the optimal rule.

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Table 1: Calibration

Technology autocorrelation	$\alpha = 0.85$
Technology standard deviation	$\sigma_\varepsilon = 0.044$
Probability of redemption	$\theta = 0.20$
Output costs	$\phi = 0.10$
Risk aversion	$\sigma = 2$
Risk free interest rate	$\rho = 0.04$
Tax rate	$\tau = 0.30$
Discount factor	$\delta = 0.90$
Hyperbolic discount factor	$\beta = 0.70$

Table 2: Descriptive Statistics

	Data	No Rule
Std. Dev. (y)	10.4	10.2
Std. Dev. (g)	19.3	16.8
Std. Dev. (spread)	76.6	82.3
Std.Dev. ($\Delta d/d$)	58.6	39.5
Corr (g, y)	0.84	0.74
Corr (spread, y)	-0.22	-0.09
Corr ($\Delta d/d$, y)	-0.27	-0.12

Table 3: Invariant Distributions for Alternative Fiscal Rules

Model Specification	Exclusion from Market (% time)	Debt if not excluded (% GDP)	Welfare (% GDP)
No Rule	3.2	60.1	0
Optimal Rule	0	50.2	0.277
Rule $d \leq 65\%$	3.2	59.8	-0.214
Rule $d \leq 60\%$	3.2	57.8	-0.163
Rule $d \leq 55\%$	0.4	55.0	0.259
Rule $d \leq 50\%$	0.0	50.0	0.276
Rule $d \leq 45\%$	0.0	45.0	0.275
Rule $d \leq 40\%$	0.0	40.0	0.212
Rule $d \leq 35\%$	0.0	35.0	0.129
Rule $d \leq 30\%$	0.0	30.0	0.024
Rule $d \leq 20\%$	0.0	20.0	-0.257
Rule $d \leq 10\%$	0	10.0	-0.656
Rule $d \leq 0$	0	0	-1.141

Table 4: Invariant Distributions for Deficit Rules

Model Specification	Exclusion from Market (% time)	Debt if not excluded (% GDP)	Welfare (% GDP)
No Rule	3.2	60.1	0
Rule $\Delta d \leq 20\%$	3.2	56.8	-0.184
Rule $\Delta d \leq 10\%$	3.2	57.4	-0.511
Rule $\Delta d \leq 5\%$	3.2	61.3	-0.946
Rule $\Delta d \leq 4\%$	2.1	63.5	-1.049
Rule $\Delta d \leq 3\%$	0	74.3	-1.056
Rule $\Delta d \leq 2\%$	0	75.0	-1.097
Rule $\Delta d \leq 1\%$	0	75.0	-1.135

Figure 1a: Default Policy Function for No Rule Economy

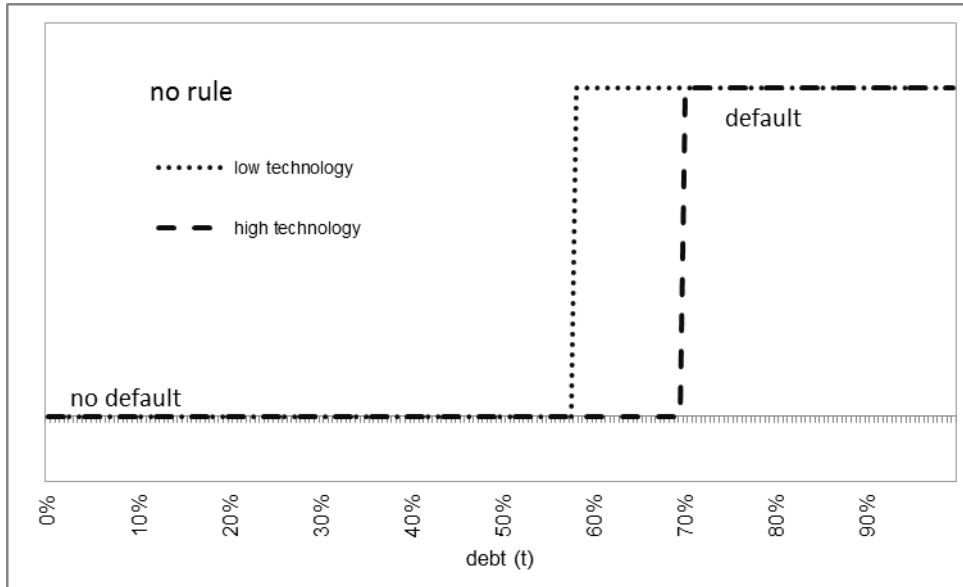


Figure 1b: Debt Policy Function for No Rule Economy

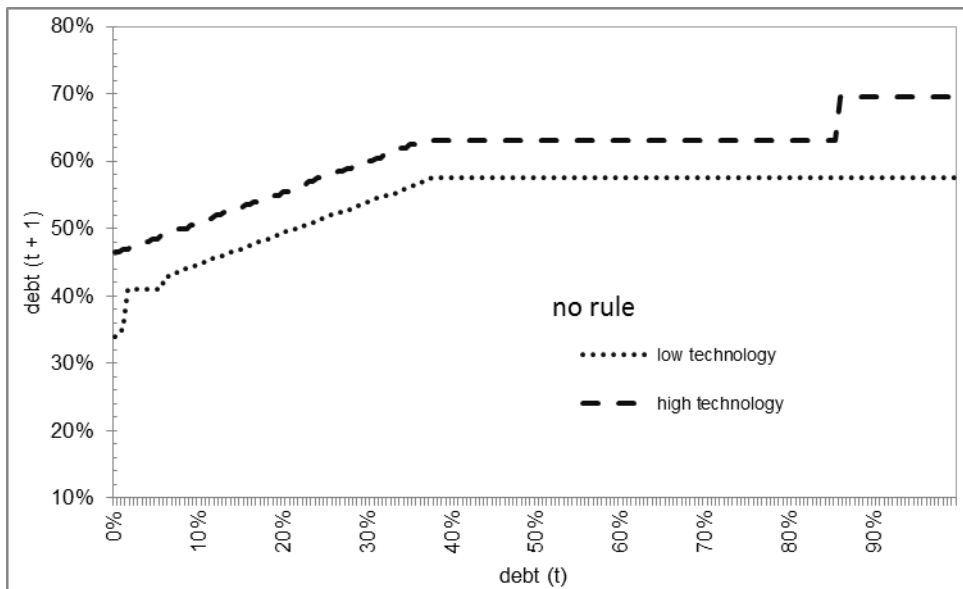


Figure 2a: Default Policy Function for Optimal Rule

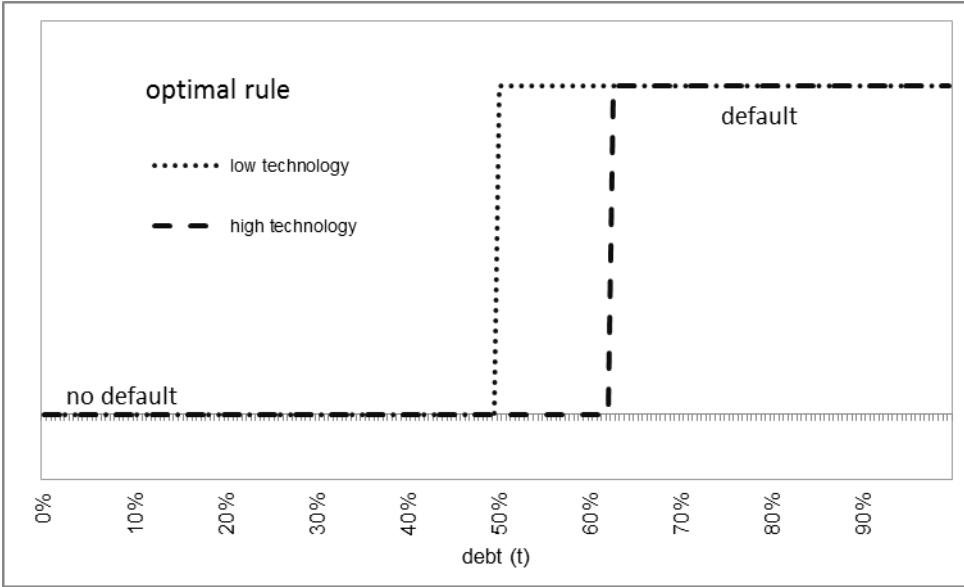


Figure 2b: Debt Policy Function for Optimal Rule

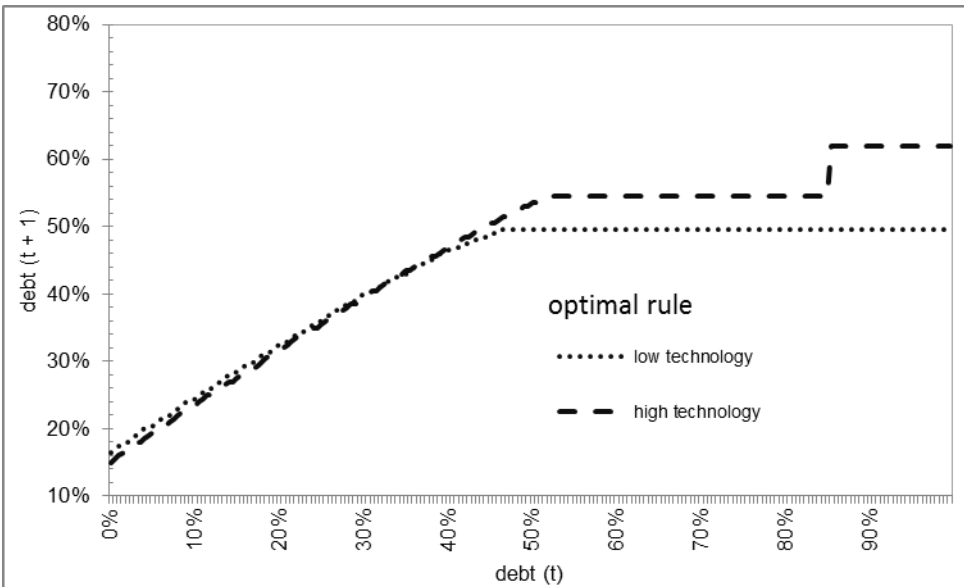


Figure 3: Debt Policy Function for Different Fiscal Rules ($z = 0$)

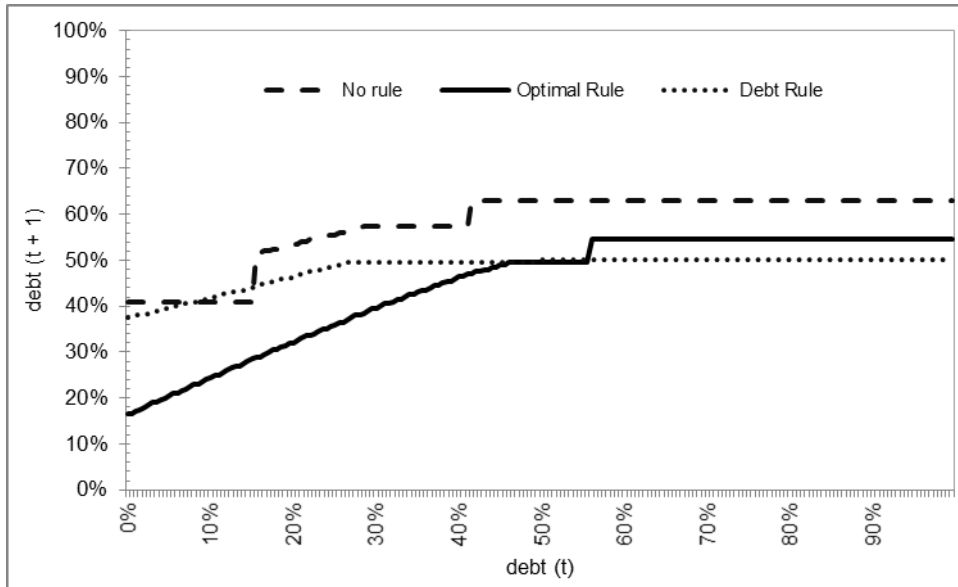


Figure 4: Debt Policy Function for Deficit Rule ($\Delta d < 2\%$, $z = 0$)

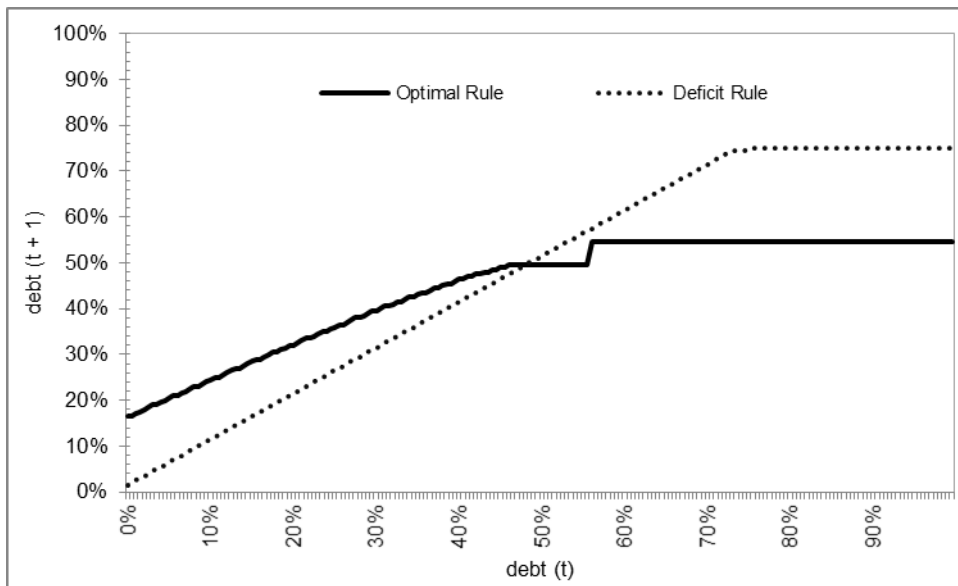


Figure 5: Consumption Profile, Optimal Rule, and Deficit Rule ($\Delta d < 2\%$, $z = 0$)

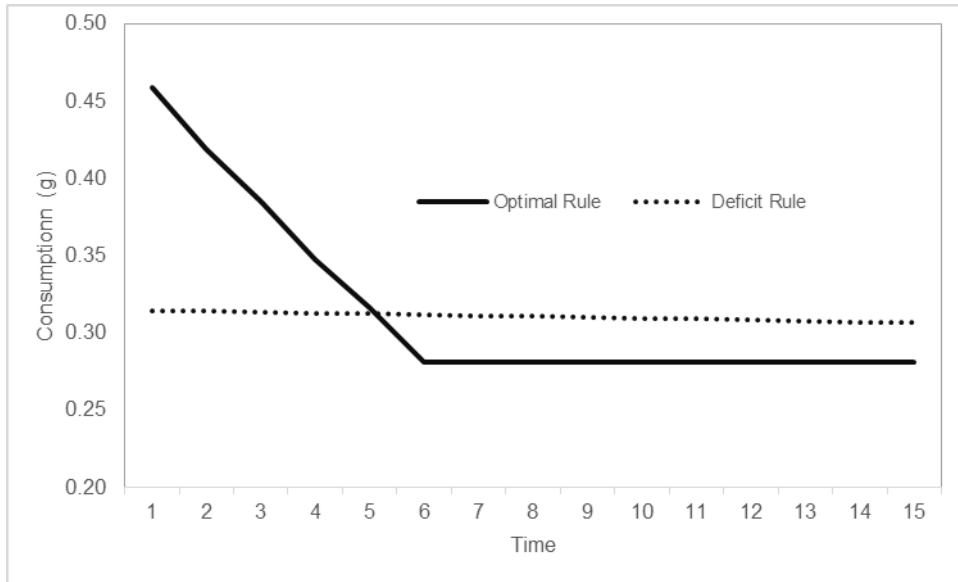


Figure 6: Fiscal Policy Counter-Cyclicality against Risk Aversion

