

Frictions and adjustments in firm-to-firm trade*

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discussion by

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Main contributions

1. Bertrand competition in a Ricardian model of firm-to-firm trade
2. Quantitative assessment of how frictions shape the incidence of cost shocks
3. Relationship formation, dissolution, switching informs search parameters of the model
4. Absolutely gorgeous data about matches
5. How do models with/without search frictions differ?
 - 5.1 Search models always have a concept of being unmatched (unemployment)
 - 5.2 How are aggregate steady-state variables affected by search?
 - 5.3 Distributional effects tend to have small effect on changes
 - 5.4 Level effects are critical for optimal tariffs

Relevant search-trade theory papers that you can leverage:

“Goods-market frictions and international trade” [Krolikowski and McCallum, 2021, *JIE*]

“Tariffs and Goods-Market Search Frictions” [Krolikowski and McCallum, 2025, *RR, JIE*]

1. Nash bargaining nests Bertrand competition

- Well known that Nash bargaining nests Bertrand (when power of exporter is zero, $\beta = 0$)
- KM2021 have Nash over quantity and negotiated price, n_{do} , which is a convex combination of the pre-tariff consumer price and the average total production cost less producers' search costs

$$n_{do} = (1 - \gamma_{do}) \left(\frac{p_{do}}{t_{do}} \right) + \gamma_{do} \left(\frac{v(q_{do}, w_o, \tau_{do}, \varphi) + w_o f_{do} - w_o l_{do} - \kappa_{do} \chi(\kappa_{do}) w_o s_{do}}{q_{do}} \right),$$

in which $\gamma_{do} \equiv \frac{(r + \lambda)(1 - \beta)}{r + \lambda + \beta \kappa_{do} \chi(\kappa_{do})} \in [0, 1]$ and consumer price $p_{do}(\varphi) = t_{do} \mu w_o \tau_{do} \varphi^{-1}$ is a markup over marginal cost including tariffs.

- Alvarez et al. [2025] use our expression in their paper
- To get Bertrand, set $\beta = 0 \Rightarrow \gamma_{do} = 1$, and assume fixed and sunk costs are zero

$$n_{do} = v(q_{do}, w_o, \tau_{do}, \varphi) / q_{do} = w_o \tau_{do} \varphi^{-1}$$

2. Quantitative assessment of how frictions shape the incidence of cost shocks

- KM2021 Proposition 2: welfare response to any foreign shock
- KM2021 Proposition 3: trade response to marginal cost shock
- KM2021 Proposition 4: trade intensive and extensive response to search cost shock
- KM2021 Appendix B.1 and B.4: shows how welfare/trade translate to price shocks
- KM2021 Table 4: calibrated price response to marginal cost shock with/without search

3. Relationship status informs search parameters

- KM2021 Appendix C.1.3 shows how relationship status maps to search parameters using the connection between Poisson point processes and the exponential distribution.
- Define $y_{ijpt} = \{0, 1\}$ i exporter, j importer country, p product, t month.
- For example, separation occurs at Poisson rate μ so the probability that separation did not (and formation does) occur during one unit of time (month) is:

$$\mathbb{P} [y_{ijpt} = 1 | y_{ijp,t-1} = 1] = e^{-\mu} \quad \text{and} \quad \mathbb{P} [y_{Fjpt} = 1 | y_{Fjp,t-1} = 0] = 1 - e^{-\gamma_{Fj}}.$$

- Can estimate a fixed effect regression

$$y_{ijpt} = \delta_{ijp} + \delta_{ijt} + \delta_{ipt} + \delta_{jpt} + \alpha y_{ijp,t-1} + \varepsilon_{ijpt}$$

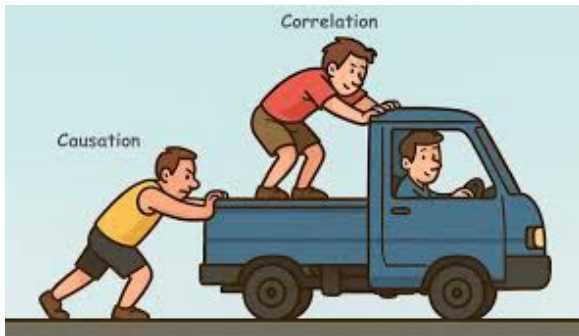
and connect this to the search parameters via

$$\mathbb{P} [y_{ijpt} = 1 | y_{ijp,t-1} = 1] - \mathbb{P} [y_{Fjpt} = 1 | y_{Fjp,t-1} = 0] = e^{-\mu} - (1 - e^{-\gamma_{Fj}}) = \alpha.$$

- Likewise for the other conditional probabilities of relationship status.

4. Absolutely gorgeous data about matches

Drastically undersold the data. Carefully push the data much harder.



5. 1. Search models always have a concept of being unmatched

- In trade and with random search and exogenous separations, this is the fraction of unmatched product varieties (KM2021 and KM2025):

$$\frac{u_{do}}{1 - i_{do}} = \frac{\lambda}{\lambda + \kappa_{do}\chi(\kappa_{do})}, \quad i_{do} = \int_1^{\bar{\varphi}_{do}} dG(\varphi) = G(\bar{\varphi}_{do}).$$

- Unmatched varieties are missing from all aggregate variables (e.g. imports):

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}} n_{do}(\varphi) q_{do}(\varphi) dG(\varphi)$$

- Compared to models without search, fraction of unmatched varieties:
 - alters the **level** of aggregate variables
 - if matched rate responds endogenously in steady-state, alters the **changes** of aggregate variables

5. 2. How are aggregate steady-state variables affected by search?

- Consider aggregate imports with/without search (would be helpful if you derive it):

$$IM_{Fj}^s = (1 - u_{Fj}) N_F \int_0^{\bar{c}} p_{Fj}(c) q_{Fj}(c) dL(c) \quad IM_{Fj}^{ns} = N_F \int_0^{\bar{c}} p_{Fj}(c) q_{Fj}(c) dL(c),$$

in which $u_{Fj} = \mu / (\mu + \gamma_{Fj})$, $\bar{c}(\tau_{Fj}, \tau_{\bar{F}j}, \underline{z})$, and $L_j(c | \mu, \gamma_{Fj}, \gamma_{\bar{F}j}, \tau_{Fj}, \tau_{\bar{F}j})$.

- Consider the elasticity

$$\frac{\partial \ln(IM_{Fj}^s)}{\partial \ln(\tau_{Fj})} = \frac{\partial \ln(1 - u_{Fj})}{\partial \ln(\tau_{Fj})} + \frac{\partial \ln(N_F)}{\partial \ln(\tau_{Fj})} + \frac{\partial \ln\left(\int_0^{\bar{c}} p_{Fj}(c) q_{Fj}(c) dL(c)\right)}{\partial \ln(\tau_{Fj})}$$

- First two terms exogenous so apply Leibniz integral rule to

$$\frac{\partial \ln(IM_{Fj}^s)}{\partial \ln(\tau_{Fj})} = \frac{\partial \ln\left(\int_0^{\bar{c}} p_{Fj}(c) q_{Fj}(c) dL(c)\right)}{\partial \ln(\tau_{Fj})}$$

5. 3. Distributional effects tend to have small effect on changes

- In your setting, response differs with/without search because of the change in the shape of the cost distribution, $dL(c)$, not the exogenous matched rate, $(1 - u_{Fj})$
- KM2021 and KM2025 have changes in that distribution too, but find it has small effects
- KM2021 Appendix A.14.1 bounds the size of the contribution of the distribution to aggregates
 - Intuitively, the cost without search sets a lower bound and the most the producer will pay sets the upper bound
- Conjecture that in your model, changes effects on aggregates via the change in distribution $dL(c)$ are bounded and might also be small so that, for example

$$\frac{\partial \ln \left(IM_{Fj}^s \right)}{\partial \ln \left(\tau_{Fj} \right)} \approx \frac{\partial \ln \left(IM_{Fj}^{ns} \right)}{\partial \ln \left(\tau_{Fj} \right)}$$

- KM2021 repeatedly find that, unless the matched rate is affected, models with search have changes in aggregate variable that are similar to changes without search

5. 4. Level effects are critical for optimal tariffs, KM2025

- Search always affects levels and levels determine optimal tariffs for large open economies (LOE)
- Domestic consumption share in o is a function of unilateral tariffs, t_{do}^u , set by d on o :

$$x_{oo}(t_{do}^u) \equiv C_{oo} / (C_{oo} + C_{do}/t_{do}^u), \quad C_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}} p_{do}(\varphi) q_{do}(\varphi) dG(\varphi)$$

- Without selection, all varieties try to match, PPF is linear [Gros, 1987]:
 - If d is small: $(x_{oo}(t_{do}^u) = 1): t_{do}^u = \mu = \sigma / (\sigma - 1)$
 - If d is large: $t_{do}^u = 1 + 1 / [(\sigma - 1) x_{oo}(t_{do}^u)]$
- With selection and Pareto Melitz [2003] model:
 - If d is small: $(x_{oo}(t_{do}^u) = 1): t_{do}^u = 1 + 1 / (\mu\theta - 1)$ Demidova and Rodríguez-Clare [2009]
 - If d is large: $t_{do}^u = 1 + 1 / (\mu\theta - 1) x_{oo}(t_{do}^u)$ Felbermayr et al. [2013]

Summary

- Leverage prior search-trade theory results more
- Push the data harder
- Tell us when and why search frictions give predictions that differ from models without search
 - With exogenous matched rates, adding search often has small effects on changes we already understand (for example, change in welfare, trade elasticity, intensive, extensive margin etc.).
 - But adding search always affects levels, which matters for other conclusions (like optimal tariffs)

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