Non-Linearities and Fiscal Policy

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Abstract

Empirical evidence shows that fiscal multipliers depend on the state of the cycle, the nature of fiscal policy and the level of debt. In other words, evidence points to non-linearities in the effects of fiscal policy. This paper provides a framework to simultaneously assess the relevance of different sources of non-linearities. The empirical analysis, which uses a panel of 13 countries between 1980 and 2014, finds that fiscal consolidations based on tax increases are in general self-defeating, in that they result in an increase of the debt-to-GDP ratio. Increasing taxes in periods of expansion has the most recessionary effect on the economy. Cutting public expenditure has a less pronounced effect on economic activity and can stabilize debt. This paper also discusses the econometrics of non-linearities. Though the literature has often adopted the local projections approach to derive impulse response functions, I address the potential pitfalls of this method both analytically and econometrically.

Keywords: Non-linearities, Debt, Recessions, Expansions, Fiscal consolidations, Narrative approach, Interacted-STVAR, Local Projections

JEL Codes: C33, E62, H60, H63

1 Introduction

The Global Financial Crisis and the Great Recession have triggered a renewed interest in the effects of fiscal policy. At the same time, discussions have increased regarding countries with high public debt under fiscal austerity programs. One of the main topics of policy debates relates to the implications that these fiscal austerity measures have for the macroeconomy, especially when a country is in an economic downturn. This paper assesses the role of the level of *public debt* in determining the size of fiscal multipliers, in a model that also accounts for the *state of the cycle* and the *type of fiscal stabilization* when mostly based on tax increases or on spending cuts.

Until recently, most of the literature estimated a single fiscal multiplier without taking into account the state of the economy. Recent work (Auerbach and Gorodnichenko (2012, 2013), Ramey and Zubairy (2016)) investigates whether estimates of fiscal multipliers differ depending on the state of the business cycle. However, there are few theoretical studies that link the size of the multiplier to economic downturns. A few exceptions are Michaillat (2014) on the labor market and Canzoneri, Collard, Dellas and Diba (2016), who make use of a costly financial intermediation as a financial friction and show that fiscal multipliers are state-dependent. Some New Keynesian models, such as Christiano, Eichenbaum, and Rebelo (2011) and Woodford (2010), highlight the importance of the phase of the business cycle and study the multiplier effect when the economy is near the zero lower bound.

Alesina, Azzalini, Favero, Giavazzi and Miano (2016) shift the focus from the state of the cycle, arguing that it is not simply "when" a fiscal adjustment happens that matters (i.e. recessions versus expansions), but also "how" it happens (i.e. expenditure-based versus tax-based). Accounting for heterogeneous effects is critical, since the size of the multiplier can be more relevant in circumstances of economic downturns, even more so when a country has a high level of debt.

The main goal of this paper is to simultaneously study these non-linearities and contribute to the ongoing debate about the time-varying effect of fiscal shocks on the macroeconomy. Therefore, we propose a general model that provides enough flexibility to account for different non-linearities. In this paper, I focus on three non-linearities. The first arises from the state-of the economy, i.e. *recessions versus expansions*. The second is the composition of the fiscal consolidation, i.e. *tax-based versus expenditure-based*. The third non-linearity arises from the government's fiscal position, i.e. *high-debt ratio versus low-debt ratio.*

The recent literature on state-dependent multipliers (e.g. Auerbach and Gorodnichenko (2012) (AG in what follows)) uses the smooth transition vector autoregression (STVAR) model.[1] This is a regime-switching model, based on a logistic distribution

[1] In AG (2012) the identification of exogenous shifts in fiscal variables is obtained using the Blanchard-Perotti identification assumptions.

that controls for the transition from one regime to the other, with weights computed as a moving average of GDP growth. Nickel and Tudyka (2014) estimate fiscal multipliers taking into account countries' fiscal positions. They augment a panel vector autoregression model and introduce debt through an interaction term, the interacted panel vector autoregression. I propose a blend of these two models (STVAR and IPVAR) the ISTVAR. This model is a flexible way to condition endogenously on countries' government debt and examine the relevance of the instrument of stabilization that the government uses, taking into account the state of the economy. Differently from Nickel and Tudyka (2014), I endogenously track the dynamics of debt by explicitly modelling the evolution of the government debt-to-GDP as a function of the interest rate payments on the debt and the primary government deficit (Favero and Giavazzi (2007)). Therefore, I contribute to the state-dependent literature of fiscal policy not only by looking into the stabilization instrument that the fiscal authority uses, but by including debt itself.

I study cases of high and low debt as well as the potential of heterogeneous effects of tax- and expenditure-based fiscal consolidations implemented in bad and good times. My results indicate that there are heterogeneous implications at different stages of the business cycle when I condition on debt and account for the composition (expenditure-based, tax-based) of fiscal adjustments. The question is which heterogeneity is more relevant: Is it the *"how"* — the way the stabilization is performed? Is it the *"when"* —the phase of the cycle? Or is it the *"initial condition"* of the economy —namely, debt— that plays the most pivotal role and potentially constrains fiscal policy under all the different dimensions?

My findings indicate that, first of all, the initial level of debt and the composition of the fiscal adjustment are the most relevant non-linearities. Second, I find that the effects of tax-based adjustments are on average more recessionary when they are implemented during an economic upturn. The reason is that, when the economy is in recession, the probability of being in a recessionary regime is already close to 1. When debt is high, tax-based consolidations appear to be self-defeating. Instead of reducing the deficit, they deliver on average higher debt ratios because the negative effect on GDP growth is larger from the budget changes. Third, expenditure cuts, have a less pronounced effect on output. But, importantly, they are able to stabilize debt independently of the state of the cycle.

Existing empirical evidence (e.g. Favero and Giavazzi (2007)) shows that omitting debt can bias the evaluation of the output effects of fiscal policy. In my context, when the state of public finances is weak, this triggers a fiscal consolidation. On the one hand, this consolidation episode improves the primary balance of the government. On the other hand, it has a negative effect on the output growth. In addition, in future periods, this adjustment may constrain the future path of taxes and spending, since the government's budget constraint should eventually be respected. Including debt allows me to account precisely for these different channels and observe whether the fiscal authority succeeds to meet its objective (i.e. to shore up fiscal sustainability) depending upon the instrument of stabilization, the initial condition of debt and the state of the economy.

The second contribution of this paper is a discussion of the *econometrics* of non-linearities in fiscal policy. Most of the recent fiscal policy literature adopts the Jordà (2005) local projections approach to derive impulse response functions (IRFs). The main advantage of the local projections method is that it is a very simple, non-parametric way to estimate impulse responses. In addition, it is robust to misspecifications. It is also argued that this approach is flexible enough to accommodate non-linear specifications. In this paper, I argue that the way the fiscal policy literature uses this method to account for non-linearities may be problematic. When non-linearities are modelled through a logistic function, there is an endogenous feedback in the system, which makes the model history-dependent.[2] Therefore, the derivation of IRFs from local projections is not simply an extension of the linear case, which is history-independent. I discuss the potential pitfalls generated by the way this estimation method has been used in the presence of non-linearities. I do so both econometrically and analytically.

Econometrically, I use the STVAR model. Unlike AG (2013), I do not use the local projections approach.[3] Instead, I estimate a STVAR and use generalized impulse response functions, which allow for the endogenous transition of the economy from one state to the other. When AG derive their IRFs for each state, they assume that the economy always remains in the same state. I show evidence that the dynamics of the IRFs from the STVAR approach are different compared to the local projections approach. I run a Monte Carlo experiment to support my argument. Analytically, I prove, within a simple non-linear model, that the two approaches are not equivalent.

I also conduct out-of-sample simulations and evaluate the performance of the IRFs derived from my model. This allows me to compare the simulated to the actual effects of new episodes for the years 2009-2013, which were years of crisis. More precisely, I use data up to 2008 to estimate my models, and then I simulate them out-of-sample over the years 2009-2013. In this way, I am able to compare the actual realization of GDP growth to simulated outcomes, conditioning on the actual fiscal changes that were adopted. The out-of-sample simulations of my STVAR baseline model perform rather well.

To sum up, this paper contributes to the literature in two ways: First, I study relevant type non-linearities in a general regime-switching model that allows responses to vary with the state of the cycle, the composition of the fiscal adjustment and the

[2] Not just a logistic function, but more in general a function of an endogenous variable.

[3] I mainly refer to Auerbach and Gorodnichenko (2013) who were the first to make use of this approach in fiscal policy. They claim that the IRFs calculated with the local projections method are equivalent to the ones derived from the conventional approach (STVAR). Going beyond this literature, this methodology has been applied recently also in the study of state-dependent monetary policy.

degree of indebtness.[4] Second, I discuss the use of local projections in the state-dependent literature.

The paper is structured as follows: In the next section of the paper, I motivate my research and provide a literature review. In Section 3 I present a general model which portends what follows. Section 4 presents the data and discusses the three types of non-linearities. The proposed model specification is introduced in Section 5. The results are included in Section 6. Section 7, provides the discussion related to the derivation of the impulse response functions from the different methodologies together with the Monte Carlo experiment. Section 8 provides some robustness checks. Section 9 concludes.

2 Related Literature

The size of the fiscal multiplier has been broadly studied in the theoretical and empirical literature. Quantification is controversial. Answers vary and depend, to some extent, on the methodology, the nature of the shock, the identification scheme and the data.

There are two main strands of identification in the empirical literature: the structural VAR approach (e.g. Blanchard and Perotti (2002), Mountford and Uhlig (2009)) and the narrative approach (e.g. Ramey and Shapiro (1998), Romer and Romer (2010)). An issue of the standard structural identification (SVAR) approach is the problem of non-fundamentalness in the estimation of fiscal shocks since this approach relies on current and past shocks. This problem is related to the fiscal foresight phenomenon. In this approach the information of news shocks and their future changes is not embedded. Therefore, the sets of information held by the econometrician and economic agents are not aligned. On the other hand, the narrative approach allows for the direct identification of exogenous shocks from past budget accounts, fiscal changes that were announced as a response to past economic conditions. For this reason, the narrative approach provides a way to take into account the information that influences the expectations of economic agents.

Devries, Guajardo, Leigh, and Pescatori (2011) constructed an important dataset for 17 OECD countries for the period 1978-2009 of tax and spending changes, similar to the Romer and Romer (2010) approach. Their narrative record includes contemporaneous changes with the aim to reduce the budget deficit. These changes are considered exogenous because they are measures taken as a response to past economic conditions

[4] My paper focuses on contractionary shocks, I do not study cases of fiscal stimulus. As Barnichon and Matthes (2015) point out, contractionary multipliers are different from expansionary multipliers, hence the sign matters.

and not to prospective ones. Guajardo, Leigh, and Pescatori (2014) study the effects of these unanticipated narratively identified shocks on macroeconomic variables. Alesina, Favero and Giavazzi (2015) extend the Devries, Guajardo, Leigh, and Pescatori (2011) dataset and distinguish between expected and unexpected fiscal corrections. This allows them to study the impact of unanticipated and anticipated fiscal changes on the macroeconomy.

Until recently the literature, as the aforementioned papers, had focused on the effect of a single multiplier, and had not distinguished between the phases of the business cycle. Recent studies have relaxed the assumption of a homogeneous multiplier across different states of the economy and seek to study non-linearities of the multiplier in different regimes. AG (2012) employed a STVAR model to study the size of the fiscal multiplier in recessions and expansions in an SVAR (Blanchard and Perotti (2002)) identification context for the US economy. AG (2013) employ a similar methodology by using narrative data of OECD countries. They estimate their model with the local projections method (Jordà (2005)).[5] They discuss a simple linear model and show that the IRFs that one derives from the conventional approach are equivalent to the IRFs that one can get from the local projections method. They compute the local projections for each horizon as a separate regression. They claim that this permits them to construct IRF and accommodate non-linearities without imposing dynamic restrictions as is the case in other regime switching models. This is what motivates the methodological part of my contribution, in which I discuss more in depth the way AG use local projections. The conclusion in both papers (AG (2012, 2013)) is that the multipliers in different regimes differ. Ramey and Zubairy (2014, 2016) also adopt the local projections method in a state-dependent model to examine the possibility of a different response of government spending changes in periods of recessions compared to expansions. Ramey and Zubairys' results show that there is no evidence of heterogeneity. They both focus on government spending shocks. However, it is important to mention that while the state variable for Auerbach and Gorodnichenko is a function of an endogenous variable (moving average of GDP), for Ramey and Zubairy their state is a dummy variable (which is 1 if the unemployment rate is above a specific threshold, i.e. 6.5%). The importance of this point will be clear when we will discuss the use of the local projections method.

The above seminal papers have motivated many recent studies in this literature. Barnichon and Matthes (2016), for example, study whether the sign of a government spending shock matters. They find that an expansionary multiplier is below one, while the contractionary is above 1. Our paper contributes to the empirical literature of the study of fiscal multipliers by examining the potential for heterogeneous effects when one considers three dimensions of non-linearities related to "how", the "when" and the "initial condition" that an episode of fiscal adjustment is implemented. Several studies

[5] I will provide a brief review of the paper and the methodology in Section 7.

(e.g. Favero and Giavazzi (2007), Ilzetzki, Mendoza and Vegh (2013), Corsetti, Meier and Müller (2013)) have highlighted the importance of the government's fiscal position of a country. Omitting this dimension can bias the effects of fiscal shocks. Nickel and Tudyka (2014) estimate fiscal multipliers and take into account the countries' fiscal position. They use an interacted panel vector autoregression in a sample of European countries. Huidrom, Kose, Lim and Ohnsorge (2016) is a very recent paper that jointly accounts for the fiscal position and the business cycle, as we do. The main differences rely on the fact that I include debt endogenously in my model, and on the composition of the stabilization episodes. They use a Blanchard and Perotti (2002) identification scheme.

I use the idea of the IPVAR model into the STVAR model, which I call ISTVAR. The implementation of this model allows me to study the three different type of asymmetries (i.e. recessions and expansions, expenditure-based and tax-based, high-debt and low-debt) simultaneously. I track endogenously the dynamics of public debt-to-gdp ratio by explicitly including an equation of debt. I follow the paper of Favero and Giavazzi (2007) and model debt as a function of the average cost of debt and the primary deficit.

3 The General Model

My goal in this paper is to provide a general encompassing framework to simultaneously assess the relevance of different sources of non-linearities. To study non-linearities in the effects of fiscal policy I need a dynamic model which can account for:

1. The behavior of the macroeconomic variables of interest (Y_t) and
2. the behavior of the policy variables under study (P_t).

The macroeconomic variables (e.g. real gdp growth, consumption, etc.) are typically assumed to be a function of both their own past values, the past values of the policy variables and any exogenous adjustments or deviations of the fiscal authority from its rule. These functions can potentially be non-linear and depend on different economic conditions.

At the same time, the policy variables respond to the change of the fiscal authority rule, as well as through a potential feedback effect from the past policy decisions together with the effect arising from the response of the macroeconomic variables. A general framework that can describe the joint evolution of the two sets of variables is:

$$Y_t = f_1\left(Y_{t-1}, P_{t-1}, shock_t; \Phi_1\right) + u_{1t} \tag{1}$$

$$P_t = f_2\left(Y_{t-1}, P_{t-1}, shock_t; \Phi_2\right) + u_{2t}.$$

Y_t is a vector of macroeconomics variables for $t = 1, 2, ..T$ years, whereas P_t represents the set of policy variables. This, for example, can be a fiscal policy rule as a reaction function to a monetary policy shock. In my study the policy rule is the debt-to-gdp ratio. Φ_j, with $j = 1, 2$, are the parameters that we need to estimate. f_j, are functions that need to be defined according to the question under study, to account for either linear or non-linear responses. The choice of the functions clearly depends on the question of interest. At the same time, it depends on the policy rule and the number of macroeconomic variables included in the system (and vice-versa). The reason is that the scarcity of the data, especially when one uses a narrative record of identified shocks, puts some limits in the degrees of freedom and the number of parameters that can be estimated.

Once all the necessary components of the model are specified, one can proceed with the estimation of the model (e.g. via seemingly unrelated regression equations or maximum likelihood) and the derivation of impulse response functions. The derivation of the impulse response functions can be done through the generalized impulse response function, which I discuss in Section 5. The last step, concerns the calculation of fiscal multipliers as the ratio of the integral of the output response to the integral of the policy adjustment.

The above general encompassing model sets the base for the analysis that follows.

4 Data and Non-Linearities

4.1 Data

I make use of the narrative record initially constructed by Devries, Guajardo, Leigh, and Pescatori (2011) and extended by Alesina, Favero and Giavazzi (henceforth AFG). The dataset consists of a time series of fiscal consolidations of 17 OECD countries. The countries included in the initial data are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, the Netherlands, Portugal, Spain, Sweden, United Kingdom, United States, for the period 1980 to 2014. Motivated by the literature of the narrative identification approach of Ramey and Shapiro (1998) and Romer and Romer (2010), AFG examine historical records available in official documents (Budget Reports, Budget Speeches, Central Banks Reports, Convergence and Stability Programs submitted by EU governments to European Commission, IMF Reports, OECD economic surveys) to identify the size, timing and principal motivation behind any fiscal action taken by each government. The fiscal alterations are measured as a percentage of GDP. As in Devries, Guajardo, Leigh, and Pescatori (2011), the focus is restricted to the identification of fiscal changes that are exogenous to the economic cycle, as well as changes that are motivated by the willingness to reduce government deficit. This implies that a fiscal consolidation with the

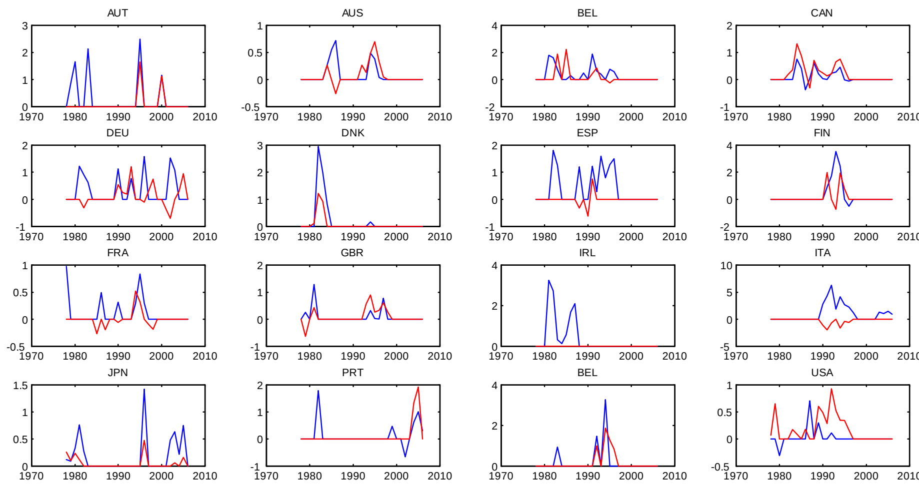

Figure 1: Country-specific narrative unanticipated (blue) and anticipated (red) fiscal adjustments.

goal of restraining domestic demand or any other countercyclical policy is not included in the dataset. Figure 1 shows the aggregate narratively identified unanticipated and anticipated episodes for each country.

The classification of fiscal consolidations as tax-based (TB) or expenditure-based (EB) is based on the spirit of the work of Alesina, Favero and Giavazzi (2015) and is going to be discussed in the next section. The difference is that I do not include all the path of future announcements of fiscal adjustments. I consider just the unanticipated and anticipated legislative announcements that are implemented the same year.[6, 7] Figure 2 depicts the EB and TB fiscal narratively identified shocks together with the GDP growth. The initial sample consists of 17 countries, but after performing some exogeneity tests I drop 3 countries, Finland, Netherlands and Sweden. In addition, I drop Germany because data are available after 1991, because of the unification, and this restricts my analysis. My final sample includes a total of 74 episodes for taxes and 101 episodes for government spending. The main macroeconomic variables of interest in my baseline specification are real GDP, government spending, which is primary government spending (total government spending net of interest payments on debt),

[6] The challenge of the narrative data is that there is often lack of information. Governments do not make legislative announcements in a frequent basis, hence there are many "zeros" in the data. This phenomenon is even stronger when one accounts for the future implementation of fiscal changes. The exclusion of the future announcements does not create any bias. Notice that in general most of the plans of announcements have a one year horizon (on average TB plans last around 1.5 years, and EB plans 1.8 years), which is the information that I include in my sample.

[7] This is in line with the work of Devries, Guajardo, Leigh, and Pescatori (2011).

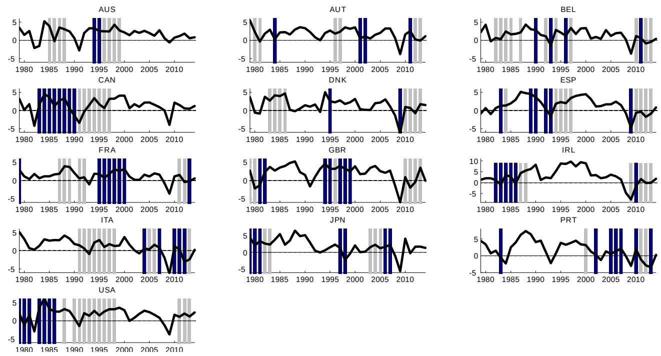

Figure 2: EB (blue) and TB (grey) episodes and the per capita GDP growth series (black line).

government revenues (current receipts), the average cost of debt and inflation. The frequency of observations is annual. My primary data source is the OECD.

In my general model, public debt plays the role of the main policy variable. In the next subsection, I present the construction of this series. I use the general government debt as a percent of GDP from the WEO of the IMF as a reference series. The histogram of Figure 3 shows the distribution of the government debt data of my sample. In my analysis, those data serve as the initial values of the debt ratio.

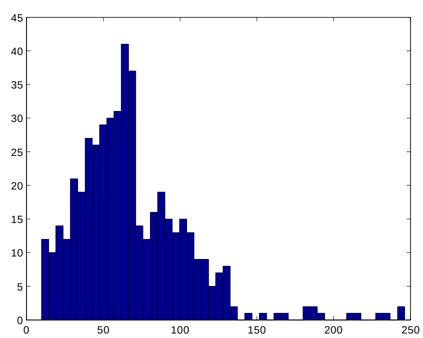

Figure 3: The distribution of fiscal position during the period 1980-2014. Debt-to-GDP ratio (x-axis), Frequency in % (y-axis).

4.2 Non-Linearities

4.2.1 State of the Economy

The first asymmetry of interest in my study arises from the state of the economy, namely if an economy is in recession or expansion. I follow Auerbach and Gorodnichenko (2012, 2013) and I use a regime indicator that is based on a logistic function, which I denote as $F(z_{it})$. $F(z_{it})$ indicates the probability of being in a recessionary regime. This is the key ingredient which allows me to smoothly endogenize in my model the possibility of the economy to move from one state to another and at the same time to track the feedback (as Caggiano, Castelnuovo, Colombo and Nodari (2015)) after a fiscal adjustment.

In my baseline specification the logistic function is a function of the two-years moving average of GDP growth and takes the following form

$$F(z_{it}) = \frac{\exp[-\gamma_i z_{it}]}{1+\exp[-\gamma_i z_{it}]}, \quad \gamma_i > 0. \tag{2}$$

Following Auerbach and Gorodnichenko, I denote by z the growth rate of output, as a moving average of two years, i.e. $z_{it} = \frac{\Delta y_{it-1}+\Delta y_{it-2}}{2}$, where Δy_{it} is GDP growth for country $\iota = 1,..,13$ at time $t = 1,...,35$. I use as an index of the business cycle the standardized measure of z_{it}.

γ is the parameter that controls the smoothness of the transitions from one regime to another. In general, large values are associated to immediate switches, while smaller ones imply a smoother transition. γ is calibrated in a way that matches the frequency and duration of recessions in an economy. The economy spends an $x\%$ of time in a recessionary regime according to the OECD dates. My goal is to calibrate γ to match this frequency. For example, for the US $\Pr((z_{it}) > 0.8 = 0.2)$, where I define an economy to be in a recession if $F(z_{it}) > 0.8$. Thus, this implies that I need to set $\gamma = 1.5$. Therefore, the magnitude of γ is in line with estimates of logit regressions of the OECD recession dates on the measure of z for all the countries in my sample.

The OECD dates are available from the Federal Reserve Bank of Saint Louis. These dates are based on the OECD Composite Leading Indicator (CLI). The series of the CLI is based on the growth cycle approach, where business cycles and turning points are identified through a deviation from the trend method. The recession dates are available in quarterly data, are not seasonally adjusted and are recorded as a dummy variable (1: for recession, 0: for expansion). I have yearly data on the narratively identified shocks, hence I translate the quarterly recession series of each country into a yearly recession series.

The general rule that I follow to calibrate γ_i for each country is that $\Pr((z_{it}) > 0.8 = x_i)$. Figure 4 shows the comparison of the constructed transition series to the OECD re-

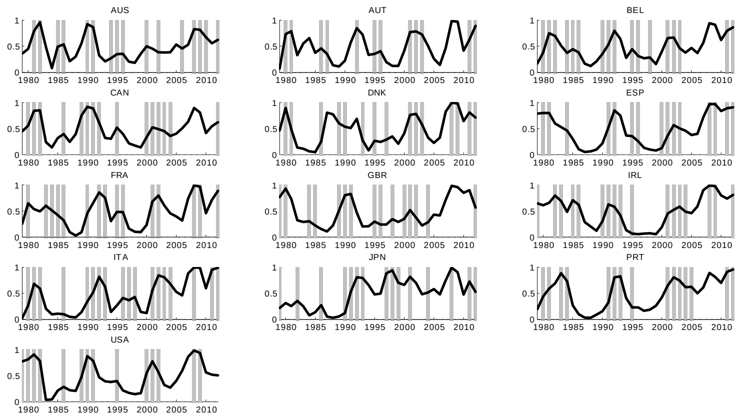

Figure 4: Recessions dates (shaded grey area) and the weight F(z) on the recession regime (black line).

cession dates associated to the economic downturns of the 13 economies of the sample. The country-specific *gammas* are presented in Table 1.

Table 1: Calibration of smoothness parameter γ

Country	Duration of Recessions	γ	Country	Duration of Recessions	γ
AUS	14%	1.14	**GBR**	19%	1.43
AUT	14%	1.53	**IRL**	14%	1.68
BEL	14%	1.13	**ITA**	22%	2.24
CAN	17%	1.09	**JPN**	17%	1.65
DNK	19%	1.72	**PRT**	22%	1.60
ESP	25%	1.70	**USA**	17%	1.56
FRA	14%	1.59			

4.2.2 Type of Fiscal Consolidation

The second asymmetry reflects the possible importance of the composition of the fiscal adjustment. Views about the relative effect of taxes or government spending differ among public debates and policymakers. Devries, Guajardo, Leigh, and Pescatori (2011) in their narrative record identify fiscal policy changes that are based either on taxes or government spending. Instead of directly including in my specification the tax and government spending adjustments I follow the Alesina, Favero and Giavazzi (2015) and take into account the fact that the different nature of changes may be correlated. Therefore, I take into account the entire fiscal adjustment (taxes and government spending together) and classify them as being tax-based if it is mainly based on tax increases, otherwise as expenditure-based.

4.2.3 Government Debt

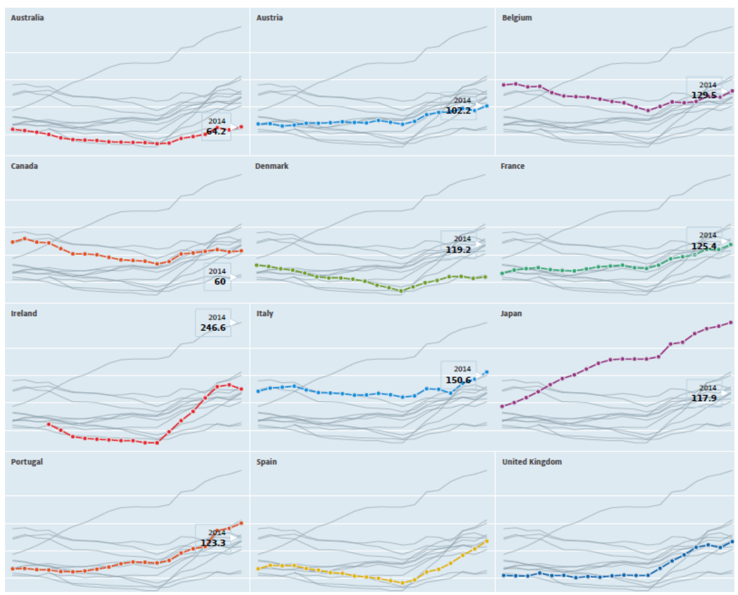

Figure 5: Debt-to-GDP ratio (Figure from the OECD website).

Figure 5 shows the government debt as a percent of GDP. From this figure it is possible to observe that countries like Belgium, Italy and Japan belong always in the group of countries with higher government debt. On the other hand, Portugal and the United States for example are cases that switch from being higher to lower (or vice versa). This means that it is not possible to fix exante the countries that would be classified as "higher-" or "lower-" debt. The ISTVAR, which is presented in Section 5.2, is a flexible way to endogenously model debt, without grouping countries into higher or lower debt.

I adopt the idea of Favero and Giavazzi (2007) to model debt in a way that mimics the government's budget constraint.

$$Debt_{it} = \frac{1 + i_{it}}{(1 + \pi_{it})(1 + \Delta y_{it})} Debt_{it-1} + (\exp(g_{it}) - \exp(\tau_{it})) . \quad (3)$$

i stands for the average cost of government debt, π is the inflation rate, g is government spending and τ is government revenues.[8] The debt-to-GDP ratio in this way is determined by the macroeconomic variables that are included in my specification. Figure 6 shows evidence that with the above equation (3) I manage to track well the debt-to-GDP ratio observed in the data. Differences may be due to presence of seigniorage, which is not considered in my framework, possible existence of stock-flow adjustments that lead to some measurement error, or due to approximation errors, since I use logarithms for the GDP growth rate and the inflation rate. For Australia, the fact that I combine different data sources to construct the series due to limited availability of data, may also explain why the implicit series does not match the observed series of the debt ratio.

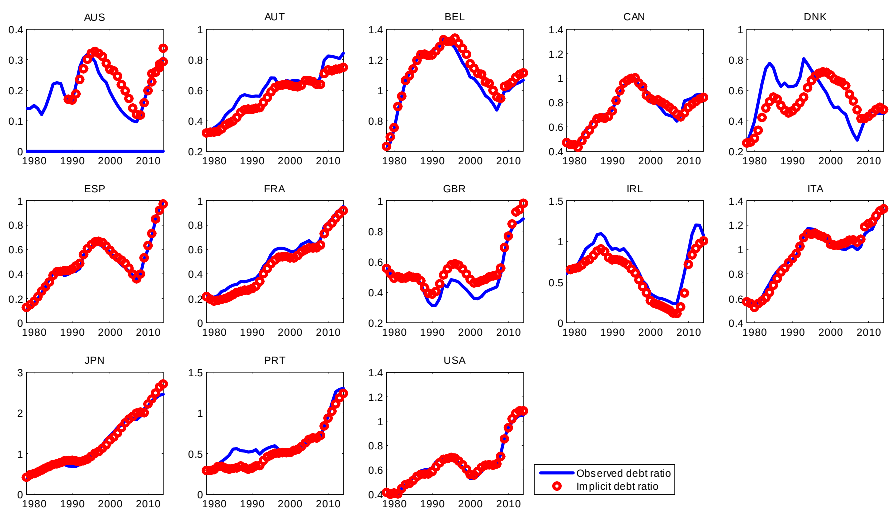

Figure 6: Observed Debt-to-GDP (data) versus the simulated series.

To understand better the insights of this identity I split (3) into two components: *a.* the so-called *snowball effect* ($\frac{1+i_{it}}{(1+\pi_{it})(1+\Delta y_{it})}Debt_{it-1}$) and *b.* the *primary balance effect*. Those represent the two channels that affect the evolution of debt following a consolidation episode. For example, suppose that the government reduces its expenditure by 1% of GDP. This implies a negative output effect and an increase of government spending. In terms of *a.*, the decrease of output growth, for a given past value of debt and a given i, implies an increase on the debt ratio. However, in terms of *b.*, the expenditure reduction has a direct impact on the primary balance, which improves,

[8]We take the exponential of the government to gdp and taxes to gdp ratios, because these variables are in logarithms.

and reduces the debt-ratio.[9] Therefore, debt will increase or decrease depending on the synergy that eventually dominates.

5 Model Specification

5.1 The Fiscal Smooth Transition VAR with Debt

My aim is to account for the three non-linearities and study their relevance on the transmission of fiscal policy. To illustrate this, I adopt the general model that I presented in Section 3, where I introduce the non-linearities of interest (i.e. state of the economy, type of fiscal consolidation, government debt). To make model (1) functional I need to specify the different sets of variables and functional forms. My baseline framework with the three non-linearities reads as follows:

$$\begin{aligned} Y_{it} = (1 - F(z_{it})) \times \left[A_E Y_{it-1} + \Theta_E Debt_{it-1} + B_{1E} e^{EB}_{i,t} + B_{2E} e^{TB}_{i,t}\right] + \\ + F(z_{it}) \times \left[A_R Y_{it-1} + \Theta_R Debt_{it-1} + B_{1R} e^{EB}_{i,t} + B_{2R} e^{TB}_{i,t}\right] + \\ + \lambda_i + \chi_t + u_{it} \end{aligned} \quad (4)$$

$$Debt_{it} = \frac{1 + i_{it}}{(1 + \pi_{it})(1 + \Delta y_{it})} Debt_{it-1} + (\exp(g_{it}) - \exp(\tau_{it}))$$

$$F(z_{it}) = \frac{\exp[-\gamma z_{it}]}{1 + \exp[-\gamma z_{it}]}, \quad \gamma > 0.$$

Relating to model (1): Y includes the key macroeconomic variables $Y = [\Delta y \ \Delta\tau \ \Delta g \ i \ \pi]$, where Δy is GDP growth, $\Delta\tau$ is the change of government revenues, Δg is the change of government spending, i is the average cost of government debt and π is the inflation rate. λ_i and χ_t are country and time fixed effects respectively. $u_{it} \sim N(0, \Sigma_u)$, $i = 1, ..., N$ index countries and $t = 1, ..., T$ index time. The policy variable includes the Debt ratio and is specified by (3). For this policy variable, I do not include an error term, since as I discussed in the previous Section this is an identity.[10] e^{EB}_{it} and e^{TB}_{it} stand for the narratively identified shocks (defined as *shocks* in model (1)). My specification distinguishes between the instrument of stabilization, expenditure-based

[9] The effect on output has a further indirect effect on the primary balance, that arises from the automatic stabilizers. In addition, there is potentially, a third channel through the interest rate payment. In our discussion, we will focus on the main effects on *a.* and *b.*

[10] Of course, one could include an error term to account for measurement error, given the discrepencies that are observed in a couple of countries.

($e_{i,t}^{EB} = e_{i,t}^{IMF} \cdot EB_{it}$) and tax-based ($e_{i,t}^{TB} = e_{i,t}^{IMF} \cdot TB_{it}$) narrative shocks, which are unanticipated and anticipated shocks implemented the same year.[11] In this model I assume that all the macroeconomic variables depend on the cycle, i.e. f_1 which in my case is the logistic function (2).[12] I estimate it as proposed by Granger and Terasvirta (1993).

Including debt in the study of fiscal consolidations is important. I remind my reader that the aim of the fiscal consolidations is to reduce public deficits. At the same time, the state of the public finances may not just trigger some episodes of fiscal adjustment but also have a direct impact on output growth through a different channel as I discussed in the introduction. Omitting debt may bias the magnitude of the consolidation effects.[13] The reason is that the short lags of $\Delta\tau$, Δg, i and π alone are incapable to trace the evolution of the debt ratio accurately enough. Favero and Giavazzi (2007), show that debt in (3) is the result of long and non-linear dynamics.[14] Since debt plays the role of the "initial condition" that the economy stands, the model dynamics are going to depend on it. This means that the initial value of the state of the economy and the initial value of the government's fiscal position matter for the purpose of studying the different dimensions of interest. I present results with initial values that make clear the dichotomies between a recessionary and expansionary regime (approximately 0.8 versus 0.2 as discussed in the previous section) and a high versus low debt ratio (0.3 and 0.9). It is important to stress that with this model I allow for the possible endogenous transition of the economy from one state to the other when a shock hits the economy, but also the endogenous feedback of debt. A flexible way that permits ME to actually account and track the transition of the economy and the debt dynamics is the use of the Generalized Impulse Response Functions.

My aim is to study the relevance of these non-linearities in the propagation of the fiscal adjustments.

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[11] $e_{i,t}^{IMF}$ stands for the the total adjustment, which includes tax and government spending changes. Whereas EB_{it} and TB_{it} are dummy variables for expenditure-based or tax-based cases respectively. An episode is recorded as expenditure-based (tax-based) when the total of expenditure-changes (tax-changes) dominate the total tax-changes (expenditure-changes).

[12] The linear model is a special case of STVAR for a value of $\gamma = 0$.

[13] I elaborate more on this argument in the appendix by discussing the econometrics.

[14] More precisely, they show that:

$$
\begin{aligned}
Debt_{it} &= \sum_{j=0}^{K} \left(\exp(g_{i,t-j}) - \exp(\tau_{i,t-j})\right)^{j} \prod_{j=0}^{K} \left(\frac{1 + i_{i,t-j}}{(1+\pi_{i,t-j})\,(1+\Delta y_{i,t-j})} \right) + \\
&\quad + \prod_{j=0}^{K} \left(\frac{1 + i_{i,t-j}}{(1+\pi_{i,t-j})\,(1+\Delta y_{i,t-j})} \right) Debt_{i,t-j-1}
\end{aligned}
$$

5.2 The Interacted Smooth Transition VAR

As a further extension of (4), I assume that the fiscal shocks depend on the level of debt, in that I explicitly interact the government budget constraint, the identity (3), with the identified consolidations. I employ a model which I call Interacted Smooth Transition Vector Autoregression (ISTVAR):

$$\begin{aligned} Y_{it} = (1 - F(z_{it})) \times \left[A_E Y_{it-1} + \Theta_E Debt_{it-1} + B_{1E} e_{i,t}^{EB} + B_{2E} e_{i,t}^{TB}\right] + \\ +F(z_{it}) \times \left[A_R Y_{it-1} + \Theta_R Debt_{it-1} + B_{1R} e_{i,t}^{EB} + B_{2R} e_{i,t}^{TB}\right] + \\ +\lambda_i + \chi_t + u_{it} \end{aligned} \tag{5}$$

$$B_{jS} = B_0^S + B_1^S \cdot Debt_{it-1}, \quad \text{for } S = E, R \text{ and } j = 1, 2,$$

$$Debt_{it} = \frac{1 + i_{it}}{(1 + \pi_{it})(1 + \Delta y_{it})} Debt_{it-1} + (\exp(g_{it}) - \exp(\tau_{it}))$$

$$F(z_{it}) = \frac{\exp[-\gamma z_{it}]}{1 + \exp[-\gamma z_{it}]}, \quad \gamma > 0.$$

This model makes the non-linearity that is introduced through debt stronger, since the effect from a fiscal adjustment in this case depends also on the interaction with debt. I expect that this additional component can allow for a better evaluation of the implications of adjustments in the study of high and low debt. Some additional justification behind this specification is the following:

The economic motivation that the aim of the fiscal adjustments is to decrease the public deficit, is a good argument to justify the choice of interacting these adjustments and debt.

An "ideal" specification would account for the interaction of both the set of macroeconomic variables and the set of the shocks. However, this clearly translates into a larger number of parameters to be estimated. There is where the so-called "curse of dimensionality" hits. Especially, because of the scarcity of the narrative data.

5.3 Generalized Impulse Response Functions

I now turn to the derivation of the impulse response functions, which in a non-linear environment may deem complicated. The derivation of impulse responses of the variables in Y_{it} to innovations is different from the case of a standard VAR, due to the presence of the logistic function and of the budget constraint. In my setting (model (1)), I compute the response of the output growth (or the rest of the economic aggregates) to fiscal shocks via generalized impulse response functions (Koop, Pesaran and Potter 1996), which allow to endogenize the transition from one regime to the other and to track the feedback between debt and the regime.

$$GIRF_{\Delta y}\left(h, \Omega_{t-1}, shock_t\right) = E\left(\Delta y_{t+h} | \Omega_{t-1}, shock_t = 1\right) - E\left(\Delta y_{t+h} | \Omega_{t-1}, shock_t = 0\right),$$

where Ω_{t-1} accounts for the history, $h = 0, 1, 2..., H$ are the horizons and $shock_t$ represents the shock of interest, which is either the tax-based or the expenditure-based narrative identified shock . I rely on the equation above to derive the impulse response functions.[15] The steps I follow are:

Step 1. First, assume that the structural shock of interest (i.e. EB/ TB) hits the economy, which is equal to one, while the rest of the shocks are equal to zero, and simulate the system forward.

Step 2. Then generate dynamically forward, an alternative simulation for all variables, by assuming, differently from *Step 1*, that all the shocks are equal to zero.

Step 3. To compute the impulse responses, take the difference between the above simulated values of *Steps (1. - 2.)*.

Step 4. In addition, run a correlated bootstrap method for the calculation of the confidence intervals, where I report the 16-84% confidence intervals.[16]

I repeat the above 4 steps for all the 2^3 combinations of interest.[17] This methodology produces impulse responses that allow for the feedback and dynamics of both the state variable $F(z)$ and $Debt$.

5.4 Fiscal Multipliers

The last part of the analysis involves the computation of the fiscal multipliers. I calculate the dynamic cumulative multipliers. Following Uhlig (2012), I take the integral of the generalized impulse response of output divided by the integral of the generalized impulse response of the fiscal policy variable of interest (revenues or expenditures)

[15] Applying this methodology in a VAR would produce standard impulse responses.

[16] For my bootstrap, I re-sample the residuals of the estimated non-linear VAR (e.g. model 4) allowing for the correlation between the residuals of the different countries. This generates a set of observations for $Y, F(z), Debt$, which allows me to re-estimate my model and derive the GIRFs. I rely on 1000 iterations.

[17] The combinations are: TB shock in recession ($F(z) = 0.8$) when debt is high (0.9); EB shock in recession ($F(z) = 0.8$) when debt is high (0.9); TB shock in expansion ($F(z) = 0.2$) when debt is high (0.9); EB shock in expansion ($F(z) = 0.2$) when debt is high (0.9); TB shock in recession ($F(z) = 0.8$) when debt is low (0.9); EB shock in recession ($F(z) = 0.8$) when debt is low (0.3); TB shock in expansion ($F(z) = 0.2$) when debt is low (0.3); TB shock in expansion($F(z) = 0.2$) when debt is low (0.3). I set for each case the initial values for debt, the regime indicator, and all the related initial parameters.

given the adjustment under study.[18] Basically, for the case of the government revenues this is the ratio of the changes of $\sum_{h=1}^{H} \Delta y_h / \sum_{h=1}^{H} \Delta \tau_h$, while $\sum_{h=1}^{H} \Delta y_h / \sum_{h=1}^{H} \Delta g_h$ is for the government expenditures.[19] I rescale these changes into currency equivalents, by using the ratio of the sample mean ratio of the expectation of output over taxes or government spending.[20,21]

6 Results

6.1 The Fiscal STVAR with Debt (Model 4)

In Section 4 (Figure 3), we saw how the government debt-to-GDP ratio is distributed along the different percentiles.[22] My econometric specification allows me to examine the size of the effects of the coefficients at specific values of the fiscal position from the percentiles of the sample. Initial conditions matter for the dynamics of the generalized impulse response functions. I present the impulse response functions together with the endogenous response of debt and the transition of the state for model (4).[23,24] In the Appendix I report the estimated coefficients (with their standard errors) for the equations of output growth, taxes and spending.

In Figure 7 and 8 I illustrate the cumulative impulse response functions of the main macroeconomic variables. I focus on the response of the output growth, for scenarios of low and high debt respectively.[25] In addition, the innovative feature of my model is that I can track the endogenous feedback of the debt-ratio, as well as, the endogenous response of the state indicator. Debt is low at a value of 30%, whereas debt is high at the

[18] Uhlig discounts those integrals, which I do not. This does not change my results.

[19] This implies that the available number of multipliers that one can compute is 2^3, given the different nonlinearities.

[20] Recall that the variables of interest are in logarithms and in real terms.

[21] Ramey and Zubairy (2016) are concerned with such a rescaling, especially for the case of the US and when they study a long historical dataset. I take into account their point, but I see that my results are robust.

[22] We drop Japan from our study, since it is the only country in our sample with such high debt-ratios.

[23] We do not depict he confidence intervals of the response of Fz and debt. The reason is that they are pretty narrow and we prefer to keep the picture of the graph more clear given that there are many curves presented together.

[24] Differently from AG, we allow for the endogenous transition from one regime to the other ($F(z)$). As it has been already stressed from Caggiano, Castelnuovo, Colombo and Nodari (2015), this is an important point to highlight. Therefore, we document the response of $F(z)$ starting in recession versus the one starting in expansion.

[25] I present the results for output, since later I present also the multipliers.

value of 90%.[26] My results represent the behavior of the average country in my sample. When debt is low, Figure 7 shows that the fiscal effects on output growth generated through increases in taxes are state-dependent. Adjustments that are mainly composed through taxes and are implemented in boom periods have the most recessionary effect. This is statistically different from the same type of consolidations when implemented in periods of recessions, which appear to be less recessionary. It appears that in bad times, when the economy underperforms and things go bad, they cannot go much worse. The state of the cycle seems to matter also when stabilizations are mainly implemented through increases in government spending. In this case, in period of expansions the effect on output is almost negligible. In terms of the endogenous response of the transition variable, for all the cases convergence to the assumed probability "target" is observed, which is that the economy spends on average 20% of the time in a recessionary regime. This is not the case when we turn to high debt, where the economy converges in a more recessionary target where the "target" indicates a probability of being 50-60% in a recessionary regime.

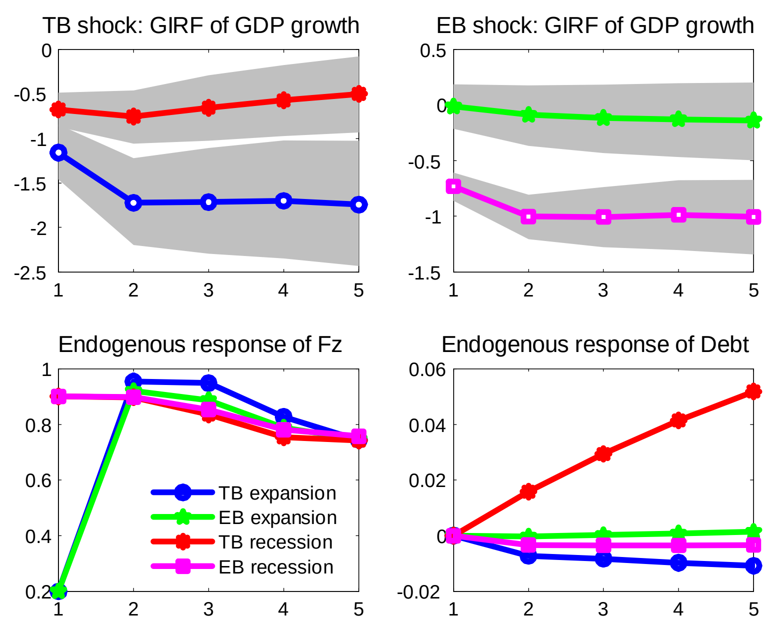

Figure 7: Cumulative GIRFs for the Fiscal STVAR with LOW Debt: The output growth response on a tax-based shock or an expenditute-based shock in recessions or expansions when the Debt ratio is low.

[26] One reason that we choose to present results for these values, is that these values are associated with the point of the tails of our sample distribution as depicted in Figure 3. In addition, the 90% value reflects the discussion of Reinhart and Rogoff (2010) that provide evidence of a negative impact of growth when the level of debt ratio is above this threshold.

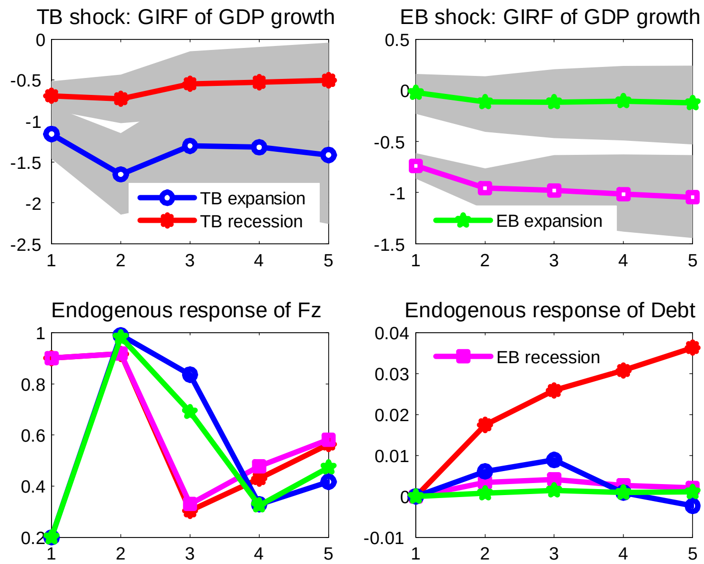

Figure 8: Cumulative GIRFs for the Fiscal STVAR with HIGH Debt: The output growth response on a tax-based shock or an expenditute-based shock in recessions or expansions when the Debt ratio is high.

In Figure 8, when the debt-ratio is high and the phase of the cycle is low, tax changes are self-defeating on average. They bring the debt-ratio to higher levels and the economy converges in a recessionary regime. Output growth falls on impact, and even if there is a sign of recovery after one year, it remains in a recessionary regime. Interestingly, on the other hand, government spending adjustments have a stabilizing feedback to debt. Especially, when an expenditure-based fiscal consolidation is implemented during good periods, this leads on average into a negligible response of the output growth. At the same time, it reduces the debt ratio. Therefore, there is a stabilizing feedback on the economic system.

6.1.1 The Cumulative Fiscal Multipliers

Figure 9 reports the dynamic response of the cumulative fiscal multipliers. The multipliers obtained when debt is low or high differ, with the government spending multipliers ranging between $[-0.5, -0.24]$, while the tax multipliers between $[-1, 0.8]$. A one percentage-point decline in government spending as a ratio to GDP is associated with an output of contraction. When debt is low the size of the contractionary effect appears to be larger. On the other hand, the tax multiplier is associated with a decrease of output on impact, which then starts to increase. This is not the case when a tax increase occurs when debt is low and when the economy is in economic upturns, where we observe a cumulative negative multiplier. In the Appendix I report the relevant cumulative multipliers with their confidence intervals. The multipliers account for both the response of GDP growth together with the response of taxes and government spending on the shock of interest, which can give a sense of the synergies that propagate back to our economic system and affect debt and output.

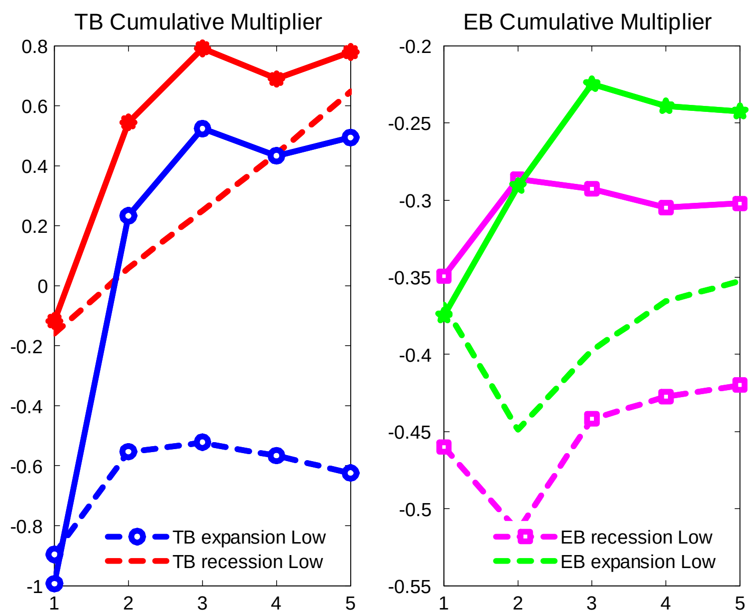

Figure 9: The Fiscal STVAR with Debt: Dynamic cumulative multipliers for H=5 horizons. Solid lines refer to cases of high debt, dashed lines refer to cases of low debt (for TB: blue in expansion, red in recession; for EB: green in expansion, magenta in recession).

6.2 The Interacted-STVAR (Model 5)

In this subsection I present the results for the ISTVAR model (5), in which when an adjustment is implemented is interacted also with the level of debt. When debt is low, from Figure 10, the fiscal effects on output growth generated through increases in taxes are state-dependent, as before. However, this is not the case for expenditure-based consolidations.When debt is high (Figure 11), compared to the previous results, it is

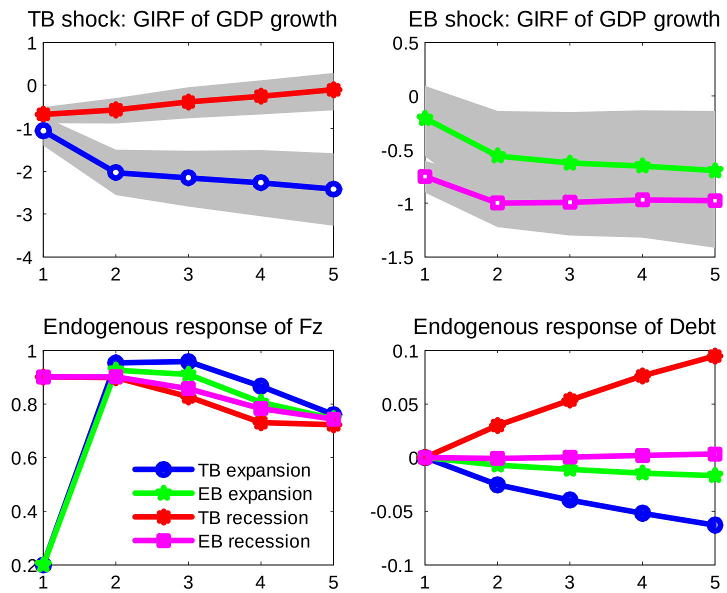

Figure 10: Cumulative GIRFS for the ISTVAR with LOW Debt: The output growth response on a tax-based shock or an expenditute-based shock in recessions or expansions when the Debt ratio is low.

interesting one to observe that now also an increase of taxes in periods of expansions is self-defeating. The tax shock increases public debt, which remains on an upward trajectory in the subsequent horizons.

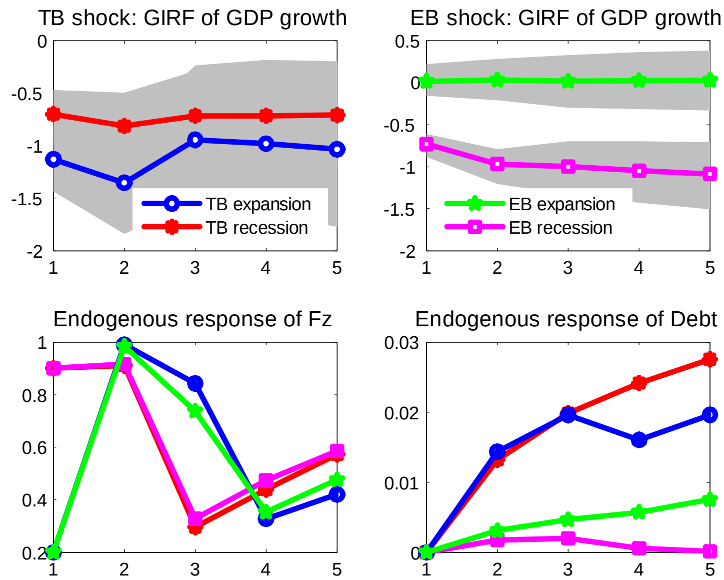

Figure 11: Cumulative GIRFS for the ISTVAR with HIGH Debt: The output growth response on a tax-based shock or an expenditute-based shock in recessions or expansions when the Debt ratio is high.

To sum up, policies implemented through expenditure-based adjustments seem to harm the economy less and are effective regarding their objective. They are able to stabilize, and even reduce debt. The picture is different when ones looks into a composition of policies based in tax increases. The distortionary flavour that taxes have, generate an opposite effect from the one that a government of a country would desire. Two factors can explain the increase in public debt, especially in periods of recessions: 1) the negative effect on the output growth and 2) the contemporaneous increase of government spending, which offsets the positive effect of higher revenues on the primary balance. This can be better understood by looking into the debt accumulation equation (3).

In general, the government can engage in decreasing the stock of public debt either by increasing its revenue, by the use of distortionary income taxation, or by reducing its expenditures, for example services that operate as a substitute for private consumption. In terms of policy implications, cutting expenditure seems to be more advisable, since it is less harmful for the economy. A cut in expenditure may reduce the distortion of taxes, since this may imply a decrease of taxation. This can be interpreted as a demand shock in the economy, and for this reason the effect on gdp growth is less pronounced.[27]

[27] Government spending is often wasteful. In this case the channel that I just described would be even stronger.

7 State-Dependent IRFs and Local Projections

In this section I discuss the potential pitfalls of the use of local projections in the state-dependent fiscal policy literature. For the purpose of this discussion I will work with a simplified model (STVAR), which accounts for two non-linearities (e.g. the state of the cycle and the type of fiscal consolidation). The goal of this section is to compare the performance of the local projections method to the GIRFS. Given this objective, I restrict our focus on the simplified model, both because it is more pedagogical and because this is the model generally used in the recent literature.

7.1 The Smooth Transition VAR

Given the objective of this section I simply assess the heterogeneity of the fiscal adjustments through state-dependence and composition. I make use of a trivariate smooth transition vector autoregressive model (for an extensive presentation see Granger and Terasvitra (1993)).

$$
\begin{aligned}
Y_{it} = (1 - F(z_{it})) \times \left[A_E Y_{it-1} + B_{1E} e_{i,t}^{EB} + B_{2E} e_{i,t}^{TB}\right] + \\
+F(z_{it}) \times \left[A_R Y_{it-1} + B_{1R} e_{i,t}^{EB} + B_{2R} e_{i,t}^{TB}\right] + \\
+\lambda_i + \chi_t + u_{it}
\end{aligned}
$$

$$
F(z_{it}) = \frac{\exp\left[-\gamma_i z_{it}\right]}{1 + \exp\left[-\gamma_i z_{it}\right]}, \quad \gamma > 0
$$

where Y_{it} includes: Δy, the GDP growth, $\Delta\tau$, the change of government revenues, Δg, the change of government spending.[28] This model specification is still distinguishing

[28]

$$
\begin{aligned}
\begin{bmatrix} \Delta y_{it} \\ \Delta\tau_{it} \\ \Delta g_{it} \end{bmatrix} = {} & [1 - F(z_{it})] \begin{bmatrix} \alpha_{11}^E & \alpha_{12}^E & \alpha_{13}^E \\ \alpha_{21}^E & \alpha_{22}^E & \alpha_{23}^E \\ \alpha_{31}^E & \alpha_{32}^E & \alpha_{33}^E \end{bmatrix} \begin{bmatrix} \Delta y_{it-1} \\ \Delta\tau_{it-1} \\ \Delta g_{it-1} \end{bmatrix} + [1 - F(z_{it})] \begin{bmatrix} \beta_1^E \\ \beta_2^E \\ \beta_3^E \end{bmatrix} e_{it}^{IMF} + \\
& + [F(z_{it})] \begin{bmatrix} \alpha_{11}^R & \alpha_{12}^R & \alpha_{13}^R \\ \alpha_{21}^R & \alpha_{22}^R & \alpha_{23}^R \\ \alpha_{31}^R & \alpha_{32}^R & \alpha_{33}^R \end{bmatrix} \begin{bmatrix} \Delta y_{it-1} \\ \Delta\tau_{it-1} \\ \Delta g_{it-1} \end{bmatrix} + [F(z_{it})] \begin{bmatrix} \beta_1^R \\ \beta_2^R \\ \beta_3^R \end{bmatrix} e_{it}^{IMF} + \\
& + \lambda_i + \chi_t + \begin{bmatrix} u_{it}^y \\ u_{it}^\tau \\ u_{it}^g \end{bmatrix}
\end{aligned}
$$

between EB and TB narrative shocks.[29]

Following the previous discussion, I estimate the response of the output growth (or the rest of the economic aggregates) to fiscal shocks with the method of the generalized impulse response functions, which allow to endogenize the transition from one regime to the other.

7.2 Local Projections Method

In this subsection I first present the local projections approach proposed by Oscar Jordà (2005) to derive the impulse responses. This requires to run a series of regressions for each horizon. The advantage of the local projections method lies on the fact that it does not constrain the shape of the IRF. I will first briefly present this approach as was introduced by Oscar Jordà for linear specifications. Along with this brief presentation I will refer to the limitations of the approach, already pointed out originally by the author, in the case of a non-linear specification. Then, I will move to the discussion of the state-dependent case as was first used in the fiscal policy literature by Auerbach and Gorodnichenko (2013).

The simplicity of this methodology has attracted many supporters, including researchers in fiscal policy and monetary policy. It is a flexible, model-free methodology, robust to misspecifications, from which one easily can derive the IRFs by running a series of regressions. In the linear case the model is

$$y_{it+h} = a_h x_{it-1} + \beta_h e_{i,t}^{IMF} + \lambda_i + \chi_t + \varepsilon_{it+h},$$

for $h = 0, 1, 2, ...$ horizons, where y is the variable of interest, x is a vector that includes the one lag of the growth of GDP, taxes and government spending. $e_{i,t}^{IMF}$ stands for the identified narrative shock (EB/ TB). The set of regressions in the above linear specification imply that the IRFs for this case are the collections of the β coefficients of each period.

AG allow for state-dependence in a straightforward way. In this case, the model becomes

$$\begin{aligned}\Delta y_{it+h} &= (1 - F(z_{it}))\left[a_{E,h} x_{it-1} + \beta_{E,h} e_{i,t}^{IMF}\right] + \\ &+ F(z_{it})\left[a_{R,h} x_{it-1} + \beta_{R,h} e_{i,t}^{IMF}\right] + \\ &+ \lambda_i + \chi_t + \varepsilon_{it+h}.\end{aligned}$$

[29]However, as a matter of ease and compactness for the discussion in the next section, I will keep notation simple and I will denote just the aggregate fiscal shock (for the disaggregate case, the extension is straightforward).

AG (2013) advocate that the local projections representation is equivalent to a moving average representation. This is the point of our potential criticism. The authors discuss the linear case both mathematically and graphically. They show that the IRFs that one recovers from the local projections approach are similar to the IRFs of the conventional approach for the linear case. Then they assume that the same should hold true for the non-linear case, and directly apply their version of local projections for their non-linear model. Basically, theirs IRFs are derived again as a sequence of the β_h's, estimated in a series of single regressions for each horizon. The IRFs for AG when a shock hits the economy in periods of recessions is the collection of the estimated $\beta_{R,h}$ and for expansions is $\beta_{E,h}$.

7.3 The Moving Average Representation

To compute the IRFs one typically proceeds by constructing first the moving average representation. Recently, it has been assumed in this literature (starting from AG) that the local projections approach is equivalent to the moving average representation of the series.

In this section I prove that the IRFs of a simple non-linear AR(1) differ from the IRFs recovered from the local projections approach.

Let me assume a non-linear model, of a similar spirit as my baseline model.

$$y_t = \alpha_t y_{t-1} + u_t \qquad \forall t \in \mathbb{N}_0. \tag{6}$$

In particular, one can have

$$\alpha_t = \frac{e^{-\frac{\gamma}{2}(y_{t-1}+y_{t-2})}}{1+e^{-\frac{\gamma}{2}(y_{t-1}+y_{t-2})}} A_1 + \frac{1}{1+e^{-\frac{\gamma}{2}(y_{t-1}+y_{t-2})}} A_2 \qquad \forall t \in \mathbb{N}_0.$$

Lemma 1 *For each* $t \in \mathbb{N}_0$

$$y_t = \sum_{\tau=0}^{t} \phi_{\tau,t} u_\tau + \phi_{-1,t} y_{-1}$$

where

$$\phi_{\tau,t} = \prod_{s=\tau+1}^{t} \alpha_s \qquad \forall \tau \in \{-1, 0, ..., t\}.$$

By convention, I assume that $\prod_{s=\tau+1}^{t} \alpha_s = 1$, if $\tau + 1 > t$. In particular, $\phi_{t,t} = 1$.

Assume now that

$$y_t = \alpha_t y_{t-1} + \beta_t e_t^{IMF} + u_t \qquad \forall t \in \mathbb{N}_0. \tag{7}$$

In particular, one can have

$$\begin{aligned} \alpha_t &= \frac{e^{-\frac{\gamma}{2}(y_{t-1}+y_{t-2})}}{1+e^{-\frac{\gamma}{2}(y_{t-1}+y_{t-2})}} A_1 + \frac{1}{1+e^{-\frac{\gamma}{2}(y_{t-1}+y_{t-2})}} A_2 \\ \beta_t &= \frac{e^{-\frac{\gamma}{2}(y_{t-1}+y_{t-2})}}{1+e^{-\frac{\gamma}{2}(y_{t-1}+y_{t-2})}} B_1 + \frac{1}{1+e^{-\frac{\gamma}{2}(y_{t-1}+y_{t-2})}} B_2 \forall t \in \mathbb{N}_0. \end{aligned}$$

Lemma 2 *For each $t \in \mathbb{N}_0$*

$$y_t = \sum_{\tau=0}^{t} \phi_{\tau,t} u_\tau + \sum_{\tau=0}^{t} \beta_\tau \phi_{\tau,t} e_\tau^{IMF} + \phi_{-1,t} y_{-1}$$

where

$$\phi_{\tau,t} = \prod_{s=\tau+1}^{t} \alpha_s \qquad \forall \tau \in \{-1, 0, ..., t\}.$$

By convention, I assume that $\prod_{s=\tau+1}^{t} \alpha_s = 1$, if $\tau + 1 > t$. In particular, I have that $\gamma_{t,t} = 1$.

The proofs are provided in the appendix.

What is important to realize from the above result is that the computation of the IRFs in the conventional approach depends on the time-varying coefficients which are functions of the lags of the left-hand-side variable. The IRFs are history-dependent. This is different from the linear case, where in the above setting would imply that $\phi_{\tau,t} = \prod_{s=\tau+1}^{t} \alpha$.[30] Therefore, in the non-linear setting the local projections method as has been widely used in the literature is not equivalent to the moving average

[30] It may be easier for the reader to notice that in the linear case if one starts from the baseline model, by substituting, one would have:
$y_0 = \alpha y_{-1} + \beta e_0^{IMF} + u_0$
$y_1 = \alpha y_0 + \beta e_1^{IMF} + u_1 = \alpha^2 y_{-1} + \alpha\beta e_0^{IMF} + \alpha u_0 + \beta_1 e_1^{IMF} + u_1$
$y_2 = \alpha y_1 + \beta e_2^{IMF} + u_2 = \alpha^3 y_{-1} + \alpha^2 \beta e_0^{IMF} + \alpha^2 u_0 + \alpha\beta e_1^{IMF} + \alpha u_1 + \beta e_2^{IMF} + u_2$ etc., whereas in the state-dependent case:
$y_0 = \alpha_0 y_{-1} + \beta_0 e_0^{IMF} + u_0$
$y_1 = \alpha_1 y_0 + \beta_1 e_1^{IMF} + u_1 = \alpha_1\alpha_0 y_{-1} + \alpha_1\beta_0 e_0^{IMF} + \alpha_1 u_0 + \beta_1 e_1^{IMF} + u_1$
$y_2 = \alpha_2 y_1 + \beta_2 e_2^{IMF} + u_2 = \alpha_2\alpha_1\alpha_0 y_{-1} + \alpha_2\alpha_1\beta_0 e_0^{IMF} + \alpha_2\alpha_1 u_0 + \alpha_2\beta_1 e_1^{IMF} + \alpha_2 u_1 + \beta_2 e_2^{IMF} + u_2$

representation, which means that the IRFs are not equivalent. This is easy to observe if one derives the IRFs that correspond to the local projections approach à la AG.

I write the baseline model specification in a simpler way. More precisely, I drop the i subscript, the country and time dummies, and I just include the lag just of the output growth.

$$\begin{aligned}\Delta y_{t+h} &= (1-F(z_t))\left[a_{E,h}\Delta y_{t-1}+\beta_{E,h}e_t^{IMF}\right]+\\ &\quad +F(z_t)\left[a_{R,h}\Delta y_{t-1}+\beta_{R,h}e_t^{IMF}\right]+\\ &\quad +\varepsilon_{t+h}.\end{aligned}$$

7.4 IRFs à la Auerbach and Gorodnichenko

$$\Delta y_t = (1-F(z_t))\left[a_{E,0}\Delta y_{t-1}+\beta_{E,0}e_t^{IMF}\right]+F(z_t)\left[a_{R,0}\Delta y_{t-1}+\beta_{R,0}e_t^{IMF}\right]+\varepsilon_t$$

$$\Delta y_{t+1} = (1-F(z_t))\left[a_{E,1}\Delta y_{t-1}+\beta_{E,1}e_t^{IMF}\right]+F(z_t)\left[a_{R,1}\Delta y_{t-1}+\beta_{R,1}e_t^{IMF}\right]+\varepsilon_{t+1}$$

$$\vdots$$

$$\Delta y_{t+h} = (1-F(z_t))\left[a_{E,h}\Delta y_{t-1}+\beta_{E,h}e_t^{IMF}\right]+F(z_t)\left[a_{R,h}\Delta y_{t-1}+\beta_{R,h}e_t^{IMF}\right]+\varepsilon_{t+h}.$$

Following AG this implies that the impulse response function of a fiscal consolidation in recession is $IRF^{\Delta y,R}=\left\{\beta_{R,h}\right\}_{h=0}^{H}$ with $H=5$. Similarly in expansion is $IRF^{\Delta y,E}=\left\{\beta_{E,h}\right\}_{h=0}^{H}$. In this case, the impulse responses depend just on the first two initial lags of the LHS variable. They do not take into account both the change of the dependent variable and the possibility of the endogenous transition from one state to the other. The correct way of constructing the impulse response functions thinking the moving average representation would be for $h=1$:

$$\begin{aligned}\Delta y_{t+1} &= (1-F(z_{t+1}))(1-F(z_t))\beta_{E,1}e_t^{IMF}+(1-F(z_{t+1}))F(z_t)\beta_{ER1,1}e_t^{IMF}+\\ &\quad +F(z_{t+1})(1-F(z_t))\beta_{R,1}e_t^{IMF}+(1-F(z_t))F(z_{t+1})\beta_{ER2,1}e_t^{IMF}+\alpha_1\alpha_0\Delta y_{t-1}+\varepsilon_{t+1}.\end{aligned}$$

Hence, one can conclude that in the non-linear case the local projections do not recover the same IRFs as the conventional approach.

7.5 Monte Carlo Experiment

The goal in this subsection is to conduct a Monte Carlo experiment. The reason is that I would like to compare the IRFs derived by the STVAR versus the IRFs from the local projections method as has been used in the literature. First, I assume that the STVAR baseline model is the true model (data generating process DGP). This means that I estimate the model and use the fitted values of the STVAR to generate data. Then I proceed by applying the local projections method on the generated data.

The algorithm for the Monte Carlo experiment is the following:

Step 1. I first draw an error $\sim N(0, \Sigma)$.

Step 2. For time t , from the DGP I generate $\begin{bmatrix} \Delta y_{it} \\ \Delta \tau_{it} \\ \Delta g_{it} \end{bmatrix}$, by taking $t-1$ and $t-2$ as given.

Step 3. I get the new $F(z_{it})$.

Step 4. Repeat 1-3.

In the first round in *Step 2*, I take the initial two lags as given. The reason is that the lags of the dependent variable are included both in the controls and in the regime indicator.

Next, I apply the local projections approach in the data generated to derive the impulse responses of the variable of interest (e.g. output growth).

Recall that when the impulse responses are estimated by the local projections method, they depict the average behavior of the economy for each sample from t to $t+h$ depending on the shock and the initial state.

7.6 IRFs from the STVAR versus Local Projections

Before moving to the main findings of this discussion, I report the results that I get from the estimation of a linear VAR. The IRF for the output growth (Figure 17 in the Appendix) show that a 1% fiscal shock has a recessionary effect, with the fiscal adjustments based mainly on spending cuts being less costly in terms of short-run output losses. I get similar results if we use the local projections method, the IRFs from the two different methods are not statistically different. This is in line with what Auerbach and Gorodnichenko (2013) have discussed.

Moving to the STVAR model, first, I discuss the results that I acquire by using the local projections approach on the data. Then, I will look into the results drawn from the Monte Carlo experiment. To derive the IRFs with the local projections method, one need to account for the serial correlation generated in the regressions from $h > 0$

and correct the standard errors. Following Ramey and Zubairy(2016), I use the Newey-West standard errors.

The results for the non-linear model, where I consider just the non-linearities that arise from the state of the business cycle together with the different composition of fiscal shocks (see Figure 18 in the Appendix), indicate that the shocks occurring during economic downturns seem to be statistically different compared to the fiscal shocks implemented during periods of economic upturns. When a tax-based fiscal consolidation hits the economy during "good" times, the immediate effect is more recessionary compared to the case of "bad" times. Fiscal adjustments based upon taxes when implemented in periods of expansion have overall on average the most recessionary effect. The tax-based adjustments implemented in periods of recessions are less recessionary, which is in accordance with my previous findings. In the case of adjustments composed mainly from taxes, I find evidence of state-dependency. On the other hand, expenditure-based fiscal consolidations implemented in different periods are not statistically different. This last result is in line with the findings of Ramey and Zubairy (2016).

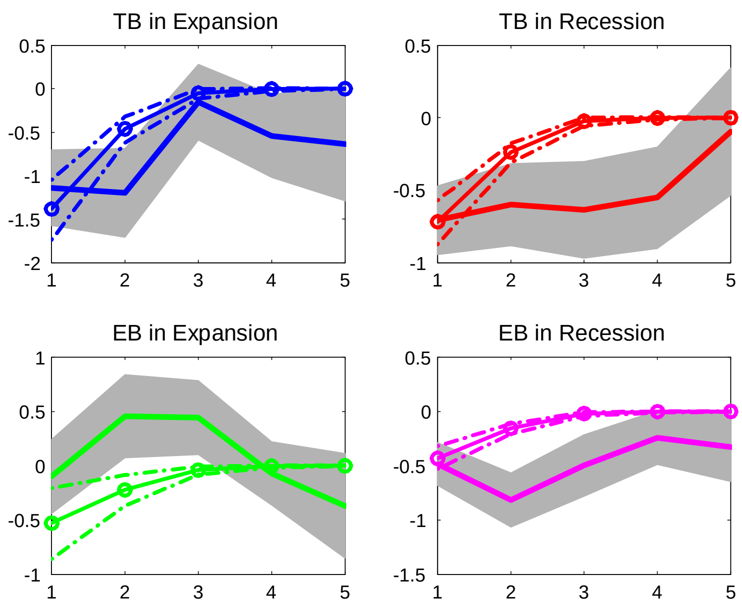

Figure 12: Data: IRFS with local projections (solid line with grey shaded area) versus STVAR.

From Figure 12 I observe that the IRFs derived from the two different methods differ. There is evidence that the two curves are statistically different. This is not the case just for the tax-based consolidations in expansions. The initial effects are identical, which is in line with what I discussed about the moving average representation and the fact that for $h = 0$ the initial effects should be the same. The response of the output growth with the local projections method is more erratic in general compared to the conventional approach. This is reasonable since the impulse responses with the local projections method are based on the estimated coefficient of the corresponding period. It important to mention, that irrespectively of the econometric discussion in a theoretical basis, the conclusions from the two different methods differ. In fact, one can notice from Figure 19 (in the Appendix) that the state-dependent evidence that I get from the GIRFs is now reversed.

Finally, I conduct out-of-sample simulations and compare the actual realization of GDP growth for the years 2009-2013 to the simulated ones derived from the STVAR and the local projections.[31]

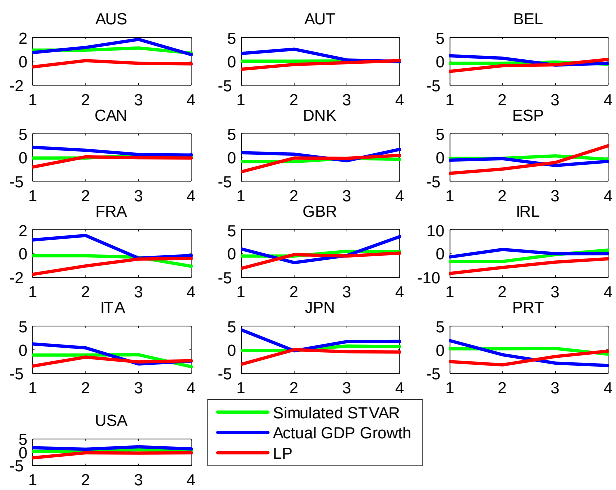

Figure 13: Out-of-sample simulation

7.7 Monte Carlo Experiment

The underline assumption is that the STVAR is the true model, which I use as the data generating process. In Figure 14, I present just the confidence intervals of the STVAR, since this is the uncertainty of the true model. The size of the initial effects is comparable to the STVAR. The derived IRFs in this case are not statistically different and the dynamics also seem not to differ much. One reason may be that the

[31]Local projections are not used in general for forecasting. I still believe that it is a reasonable way to compare the fit of the two methods.

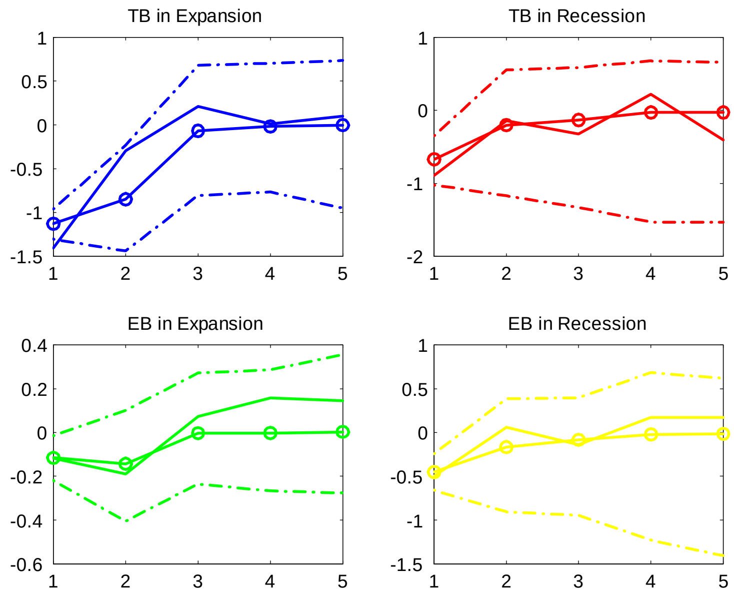

Figure 14: Monte Carlo Experiment: The solid line with the circles depicts the IRFs derived from the true model (the DGP, i.e. the STVAR) and the true confidence intervals, while the solid line depicts the IRFS derived with the local projections method.

non-linearity introduced is not that strong, since the GDP growth after a shock does not change much from one year to the other.[32,33]

8 Robustness Checks and Additional Tests

In this section I briefly discuss part of the preliminary tests needed regarding my baseline model specification and the narrative data.[34]

[32]For this reason, as a simple exercise I consider conducting the same experiment in the ISTVAR framework. In this case the non-linearity that arises from the government debt dynamics is stronger, so it could be interesting to see the difference to the existing findings.

[33]Further tests for the difference of the IRFs include the one of Olivei and Tenreyro (2010).

[34]Further details of the tests and results, as well as further robustness checks can be provided if requested.

8.1 Linearity tests

To make sure that the smooth transition regime switching models are identified, I need to conduct a hypothesis testing of H_0: Linear model versus H_1: Logistic STVAR model. I conduct two type of linearity tests. First, we follow Terasvirta and Yang (2014) and use the LM-type test to compare the residual sum of squares of the linear model to the ones of a second- or third-order approximation os the STVAR specification. Then I use a standard likelihood ratio test. Both tests are in favor of the non-linear model.

To conduct the linearity test we approximate the logistic function by a 2nd or 3rd order Taylor expansion. The non-linear performs better compared to the linear model. The values for both the Akaike criterion and the Schwarz criterion are lower for the non-linear model, which indicate that it is the preferred model.

	Linear Model	Non-Linear Model
AIC	4.15	4.06
BIC	4.61	4.58

8.2 Exogeneity of the narrative identified shocks

To investigate whether the identified adjustments are systematically uncorrelated with other developments affecting output, I use a simple test of exogeneity (Granger causality test). More precisely, I regress the narrative identified adjustments on the lag of output growth, and augment by including lagged values of the narrative measures. If the past variables are not able to predict a shift in the components of spending or taxes, then the shift is considered to be exogenous.

The results of the Granger causality tests that I run for each country and for each component show that, in most of the cases, the null hypothesis that the past variables predict the narrative measures is rejected. For Sweden and the Netherlands I am not able to reject the null hypothesis. Therefore I decide to drop these countries from our analysis.

8.3 Hypothesis Testing

Econometrically, one can examine whether the "when", the "how", or the "initial condition" matters more or less by testing different hypothesis.

Hypothesis Testing	
Cycle	$B_{1E} = B_{1R}$; $B_{2E} = B_{2R}$
Composition	$B_{1E} = B_{2E}$; $B_{1R} = B_{2R}$
Initial Condition	$B_0^E = B_1^E$; $B_0^R = B_1^R$

9 Concluding Remarks

The effect of fiscal austerity during economic downturns is the Gordian knot of policy discussions. In this paper I contribute to the empirical literature of fiscal policy in two ways. First, I study the relevance of non-linearities regarding the output effects of fiscal shocks. I propose a general encompassing model to study non-linearities. I focus on three types of non-linearities simultaneously, related to the "how" a fiscal adjustment is implemented, "when" and the initial condition of the economy. The simultaneous study of the initial condition, namely the debt ratio of the economy, with the other two non-linearities, captures the novelty of this paper. And, indeed, I find that it matters.

I examine the potential asymmetric response of fiscal consolidations by allowing for a non-linearity on the state of the economy, the composition of the fiscal adjustments and the government's budget fiscal position. I present an Interacted STVAR aiming to study these asymmetries. In general, policies implemented through expenditure-based adjustments seem to harm the economy less and work effectively. They are able to stabilize, and even reduce debt. The picture is different when I look into a composition of policies based on tax increases. The distortionary flavour that taxes potentially may have, generates an effect opposite from what the government of a country would desire. The effect of tax-based adjustments are, on average, the most recessionary. When debt is high, by increasing taxation, the fiscal authority fails to stabilize. However, when the authority decides to cut public expenditure during a good period, stabilization of debt is observed together with a negligible effect to the output growth. The heterogeneous response of the expenditure-based consolidations implemented during good periods when debt is high is an interesting policy implication that should be examined further. In addition, the evidence of the asymmetries between low and high ratios of debt should be evaluated to understand the channels of the transmission mechanism. For example, it could be important to control for a component of monetary policy, particularly when interest rates are close to the zero lower bound, and possibly study a related theoretical model.

Furthermore, I address some key problems in the econometrics of the existing literature. More precisely, I discuss the potential pitfalls of the use of the local projections method in the fiscal policy literature. I prove that the IRFs derived from the conventional approach are not the same as those derived from the local projections' approach. When I compare the IRFs derived from the two approaches, the two seem to differ and lead to completely different policy conclusions. Nevertheless, in my Monte Carlo experiments, my findings indicate that they are not statistically different.

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Appendix A: Debt

I will discuss econometrically why it is important to include debt. The reason is simple, omitting debt may deliver biased estimates. The aim is to estimate the effect of fiscal adjustments on the output growth. As I discussed in Section 3, the records of the fiscal adjustments are identified in a way that they are exogenous to the business cycle. However, they depend on one key motivation. The motivation is to decrease the government's deficit. Assume a simple model, with a focus just on output growth. Suppose that the true model is the following

$$\Delta y_t = \alpha + \beta \cdot Debt_{t-1} + \gamma \cdot e_t^{IMF} + \varepsilon_t.$$

I can reasonably assume that both $\beta < 0$ and $\gamma < 0$. The latter, can be seen also from Figure 12 for the linear case and from Figure 13 for the state-dependent case, where the effect of fiscal consolidations had a recessionary effect on output growth. At the same time, I have that

$$e_t^{IMF} = \kappa \cdot Debt_{t-1} + \upsilon_t,$$

where $\kappa > 0$, given the motivation of the fiscal adjustments. If I combine the two equations,

$$\Delta y_t = \alpha + (\frac{\beta}{\kappa} + \gamma) \cdot e_t^{IMF} + \nu_t$$

it is clear that if one considers just the fiscal adjustments , this would imply an overestimation of the effect $((\frac{\beta}{\kappa} + \gamma) < 0)$. Intuitively, in this case, one could also think that the stabilization feedback effect to the system, arising from debt, is missing.

Appendix B: Proofs of MA representation

Proof of Lemma 1. I proceed by induction.

Initial Step. $t = 0$. By (6), $y_0 = \alpha_0 y_{-1} + u_0 = u_0 + \alpha_0 y_{-1}$. At the same time, note that $\phi_{0,0} = \prod_{s=0+1}^{0} \alpha_s = 1$ and $\phi_{-1,0} = \prod_{s=-1+1}^{0} \alpha_s = \alpha_0$. I can conclude that

$$y_0 = \sum_{\tau=0}^{0} \phi_{\tau,0} u_\tau + \phi_{-1,0} y_{-1}.$$

Inductive Step. The statement is true for t. I next show it holds for $t + 1$. By (6) and inductive hypothesis, it follows that

$$\begin{aligned} y_{t+1} = \alpha_{t+1} y_t + u_{t+1} &= \alpha_{t+1} \left(\sum_{\tau=0}^{t} \left(\prod_{s=\tau+1}^{t} \alpha_s \right) u_\tau + \left(\prod_{s=-1+1}^{t} \alpha_s \right) y_{-1} \right) + u_{t+1} \\ &= \left(\sum_{\tau=0}^{t} \alpha_{t+1} \left(\prod_{s=\tau+1}^{t} \alpha_s \right) u_\tau + \alpha_{t+1} \left(\prod_{s=0}^{t} \alpha_s \right) y_{-1} \right) + u_{t+1}. \\ &= \sum_{\tau=0}^{t} \left(\prod_{s=\tau+1}^{t+1} \alpha_s \right) u_\tau + \left(\prod_{s=0}^{t+1} \alpha_s \right) y_{-1} + u_{t+1} \\ &= \sum_{\tau=0}^{t+1} \left(\prod_{s=\tau+1}^{t+1} \alpha_s \right) u_\tau + \left(\prod_{s=0}^{t+1} \alpha_s \right) y_{-1} \\ &= \sum_{\tau=0}^{t+1} \phi_{\tau,t+1} u_\tau + \phi_{-1,t+1} y_{-1}. \end{aligned}$$

By induction, the statement follows. ■

Proof of Lemma 2. For each $t \in \mathbb{N}_0$ define

$$\tilde{u}_t = \beta_t e_t^{IMF} + u_t. \tag{8}$$

By (7), I have that

$$y_t = \alpha_t y_{t-1} + \tilde{u}_t \qquad \forall t \in \mathbb{N}_0.$$

By Lemma 1, I can write

$$y_t = \sum_{\tau=0}^{t} \phi_{\tau,t} \tilde{u}_\tau + \phi_{-1,t} y_{-1} \qquad \forall t \in \mathbb{N}_0 \tag{9}$$

where

$$\phi_{\tau,t} = \prod_{s=\tau+1}^{t} \alpha_s \qquad \forall \tau \in \{-1, 0, ..., t\}.$$

By plugging (8) in (9), I can conclude that for each $t \in \mathbb{N}_0$

$$\begin{aligned} y_t &= \sum_{\tau=0}^{t} \phi_{\tau,t} \left(\beta_\tau e_\tau + u_\tau \right) + \phi_{-1,t} y_{-1} \\ &= \sum_{\tau=0}^{t} \phi_{\tau,t} u_\tau + \sum_{\tau=0}^{t} \beta_\tau \phi_{\tau,t} e_\tau^{IMF} + \phi_{-1,t} y_{-1}, \end{aligned}$$

proving the statement.

Appendix: Tables and Figures

Horizon	Expansion	CI low Exp	CI up exp	Recession	CI low rec	CI up Rec
1	0.9800	0.5900	1.7900	0.1200	-0.3100	0.6500
2	-0.2300	-1.3600	0.8300	-0.5400	-2.2000	1.2400
3	-0.5200	-1.5900	0.3600	-0.7800	-1.8600	0.3500
4	-0.4300	-1.9700	1.0800	-0.6800	-1.3800	0.0100
5	-0.4900	-1.8700	0.9200	-0.7700	-1.5500	-0.1300

High Debt: The TB Cumulative Multiplier in recessions and expansions with the related upper or lower confidence band.

Horizon	Expansion	CI low Exp	CI up Exp	Recession	CI low Rec	CI up Rec
1	-0.3700	-0.6300	-0.1900	-0.3400	-0.4700	-0.2500
2	-0.2900	-0.5500	-0.0300	-0.2800	-0.4400	-0.1200
3	-0.2200	-0.4100	-0.0400	-0.2900	-0.4500	-0.1400
4	-0.2400	-0.4000	-0.0700	-0.3000	-0.4600	-0.1700
5	-0.2400	-0.3900	-0.0700	-0.3000	-0.4600	-0.1600

High Debt: The EB Cumulative Multiplier in recessions and expansions with the related upper or lower confidence band.

Horizon	Expansion	CI low Exp	CI up Exp	Recession	CI low Rec	CI up Rec
1	0.8831	0.5900	1.4000	0.1574	-0.0600	0.3800
2	0.5458	0.3600	0.8200	-0.0581	-0.3500	0.2000
3	0.5143	0.3100	0.7700	-0.2459	-0.6300	0.0700
4	0.5586	0.3500	0.8300	-0.4339	-1	-0.0600
5	0.6161	0.3900	0.9000	-0.6386	-1.4500	-0.1600

Low Debt: The TB Cumulative Multiplier in recessions and expansions with the related upper or lower confidence band.

Horizon	Expansion	CI low Exp	CI up Exp	Recession	CI low Rec	CI up Rec
1	-0.3619	-0.6200	-0.1700	-0.4527	-0.6300	-0.3300
2	-0.4415	-0.7500	-0.2200	-0.5062	-0.8100	-0.3100
3	-0.3912	-0.6500	-0.1800	-0.4348	-0.7400	-0.2300
4	-0.3598	-0.5900	-0.1700	-0.4206	-0.7100	-0.2500
5	-0.3466	-0.5600	-0.1700	-0.4132	-0.6600	-0.2700

Low Debt: The EB Cumulative Multiplier in recessions and expansions with the related upper or lower confidence band.

Variable	Coefficient (DeltaY)	Std. Error	T-statistic	Coefficient (DeltaT)	Std. Error	T-statistic	Coefficient (DeltaG)	Std. Error	T-statistic
(1-F)DeltaY	0.4167	0.0893	4.6645	0.0636	0.1313	0.4843	-0.1944	0.1846	-1.0531
(1-F)DeltaT	-0.2313	0.0802	-2.8854	-0.0417	0.1178	-0.3539	0.1177	0.1656	0.7106
(1-F)DeltaG	-0.2070	0.0610	-3.3965	0.0413	0.0896	0.4611	0.2633	0.1259	2.0909
(1-F)i	0.1325	0.0694	1.9099	0.1758	0.1020	1.7241	0.1991	0.1433	1.3890
(1-F)pi	-0.0017	0.0796	-0.0217	0.1477	0.1170	1.2623	-0.1581	0.1644	-0.9611
(1-F)Debt	-0.0083	0.0081	-1.0231	-0.0204	0.0119	-1.7193	-0.0578	0.0167	-3.4652
(1-F)eTB	-0.0129	0.0046	-2.8207	0.0231	0.0067	3.4466	-0.0190	0.0094	-2.0102
(1-F)eEB	0.0020	0.0035	0.5730	-1.1450e-04	0.0051	-0.0224	-0.0036	0.0072	-0.5057
(1-F)DeltaY	0.2041	0.1107	1.8435	0.4104	0.1628	2.5214	-0.3918	0.2287	-1.7129
(1-F)DeltaT	-0.1169	0.0750	-1.5593	-0.0149	0.1102	-0.1353	0.1605	0.1549	1.0362
(1-F)DeltaG	0.0991	0.0348	2.8495	-0.0526	0.0511	-1.0296	-0.3171	0.0718	-4.4155
(F)i	-0.0894	0.0671	-1.3332	-0.0674	0.0986	-0.6841	0.0749	0.1385	0.5405
(F)pi	-0.0202	0.0696	-0.2902	-0.0138	0.1023	-0.1350	0.0153	0.1437	0.1065
(F)Debt	0.0286	0.0078	3.6878	0.0143	0.0114	1.2589	-0.0606	0.0160	-3.7854
(F)eTB	-0.0061	0.0026	-2.3323	0.0086	0.0039	2.2436	0.0358	0.0054	6.6153
(F)eEB	-0.0083	0.0020	-4.1784	0.0049	0.0029	1.6636	-0.0096	0.0041	-2.3383

Estimation Results of the Fiscal STVAR with Debt (regressors of lag(1) - except from the identified shocks EB,TB).

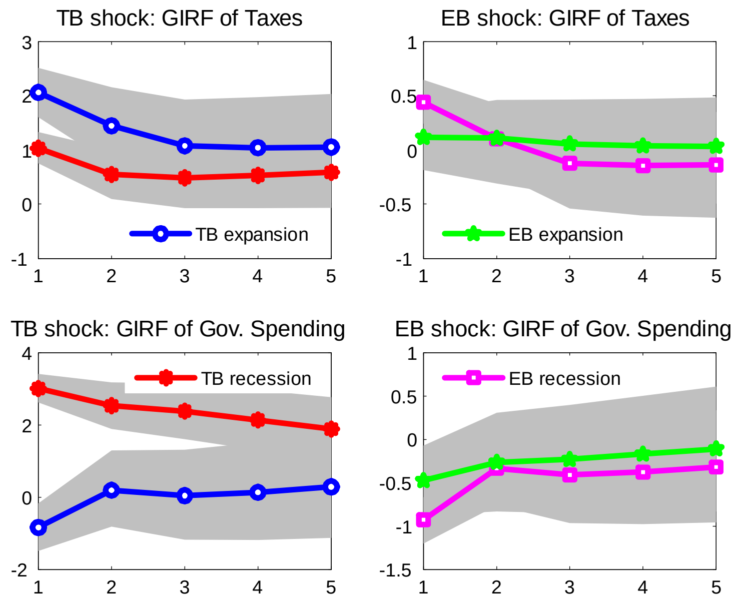

Figure 15: Cumulative GIRFs for the Fiscal STVAR with LOW Debt: The responses of taxes and government spending on a tax-based shock or an expenditute-based shock in recessions or expansions when the Debt ratio is low.

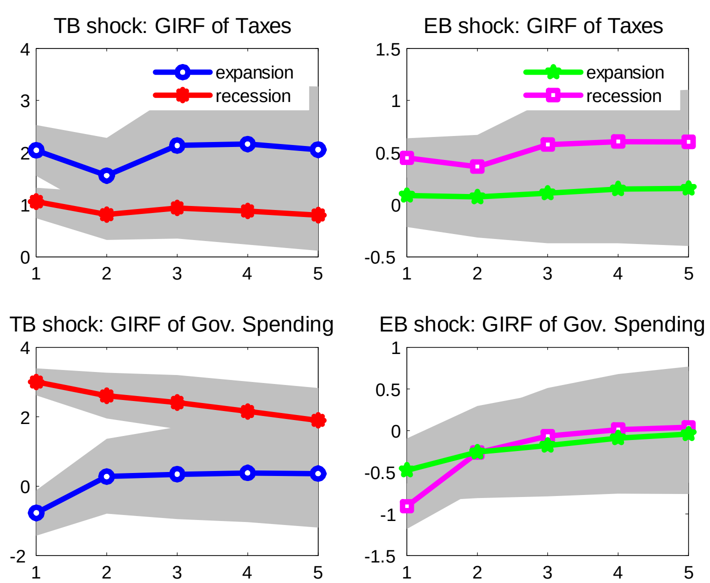

Figure 16: Cumulative GIRFs for the Fiscal STVAR with HIGH Debt: The responses of taxes and government spending on a tax-based shock or an expenditute-based shock in recessions or expansions when the Debt ratio is high.

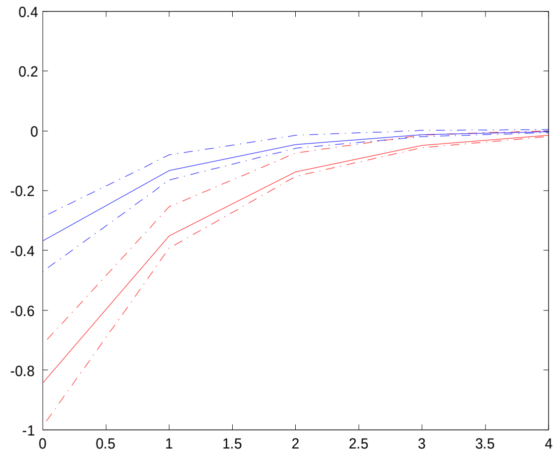

Figure 17: GIRF for the Linear 3-variate VAR: The output responce on an tax-based (red); expenditure-based (blue) fiscal adjustment

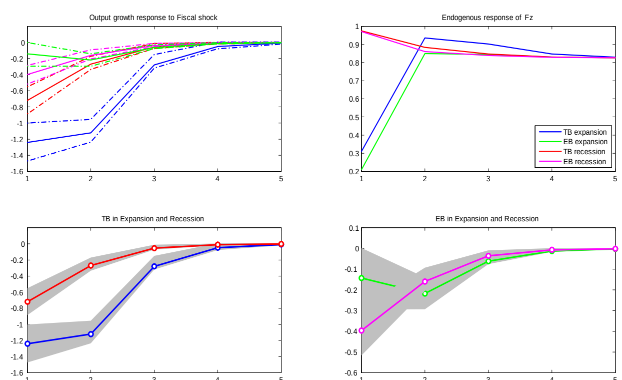

Figure 18: GIRF for the STVAR: EB and TB fiscal adjustments in Recessions and Expansions and the endogenous response of Fz

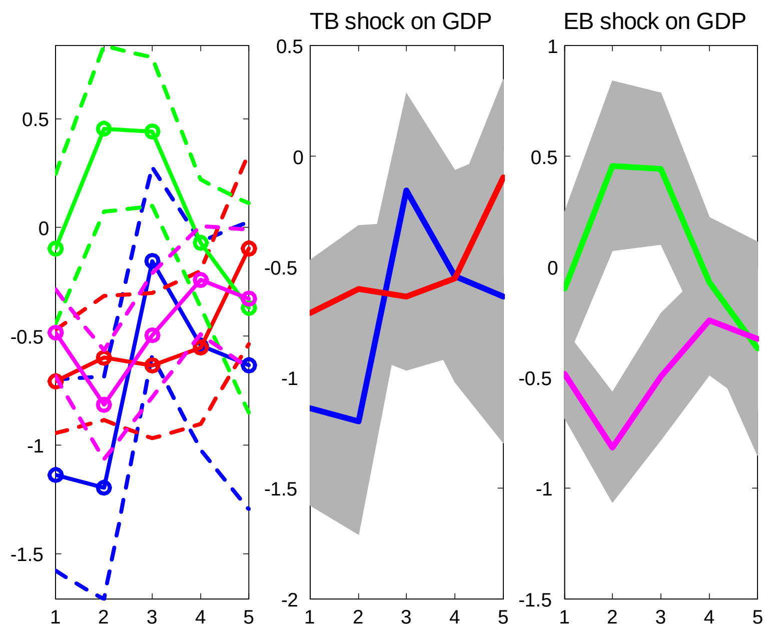

Figure 19: Data: IRFs derived with local projections.