# Taxing Sudden Capital Income Surges[†]

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**Abstract**

We study the impact of flat and progressive capital taxes in a continuous-time heterogeneous-agent incomplete markets model with sudden surges in capital income. We find that simple flat taxes on all capital income can increase the skewness and kurtosis of the wealth distribution relative to the labor income distribution. But progressive taxes by taxing sudden capital income surges at a higher rate can reduce wealth inequality and cause less distortion than flat taxation, provided that the government uses tax revenues to finance more public debt.

**Keywords**: Wealth distribution; Exponential tail; Heterogeneous agents; Incomplete markets; Capital taxation.

*JEL Classifications: C61, D83, E21, E22.*

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# 1 Introduction

In this paper, we examine the aggregate and distributional effects of capital income taxation when capital income can experience sudden surges. By examining 100 of richest Americans listed in the Forbes magazine, Graham (2021) argues that "[b]y 2020 the biggest source of new wealth was what are sometimes called 'tech' companies. Of the 73 new fortunes, about 30 derive from such companies. These are particularly common among the richest of the rich: 8 of the top 10 fortunes in 2020 were new fortunes of this type." Halvorsen, Hubmer, Ozkan, and Salgado (2023) study the Norwegian administrative data and find that at least a quarter of wealthiest people start with debt but experience rapid wealth growth early in life. Given the importance of capital income surges in shaping wealth inequality, it is crucial to understand the effects of taxes on normal capital income and on sudden capital income surges.

Building on the model of Benhabib, Cui, and Miao (2024, BCM thereafter), we introduce progressive capital taxes and fiscal policy. BCM provide a tractable general-equilibrium model that accounts for the US distributions of earnings and wealth since 2000 with a focus on how *sudden new fortunes* generated from investment affect the aggregate and the wealth distribution. By contrast, our focus is to compare the aggregate and distributional effects of flat capital taxes with those of progressive capital taxes.

Like the BCM model, our model departs from the standard Bewley-Huggett-Aiyagari (BHA) model by introducing two key ingredients.[1] First, the model separates illiquid capital assets (which incur maintenance costs) from liquid safe assets (bonds) similar to Kaplan, Moll, and Violante (2018). Second, the model introduces idiosyncratic investment risks in the form of Poisson jumps of capital income, which apply only to new capital investments, but not to the rate of return on capital already in place. At each point in time, each household has a chance of investing in a risky project or conducting innovations/R&D. Such activities arrive as rare events and may generate large random capital income. These jumps are critical to account for the top wealth shares.[2] We adopt the *hyper-exponential distribution* (HED) specification for the jump size of entrepreneurial capital income because it allows us to get analytic solutions to compute the stationary equilibrium tractably.

[1] As is well known (e.g., Benhabib and Bisin (2018) and Stachurski and Toda (2019)), a standard BHA model with infinitely-lived agents facing idiosyncratic labor income risks alone generates a counterfactual result that the tail thickness of the model output (wealth) cannot exceed that of the input (income). The reason is that usually precautionary savings compresses the input distribution. By contrast, the capital income jump risks influence precautionary savings in a different way than labor income risks, and therefore our model can generate a thicker tailed wealth distribution than the labor income distribution.

[2] This feature is consistent with the wealth accumulation of some richest Americans in recent years as mentioned before. Another feature is that the wealth distribution converges quickly since there are always some (albeit very few) people who experience this large jumps. In our simulation, the wealth distribution already converges after 15 years in the model.

A nice feature of our model is that it can be applied to study the impact of taxing capital income jumps as progressive capital taxation, in addition to the traditional flat taxes on all capital income.[3] We demonstrate two effects of capital taxation, whether it is flat or progressive. First, following BCM, the economy has a stationary equilibrium where the interest rate is lower than the subjective discount rate, as described by Aiyagari (1994). In this equilibrium, prices and aggregate quantities are determined independently of the full wealth distribution, as only its mean affects the aggregate variables. Notably, increasing the capital tax reduces both capital demand and aggregate saving incentives, leading to a lower capital level. As a result, the interest rate rises under plausible parameters, which reduces the marginal propensity to consume. Second, as the marginal propensity to consume decreases with higher capital taxes, everyone, including poorer households, borrows less or save more, driving wealth inequality in either direction.

In the presence of capital income jumps, the flat-rate capital taxation can increase the skewness and kurtosis of the wealth distribution relative to the labor income distribution. By contrast, with progressive capital taxation, taxing capital income jumps can reduce the relative skewness and kurtosis due to the effect of precautionary savings. Thus, the extent of capital income surges significantly influence how capital tax shapes wealth inequality relative to labor income inequality.

To further examine the quantitative implications of capital taxation, we calibrate our model to confront with the US data. We choose parameter values to match the US micro and macro data, and especially statistics related to the wealth and labor income distributions. The estimated labor income process (with only three parameters) turns out to match the distribution of income growth obtained from the census data closely. Finally, the separation between elasticity of intertemporal substitution (EIS) and risk aversion in agents' utility is important not only for understanding precautionary saving (Weil (1993)), but also for generating a large, realistic marginal propensity to consume (MPC) as in the data.[4] This feature is critical for the existence of a stationary equilibrium and also for matching the data. Quantitatively, by specifying two mixed exponential components for the HED, we find that our calibrated model can match the wealth distribution in the data closely. In particular, we match the wealth shares held by the top 0.1% and 1%.

We find that the impact on wealth inequality depends crucially on how tax revenues are distributed. When tax revenues are transferred to all households evenly, such a policy raises wealth inequality. But when tax revenues are used to finance more government bonds that provide liquidity for precautionary savings, such a policy reduces wealth inequality. This aspect is similar to the beneficial effect of public liquidity provision identified in previous studies, such as Aiyagari and McGrattan (1998), Angeletos, Collard, and Dellas (2020), Bayer, Born, and Luetticke (2023), and

[3]In practice, taxing capital income jumps can be implemented as capital gain taxes.

[4]See Kaplan and Violante (2021) for the impact of recursive utility on the MPC in the discrete-time BHA framework.

Bassetto and Cui (2023).

Additionally, a novel quantitative finding is that taxing the jump part of capital income, similar to progressive capital taxation, has less distortion in investment efficiency, compared to taxing all capital come at a flat rate. That is, given the same amount of tax revenues, progressive capital taxation generates less output loss than flat capital taxation does. This result comes from the fact that the investment incentive is insensitive to the variation of the jump return because it is still extremely high even with, e.g., 20% tax on it and the jump income occurs with a very small probability anyway. This new result can add new perspective to the current debate of wealth taxation. Taxing total wealth is akin to taxing all components of capital income. Our model indicates that taxing the jump component of capital income can reduce inequality while being less distortionary, so it could be a better solution.

**Related literature.** Our paper builds on the BCM model, which contributes to the macroeconomics literature on wealth inequality in the tradition of the BHA model.[5] The advantage of using the BCM framework lies in its recursive utility specification and the novel addition of capital income jumps, which allow us to generate realistic marginal propensity to consume (MPC) and realistic income and wealth distributions, as observed in the data. For instance, the model can match the wealth share of the top 0.1%. Thus, the capital taxation results highlighted in this paper have not only qualitative insights but also quantitative implications.

In our paper, taxing capital in the presence of capital income surges behaves differently than in a scenario where random returns to wealth generate a Pareto tail as in Benhabib et al. (2011). The relationship between wealth inequality and labor income inequality can be non-monotonic with respect to capital taxation when capital income jumps are present. In addition, we demonstrate that simple transfer policies can actually increase inequality, similar to Kaymak and Poschke (2016). We highlight the significance of debt management alongside progressive capital taxation to achieve efficiency and reduce inequality. This complements the research on progressive income taxation in macroeconomic models with uninsurable, idiosyncratic productivity shocks, such as the studies by Conesa and Krueger (2006), Bakis et al. (2015), and Heathcote et al. (2017). Also, previous work, such as by Panousi (2015), studies capital taxation in terms of redistribution under investment risk. Our model highlights the benefit of progressive capital taxation under jump investment risk.

In an overlapping-generations setting, Conesa, Kitao, and Krueger (2009) do not find progressive capital taxation is needed when labor income tax is progressive. The benefit of progressive capital tax in our model comes from the presence of sudden capital income, although our policy analysis is positive in nature. The benefit of taxing sudden capital income shares have similar fea-

[5] See Heathcote, Storesletten, and Violante (2009), Guvenen (2011), Quadrini and Rios-Rull (2015), Krueger, Mitman, and Perri (2015), and Benhabib and Bisin (2018) for recent surveys.

tures of taxing the top 1% labor income, as discussed by Kindermann and Kruger (2022). They show that to match the high concentration of labor earnings and wealth, their model requires that households occasionally have opportunities to earn very high wages through, e.g., attractive but rare entrepreneurial activities. These households' labor supply is insensitive to high marginal tax rates (close to 80%) as long as they have not yet accumulated substantial wealth. A strong negative income effect on leisure keeps these households working hard even with high taxes. Similarly, in our model, capital demand is insensitive to raising the tax rate on the jump component, as it occurs infrequently.

# 2 Model

We extend the BCM model by introducing progressive capital income taxation. We briefly introduce the model setup and refer the reader to BCM for a more detailed presentation.

## 2.1 Preferences

There is a continuum of infinitely-lived households, indexed by $i$ and distributed uniformly over $[0, 1]$. All households have the same recursive utility over consumption in continuous time (Duffie and Epstein (1992)). For simplicity, we omit the household specific index $i$.

Let $dt$ denote the time increment. It helps intuition much better by motivating such utility as the limit of a discrete-time model (Epstein and Zin (1989)) as the time interval shrinks to zero.[6] The continuation utility $U_t$ at time $t$ over a consumption process $\{c_t\}_{t\geq 0}$ satisfies the following recursion:

$$f(U_t) = f\left(c_t\right)dt + \exp\left(-\beta dt\right) f\left(\mathcal{R}_t\left(U_{t+dt}\right)\right), \tag{1}$$

where $\beta > 0$ denotes the rate of time preference, $f$ denotes a strictly increasing time aggregator function, and $\mathcal{R}_t$ denotes a conditional certainty equivalent. Notice that $U_t$ is ordinally equivalent to $f\left(U_t\right)$ for a strictly increasing function $f$.

We adopt the specification of Weil (1993):

$$f\left(c\right) = \frac{c^{1-1/\psi}}{1-1/\psi}, \;\; \mathcal{R}_t\left(U_{t+dt}\right) = u^{-1}\mathbb{E}_t u\left(U_{t+dt}\right), \; u\left(U_{t+dt}\right) = \frac{-\exp\left(-\gamma U_{t+dt}\right)}{\gamma}, \tag{2}$$

where $\gamma > 0$ is the coefficient of absolute risk aversion and $\psi > 0$ $(\psi \neq 1)$ is the EIS parameter. The specification of $f$ in (2) implies that consumption can never be negative.[7]

[6]See Caldara et al (2012) for a comparison of different solution methods for computing the equilibrium of dynamic stochastic general equilibrium models with recursive preferences.

[7]Moreover, the CARA specification of $u$ allows the consumption/saving problem with additive labor income risk

## 2.2 Decision Problem

At each time $t \geq 0$, each household is endowed with one unit of labor. It owns and runs a private firm, which employs labor supplied by other households in the competitive labor market but can only use the capital stock invested by the particular household. Each household faces two independent sources of idiosyncratic shocks that hit its private firm and its earnings. It can only trade riskless bonds and cannot fully diversify away idiosyncratic shocks. We focus on a stationary economy in which all aggregate (per capita) quantities and prices (wage and interest rate) are constant over time. The government imposes a tax rate $\tau_k$ on the base including capital and bonds and a tax rate $\tau_\ell$ on labor income. Besides, it can impose a tax when capital jump income is realized (see below).

Let the (normal) production function take the form

$$y_t = Ak_t^{\alpha} l_t^{1-\alpha},\ \alpha \in (0,1),$$

where $A$ represents TFP, $y_t$, $k_t$, and $l_t$ denote output, capital, and labor, respectively. Let $w$ and $R^k$ denote the *after-tax* wage rate and capital return, and $\delta > 0$ denotes the depreciation rate. Profit maximization implies

$$R^k k_t = (1-\tau_k)\max_{l_t}\left\{Ak_t^{\alpha}l_t^{1-\alpha} - \frac{w}{1-\tau_\ell}l_t - \delta k_t\right\} = (1-\tau_k)\left[\alpha A\left(\frac{(1-\alpha)A}{w/(1-\tau_\ell)}\right)^{\frac{1-\alpha}{\alpha}} - \delta\right]k_t. \quad (3)$$

The household faces idiosyncratic investment risk and labor income (earnings) risk. The effective market hours are represented by the process $(\ell_t)$, which is governed by the dynamics

$$d\ell_t = \rho_\ell (L - \ell_t)\,dt + \sigma_\ell \sqrt{\ell_t}\,dW_t^{\ell}, \quad (4)$$

where $W_t^{\ell}$ is a standard Brownian motion and $\sigma_\ell, \rho_\ell > 0$. This is the square-root process modeled in Cox et al. (1985). One can interpret $\ell_t$ as the product of labor hours and idiosyncratic labor productivity. To ensure $\ell_t$ is positive, we assume that $2\rho_\ell L \geq \sigma_\ell^2$.

Besides, the capital income is hit by a jump shock $dJ_t$, where $J_t$ is a jump process. For each realized jump, the jump size $q$ is drawn from a fixed probability distribution $\nu$ over $[0,\infty)$. Assume that all shocks are independent of each other and across households. For notation simplicity, $q$ already takes into account the capital income taxation. The before-tax jump size will be specified later.

to admit a closed-form solution (Weil (1993)). Angeletos and Calvet (2006) also consider CARA specification for $u$, but they assume that $f(c) = -\psi \exp(-c/\psi)$ is an exponential function. This specification implies that optimal consumption can be negative and cannot generate a stationary wealth distribution.

Suppose that the intensity at which a jump occurs depends upon $k_t$ and is given by $\lambda_t = \lambda_k k_t$, where $\lambda_k > 0$. Intuitively, during any time interval $[t, t+dt]$, the household receives an average capital income $\lambda_k k_t \mathbb{E}_\nu [q]\, dt$. The interpretation is that there is a rare event that the new investment earns a large return and the success probability is positively related to the capital stock. Such a return represents additional output from entrepreneurial risk-taking activities like innovations or R&D.

We specify the jump size distribution $\nu$ explicitly as a hyper-exponential distribution (HED), which is a weighted average of $n$ exponential distributions with nonnegative weights. This type of distributions is flexible and can approximate any completely monotone distributions (Feldmann and Whitt (1998) and Cai and Kou (2011)).[8] BCM find that the HED specification is useful to generate realistic wealth distribution. In this paper, this specification brings new implication for capital tax policy thanks to its simple analytical moment generating function (to be used in the household's problem). The PDF for the HED can be written as

$$f(q) = \sum_{j=1}^{n} p_j \frac{\exp(-q/\mu_j)}{\mu_j},\ q > 0, \tag{5}$$

where $p_j \in [0,1]$, $\mu_j > 0$, and $\sum_{j=1}^{n} p_j = 1$. An interpretation is that given an arrival of innovation, a fraction of $p_j$ households draw capital income jumps from the exponential distribution with mean $\mu_j$. For notation consistency, $\mu_j$ is the after-tax variable. Suppose the government can levy tax on the income jump by a rate $\tau_J$. Then, the pre-tax mean $\tilde{\mu}_j = \mu_j/[(1-\tau_k)(1-\tau_J)]$ for each component.

Capital assets are illiquid and owning $k_t$ of them incurs maintenance costs given by $\eta k_t^2/2 + \chi k_t$ per unit of time, where $\eta > 0$ and $\chi > 0$ are parameters. The household can also trade riskless bonds at the after-tax interest rate $r$ to insure against idiosyncratic shocks. Let $b_t$ denote the household's holding of bonds. Households can borrow and lend among themselves without any trading frictions so that $b_t < 0$ represents borrowing. To deliver a closed-form solution, we do not impose binding borrowing constraints, but a transversality condition on the value function must be satisfied to rule out Ponzi schemes (e.g., Merton (1971)). Let $x_t = b_t + k_t$ denote the household's wealth level. Then the entrepreneurial profits $\pi_t$ follow dynamics

$$d\pi_t = R^k k_t dt - \left(\chi k_t + \frac{\eta}{2} k_t^2\right) dt + dJ_t.$$

[8] Cai and Kou (2011) also study more general mixed-exponential distribution (MED) with possibly negative weights. The MED can approximate any distribution arbitrarily closely (Botta and Harris (1986)). Cai and Kou (2011) show that HED or MED for the jump size is useful for computing option prices given fat-tailed stock returns.

The household faces the following dynamic budget constraints

$$
\begin{aligned}
dx_t &= rb_t dt + d\pi_t + w\ell_t dt - c_t dt + \Upsilon dt \\
&= rx_t dt + \left(R^k - \chi - r\right) k_t dt - \frac{\eta}{2} k_t^2 dt + dJ_t + w\ell_t dt - c_t dt + \Upsilon dt.
\end{aligned} \tag{6}
$$

where $\Upsilon$ represents per capita government transfers (or lump-sum taxes if $\Upsilon < 0$). The household problem is to choose consumption and capital investment processes $(c_t, k_t)_{t\geq 0}$ to maximize utility $U\left(\{c_t\}_{t\geq 0}\right)$ subject to the budget constraints (6), given initial wealth $x_0 = x$ and initial labor $\ell_0 = \ell$. Let $V(x, \ell)$ denote the value function.

Suppose that $0 < r < \beta$ (which will be the case in equilibrium), using BCM's dynamic programming result, we know that value function takes the form

$$
V(x_t, \ell_t) = \theta\left(x_t + \xi_\ell \ell_t + \xi_0\right),
$$

where $\theta$, $\xi_\ell$, and $\xi_0$ are given by

$$
\theta = \left[\psi(\beta - r) + r\right]^{\frac{1}{1-\psi}}, \tag{7}
$$

$$
\xi_\ell = \frac{-(\rho_\ell + r) + \sqrt{(\rho_\ell + r)^2 + 2\sigma_\ell^2 \theta\gamma w}}{\theta\gamma\sigma_\ell^2} > 0, \tag{8}
$$

$$
\xi_0 = \frac{1}{r}\left\{\eta k - \frac{\eta}{2}k^2 + \Upsilon + \xi_\ell \rho_\ell L\right\}. \tag{9}
$$

Then the optimal consumption rule and capital demand (after using the HED specification) are given by

$$
c_t = \theta^{1-\psi}\left(x_t + \xi_\ell \ell_t + \xi_0\right), \tag{10}
$$

$$
k_t = k \equiv \frac{1}{\eta}\left(R^k - \chi - r + \lambda_k \sum_j \frac{p_j}{\mu_j^{-1} + \gamma\theta}\right), \tag{11}
$$

To understand the consumption rule in (10), we need to introduce the concept of human wealth, which is defined as the (after-tax) expected present value of future labor income. For our incomplete markets model with uninsured risk, there is no unique stochastic discount factor used to discount future labor income. The literature typically uses the interest rate $r > 0$ as the discount rate. Formally, we define human wealth as

$$
h_t \equiv \mathbb{E}_t\left[\int_t^\infty e^{-r(s-t)} w\ell_s ds\right] = \frac{w}{r + \rho_\ell}\left(\ell_t + \frac{\rho_\ell L}{r}\right). \tag{12}
$$

Then we can rewrite (10) as

$$c_t = \vartheta\left(x_t + a_h h_t + \Gamma\right), \tag{13}$$

where we define

$$\vartheta \equiv \psi\left(\beta - r\right) + r > 0 \tag{14}$$

$$a_h \equiv \frac{\left(r + \rho_\ell\right)\xi_\ell}{w} \in \left(0, 1\right), \tag{15}$$

$$\Gamma \equiv \frac{\eta k^2}{2r} + \frac{\Upsilon}{r}. \tag{16}$$

The moment-generating function of HED simplifies the derivation.[9] The variable $\vartheta$ represents the marginal propensity to consume (MPC), which is important to understand the consumption behavior and the wealth distribution. The assumption of $0 < r < \beta$ ensures that the MPC is positive. which also shows that the MPC increases with the EIS parameter $\psi$. This assumption will be satisfied in general equilibrium. As is well known, the MPC is equal to $r$ in the standard time-additive CARA utility model (e.g., Caballero (1990) and Wang (2007)) without financing constraints. Importantly, recursive utility in our model helps generate a MPC higher than $r$. Finally, the consumption rule is linear in wealth and it is important for aggregation and useful to analyze wealth distribution and stationary equilibrium.

## 2.3 Government

Let $G$ be the exogenous constant government expenditure. The government has an exogenous fixed bond supply $B$ at each time. The residual $\Upsilon$ is used as lump-sum transfer. When $\Upsilon < 0$, it becomes lump-sum tax. In a stationary equilibrium, the government budget constraint is given by

$$G + \Upsilon + rB = \frac{\tau_k}{1 - \tau_k}\left(R^k + \frac{\tau_J}{1 - \tau_J}\lambda_k \sum_j p_j \mathbb{E}[q_j]\right)K + \frac{\tau_\ell}{1 - \tau_\ell} wL, \tag{17}$$

where $K$ denotes aggregate capital stock and $q_j$ is the stochastic jump size conditioning on drawing $j$th component. As mentioned above, the government tax capital and labor income at the flat rates $\tau_k$ and $\tau_\ell$, respectively. The government also tax additional capital jump income at the rate $\tau_J$.

[9] HED's moment generating function

$$\mathbb{E}_\nu \exp\left(tq\right) = \sum_{j=1}^{n} \frac{p_j}{1 - \mu_j t}, \text{ for } t < \min_j \{1/\mu_j\}.$$

## 2.4 Stationary Equilibrium

We now add household-specific index $i$ and conduct aggregation. Aggregate consumption, labor, capital, wealth, and output are given by

$$
\begin{aligned}
C_t &\equiv \int c_t^i di,\ L \equiv \int \ell_t^i di,\ \ K_t \equiv \int k_t^i di,\ X_t \equiv \int x_t^i di, \\
Y_t &\equiv \int y_t^i di + \lambda_k \sum_j \frac{p_j \mathbb{E}[q_j]}{(1-\tau_J)(1-\tau_k)} = AK_t^\alpha L^{1-\alpha} + \lambda_k \sum_j \frac{p_j \mathbb{E}[q_j]}{(1-\tau_J)(1-\tau_k)}.
\end{aligned}
$$

Aggregate output $Y_t$ consists of two components: total output generated by firm production $\int y_t^i di$ and extra output generated by business risk taking. After aggregation, we are ready to define equilibrium in the steady state.

**Definition.** *Given constant government policy* $(G, B, \tau_k, \tau_\ell, \tau_J)$*, a* stationary competitive equilibrium *consists of constant wage* $w$ *and interest rate* $r$*, individual choices* $\{c_t^i, k_t^i, l_t^i\}_{t\geq 0}$ *for* $i \in [0,1]$*, a transfer policy* $\Upsilon$*, and constant aggregate quantities* $C$*,* $Y$*, and* $K$*, such that (i) given* $(w, r)$*, the processes* $\{c_t^i, k_t^i, l_t^i\}_{t\geq 0}$ *are optimal choices for each household* $i$*; (ii) the bond, capital, and labor markets all clear*

$$
\int b_t^i di = B,\ X = K + B,\ \ \int l_t^i di = L,
$$

*and (iii) finally the government budget* (17) *holds.*

By (13), aggregate consumption is given by

$$
C = \vartheta\left(K + B + a_h H + \Gamma\right), \tag{18}
$$

where $\Gamma$ is defined in (16) with $k = K$ and we use (12) to derive aggregate human wealth as

$$
H \equiv \int h_t^i di = \frac{wL}{r}. \tag{19}
$$

According to the constant-returns-to-scale technology in (3), we can show that the capital/labor ratio is identical for all households. Thus we have

$$
R^k = (1-\tau_k)[\alpha A K^{\alpha-1} L^{1-\alpha} - \delta], \tag{20}
$$

$$
w = (1-\tau_\ell)(1-\alpha) A K^\alpha L^{-\alpha}, \tag{21}
$$

and $AK^\alpha L^{1-\alpha} = \int y_t^i di$. We can also derive the resource constraint

$$
C + G + \delta K + \frac{\eta}{2} K^2 + \chi K = Y, \tag{22}
$$

where $Y$ is aggregate output in stationary equilibrium.

# 3 Calibration

In this section we calibrate our model and examine its quantitative implications for the aggregate economy and for the income and wealth distributions. We solve for the stationary equilibrium numerically and suppose that one unit of time in our model corresponds to one year.

Both capital tax and labor income tax rates are set to $\tau_k = \tau_\ell = 0.25$, so that the government collects 25% of output as tax revenues. We have experimented with different tax rates, and after recalibration the distributional statistics below are similar. The tax rate on jump income $\tau_J$ is set to zero in the benchmark. The rest of parameter choices follows BCM.

Table 1: Calibrated Parameter Values

| | Value | Explanation/Target | | Value | Explanation/Target |
|---|---|---|---|---|---|
| $\beta$ | 0.1417 | MPC = 0.20 | $B$ | 1.6101 | $B/Y = 0.81$ |
| $\gamma$ | 4.100 | relative risk aversion 5 | $G$ | 0.3777 | $G/Y = 0.19$ |
| $\psi$ | 1.5 | EIS | $\tilde{\mu}_2$ | 414.54 | top $0.1\%$ wealth share |
| $\alpha$ | 0.3300 | capital share | $\tilde{\mu}_1$ | 0.1050 | top $20\%$ wealth share |
| $\delta$ | 0.1251 | $I/Y = 0.16$ | $p_2$ | 0.0048 | average innovation return 14% |
| $A$ | 1.3120 | $w = 1$ | $p_1$ | 0.9952 | $1 - p_2$ |
| $L$ | 0.8000 | estimated | $\eta$ | 0.0049 | $R^k - r = 3.0\%$ |
| $\rho_\ell$ | 0.0030 | estimated | $\chi$ | 0.0175 | interest rate $r = 2.5\%$ |
| $\sigma_\ell$ | 0.1097 | estimated | $\lambda_k$ | 0.0500 | innovation probability |
| $\tau_\ell = \tau_k$ | 0.25 | average tax rate | $\tau_J$ | 0 | benchmark |

Consider $\{\alpha, \delta, \psi, \chi, \eta, \beta, \gamma, A, G, B\}$. We set the capital share $\alpha = 0.33$ as in the macro literature. Set the depreciation rate $\delta = 12.5\%$ to target 16% investment to output ratio in the US data. We set the EIS parameter $\psi = 1.5$ in line with the finance literature on long-run risk, and later we conduct a sensitivity analysis with respect to $\psi$. Set the linear maintenance cost parameter $\chi$, the quadratic maintenance cost parameter $\eta$, the subjective discount rate $\beta$, and the CARA parameter $\gamma$ to target the following equilibrium variables: the interest rate $r = 2.5\%$ in line with real return of government bonds,[10] the return premium $R^k - r = 4\%$,[11] the MPC $\vartheta = 0.20$ by (14),

[10] We use the average returns of 1 year Treasury and long-term treasury between 2000 and 2020. The result is robust if we target different maturities, and the target interest rate $r$ is in the range of 2% to 3%.

[11] The premium $R^k - r$ cannot be large relative to the interest rate. One reason for this premium is liquidity premium (because of adjustment costs of capital), which can be approximated by the average of the spread between $AAA$ corporate bonds and treasuries of similar maturity, which is roughly 1% after 1984 (see Krishnurmuthy and Vissing Jorgensen (2012), Del Negro et al. (2017), and Cui and Radde (2020)). Further, the private equity premium is around 2% according to Angeletos (2007) for compensating idiosyncratic risks.

in line with most of OECD aggregate MPC measures (Carroll, Slacalek, and Tokuoka (2014)), and the coefficient of relative risk aversion $\gamma C = 5$, where $C$ is the aggregate consumption level in the stationary equilibrium. We normalize the steady-state (after-tax) wage rate to one by adjusting the TFP parameter $A$.

Government spending $G$ is set so that the government expenditure to output ratio is 19% in line with the data. The debt to output ratio is 81%. These are obtained as the averages between 2000 and 2019, which will be used for our distributional statistics discussed later as well. Since the model does not feature borrowing constraints and the government has the lump-sum tax instrument, calibrating government debt to different levels has no aggregate consequence because of the Ricardian equivalence discussed previously.

Next, BCM obtain the three parameters $L$, $\rho_\ell$, and $\sigma_\ell$ in (4) by simulated method of moments (SMM). The SMM targets important moments in the social-security administrative (SSA) data analyzed by Guvenen et.al. (2021).[12] The SMM procedure uses the standard deviation, the skewness, as well as the fraction of earning changes less than 5%, 10%, and 20%.

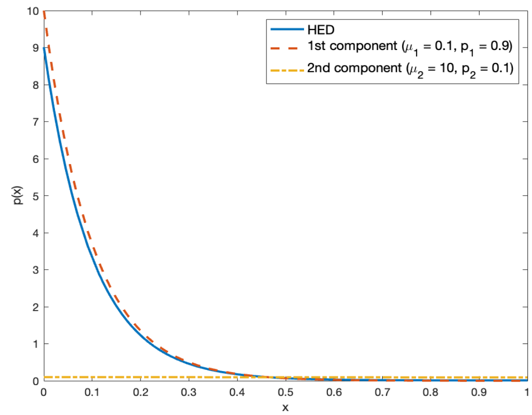
Finally, we consider the remaining parameters in Table 1 that govern the jump process. The jump intensity parameter $\lambda_k$ is set to 5%, and given the equilibrium capital stock $K$, the annual probability $\lambda_k K$ of an innovation or R&D is 12.4%. It should be noted that our model allows for both success and failure of innovations or R&D, because the jump returns may not be enough to compensate the loss arising from adjustment costs. This can happen if the jump size is close to zero. We acknowledge that the success probability varies across different sectors and industries. For example in the pharmaceutical industry, the success probability ranges from 4% to 15% across different development stages.[13]

We choose values of $\mu_1$, $\mu_2$, and $p_1$ (note $p_2 = 1 - p_1$) to target three statistics: 14% of the average pre-tax private returns to innovations and/or R&D (i.e., $\lambda_k \mathbb{E}_\nu [q] / (1 - \tau_k)$ in the model), and top 0.1% and 20% wealth shares in the US data. Griffith (2000) estimates the private returns ranging from 14% to 20% in the US. The public return can be even higher. Our target of 14% for the private return is conservative. Using administrative tax data, Smith, Zidar, and Zwick (2021) estimate that the top 0.1% wealth shares increased from 9.9% in 1989 to 15% in 2016. They also show that the most recent estimates from several approaches in the literature tell starkly different stories about the level and evolution of these wealth shares. We choose 15% in 2016 as

[12]As illustrated by Guvenen et. al. (2021), SSA and contains information for every US individual who was ever issued a Social Security number. Basic demographic variables, such as date of birth, place of birth, sex, and race, are available along with several other variables. The earnings data are derived from the employee W-2 forms, which U.S. employers have been legally required to send to the SSA since 1978. The measure of labor earnings is annual and includes all wages and salaries, bonuses, and exercised stock options as reported on the W-2 form (Box 1).

[13]Therefore, we experiment with different success probability, and recalibrate parameters. We find that the model implication for the wealth distribution statistics does not change significantly, which is consistent with the discussion above about the analytical features of the tails.

Figure 1: Example HED Probability Density Function




our target for the top $0.1\%$ wealth shares. We also choose the top $20\%$ wealth share, which is $79.8\%$ according to the average from the Survey of Consumer Finance after 2000.

To understand better the power of HED, Figure 1 shows the comparison of the probability density functions of an example HED and its two components. For illustration purpose, we set $\mu_1 = 0.1$, $\mu_2 = 10$, $p_1 = 0.9$, and $p_2 = 0.1$. So, the first component of the HED has a mean that is close to zero, but it occurs with $90\%$; the second component has a large mean, but it occurs with only $10\%$. Notice that the HED has a PDF that is very close to that of the first, the most likely component. However, the second component helps the HED to capture the far right tail. As a result, the HED offers flexibility in quantitative analysis because the first component can assist the model in capturing most part of the wealth distribution, while the second part is crucial for the right tail. This feature may explain why using other types of input distribution for the jump size leads to substantial worse quantitative performance. For example, we have tried input distributions including Gamma, Weilbull, and even Pareto, which are supposed to capture the right tail well. Also, notice that a single exponential distribution requires only one parameter, so a mixture of other distributions, instead of exponentials as in the HED, will need many more parameters.

Note that to compute the wealth shares in our model, we run 100 simulations and compute the average. For each simulation, we discretize the equilibrium wealth process $x_t$ (26). The time increment represents one week. In the end, we run 100 simulations of the wealth process $x_t$, each simulation having 15 years and 52 weeks per year and 100,000 people. Increasing simulation length and/or the number of people does not change our results significantly. In (26), important parameters that govern the wealth distributions are $\mu_x = -0.1546$, $\rho_x = 0.1750$, and $\phi = 0.7540$ according to our calibration.

Targeting the top $0.1\%$ showcases the model's ability to match the right tail of the distribution, and targeting the top $20\%$ illustrates the model's ability to capture the general features of inequality (e.g., the conventional view of 80-20 rule). Our model turns out to match non-targeted statistics well. For example, according to the average between 2000 and 2019 obtained from the distributional financial account of Federal Reserve Board, the top 1% and 10% wealth shares are 31.5% and 66.7%, respectively. The corresponding model statistics are 32.2% and 62.7%. Our model generates about 1.5% wealth share for the bottom 50%, slightly below the data 1.7%. Our model also generates 33.7% wealth share for the top 50% to 10%, slightly larger than the data 32%. This discrepancy is expected, as agents in our model do not face borrowing constraints. The model's wealth Gini coefficient is 0.77, slightly below the observed 0.80, partly due to some households in the simulation being in debt and not receiving government transfers.[14] Notably, about 26% of people have negative wealth in the model, although none experience negative consumption.

# 4 Effects of Capital Taxes

In this section we study the effects of capital taxes on the aggregate economy and the wealth distribution.

## 4.1 Aggregate Investment and Saving

In this subsection, we use the asset/investment demand and supply analysis of Aiyagari (1994) to understand the aggregate equilibrium determination.

We first derive the aggregate investment demand curve. Combining equations (11) and (20) yields

$$(1-\tau_k)(\alpha(K/L)^{\alpha-1}-\delta)-\chi=\eta K+r+\lambda_k\sum_j\frac{p_j}{\tilde{\mu}_j^{-1}(1-\tau_k)(1-\tau_J)+\gamma\theta} \tag{23}$$

Because $\theta$ is a function of $r$ given in (7), we can use (23) to derive aggregate capital $K$ as a function of the interest rate $r$, denoted by $K(r)$. Then we obtain the aggregate investment demand curve $\delta K(r)$. Next we derive the aggregate saving curve. Aggregate savings $S$ can be shown as

$$\begin{aligned} S &\equiv Y-C-G-\frac{\eta}{2}K^2-\chi K \\ &= (r+\delta-\vartheta)K+wL(1-\vartheta a_h/r)+\frac{1}{2}\eta K^2\left(1-\frac{\vartheta}{r}\right) \\ &\quad +\lambda_k K\sum_j\frac{p_j\tilde{\mu}_j^2(1-\tau_k)^2(1-\tau_J)^2}{\tilde{\mu}_j(1-\tau_k)(1-\tau_J)+(\gamma\theta)^{-1}}+(1-\vartheta/r)(rB+\Upsilon). \end{aligned} \tag{24}$$

[14] There is a well-known issue of calculating the Gini coefficient for a distribution with negative values. In our case, the negative wealth of some borrowing agents is treated as zero.

where we have substituted the aggregate consumption function (18) into the above equation. Since aggregate capital $K$ is a function of $r$, aggregate output $Y$ is also a function of $r$ . As a result, $S$ is a function of $r$. Aggregate savings consist of five components. The first component $(r + \delta - \vartheta) K$ represents savings out of capital assets. The second component is precautionary savings against the Brownian labor income risk. The third component represents savings out of capital returns. The fourth component represents precautionary savings against the capital return jump risk. The last component is proportional to public savings (taxes minus government expenditure excluding lump-sum transfers/taxes).

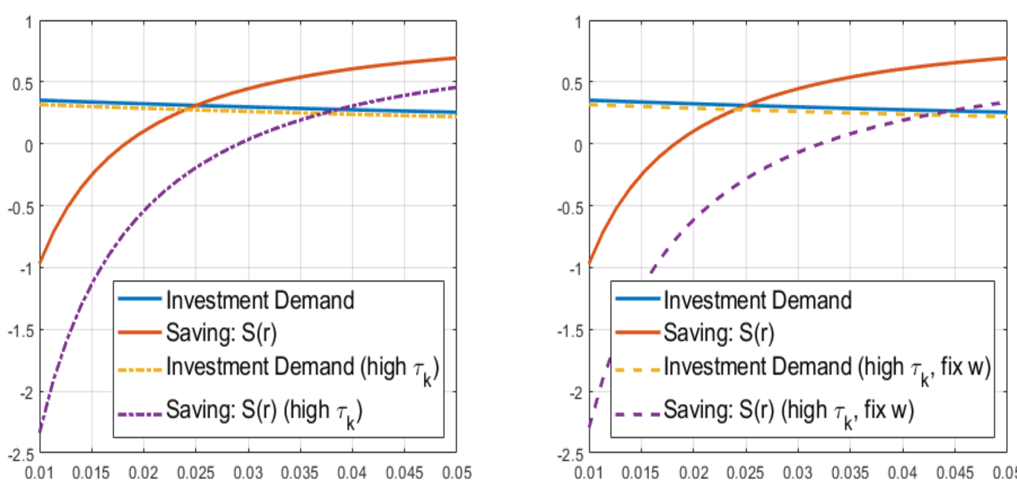
By the market-clearing condition, aggregate saving is equal to aggregate investment so that

$$S(r) = \delta K(r), \tag{25}$$

which determines the stationary equilibrium interest rate. Following the proof by BCM, we can show that the equilibrium interest rate $r$ satisfies $0 < r < \beta$.

By (23), we can also show that an increase in the capital income tax rate ($\tau_k$ or $\tau_J$) reduces the demand for capital for any given $r$. At the same time, the tax increase pushes down the saving curve according to (24). The equilibrium level of capital stock thus falls; but depending on the magnitude of reductions in demand for capital and supply of saving, the equilibrium interest rate $r$ may or may not fall.

Figure 2: Illustration: Investment and Savings functions with different $\tau_k$




Note: we use parameters obtained from calibration below. "High $\tau_k$" corresponds to an economy with 5% higher in the level of tax rate. "fix w" means fixing the wage rate.

Given the calibrated parameter values in the previous section, Figure 2 illustrates the impact of a higher capital tax rate $\tau_k$. We find that the equilibrium interest rate rises with $\tau_k$ since the fall of saving dominates that of capital demand. Similar qualitative impacts on saving and investment curves are found for adjusting tax rates $\tau_J$, and therefore we leave them out for simpler exposition.

According to our equilibrium definition, we take $B$ as given and allow $\Upsilon$ to be determined endogenously such that the government budget constraint holds. A special feature our model is that aggregate Ricardian equivalence holds in the following sense. A dollar increase in the government bond supply $B$ can be offset by $r$ dollars decrease in the government transfer. As a result, the changes in $B$ and $\Upsilon$ can offset each other so that aggregate consumption in (18) and aggregate savings in (24) do not change with $B$. By (23), the aggregate investment demand does not depend on $B$ or $\Upsilon$. Thus, the equilibrium aggregate capital stock $K$, the interest rate $r$, and the wage rate $w$ do not change with $B$ provided that $rB + \Upsilon$ is held fixed. However, debt policy $B$ has an impact on the individual decisions and shifts the wealth distribution. Every household prepares exactly enough to offset the consequent change in the lump-sum transfer/taxes and bond holdings, and this shift has unequal impact on households in an environment with uninsurable idiosyncractic risks. We will provide a further discussion on the wealth distribution later.

## 4.2 Income and Wealth Distributions

To study the wealth distribution, we substitute the optimal consumption rule (10) into the wealth dynamics (6) to derive

$$dx_t = -\rho_x x_t dt + \mu_x dt + \phi w \ell_t dt + dJ_t, \tag{26}$$

where $\rho_x \equiv \psi(\beta - r)$ and $\phi \equiv 1 - \frac{\vartheta \xi_\ell}{w}$, and

$$\mu_x \equiv \left(R^k - \chi - r\right) k - \frac{\eta}{2} k^2 - \vartheta \xi_0 + \Upsilon, \tag{27}$$

Clearly, $\rho_x > 0$ if $r < \beta$. The term $\phi$ represents the marginal propensity to save (MPS) out of labor income. We restrict parameter values such that $\phi > 0$ in equilibrium.

Let $z_t \equiv w\ell_t$ denote labor income. It follows from (4) that

$$dz_t = \rho_\ell (Z - z_t) dt + \sigma_z \sqrt{z_t} dW_t^l, \tag{28}$$

where $Z \equiv wL$ and $\sigma_z \equiv \sqrt{w}\sigma_\ell$. For $\rho_x > 0$ and $\rho_\ell > 0$, the joint wealth and labor income process $\{x_t, z_t\}$ has a limiting stationary distribution if $\mathbb{E}_\nu \ln(1+q) < \infty$ (Jin, Kremer, and Rüdiger (2020)). The assumption on the jump distribution $\nu$ means intuitively that large jumps are not strong enough to push the process eventually to infinity. By a law of large numbers, the stationary distribution of the joint process gives the cross-sectional stationary distribution of wealth and earnings.[15] Given the HED (5) for the jump size $q > 0$, it follows from Jin, Kremer, and Rüdiger (2020) that the joint process $\{x_t, z_t\}$ has a stationary distribution and its law converges to this distribution exponentially fast.

As is well known, the square-root labor income process $z_t$ has a stationary Gamma distribution, which has an exponential tail. To study the tail property of the wealth distribution, BCM analyze the exponential moments of $x_t$ as $t \to \infty$, $\lim_{t\to\infty} \mathbb{E}\left[\exp\left(\alpha x_t\right)\right]$ for $\alpha > 0$, following Keller-Ressel and Mayerhofer (2014) and Glasserman and Kim (2010).[16] Then, the effects of capital tax can be shown explicitly by modifying their result; both the stationary wealth and labor income distributions have exponential tails with the exponential decay rates given by

$$\overline{\alpha}_x \equiv \min_j \left\{ \frac{1}{(1-\tau_k)(1-\tau_J)\tilde{\mu}_j} \right\}, \quad \overline{\alpha}_z \equiv \frac{2\rho_\ell}{\sigma_\ell^2}.$$

Therefore, a higher capital tax rate in $\tau_k$ or $\tau_J$ increases the decay rate of the wealth distribution.

Given our baseline calibration in Table 1 without taxing the capital income jump, we find that $\overline{\alpha}_x = 1/[(1-\tau_k)\tilde{\mu}_2] = 0.0032$ and $\overline{\alpha}_z = 0.50$. Thus the wealth distribution has a much smaller exponential decay rate than the labor income distribution. Intuitively, the exponential decay rate of the wealth distribution depends on that of the capital return jump size distribution. If the capital income jump size is drawn from some exponential distribution with a sufficiently large mean $\mu_j$, then the exponential decay rate of the wealth distribution is given by $1/\mu_j$, which can be much smaller than the exponential decay rate of the income distribution $\overline{\alpha}_z$. The larger is $\mu_j$, the smaller is the exponential decay rate of the wealth distribution. Intuitively, the top wealth share is essentially determined by those who receive large capital income jumps. Without capital income jump risks, the wealth distribution would have a lighter tail than the income distribution.

In addition to the analysis of the tail property, we next study the skewness and the kurtosis of the wealth and labor income distributions, denoted by $Skew[x]$ and $Kurt[x]$ for any variable $x$. BCM derive the following explicit expressions for these statistics for the HED in (5):

$$\mathrm{Var}[z] = \frac{\sigma_z^2 Z}{2\rho_\ell}, \ \mathrm{Skew}[z] = \sqrt{\frac{2\sigma_z^2}{\rho_\ell Z}}, \ \mathrm{Kurt}[z] = \frac{3\sigma_z^2}{\rho_\ell Z},$$

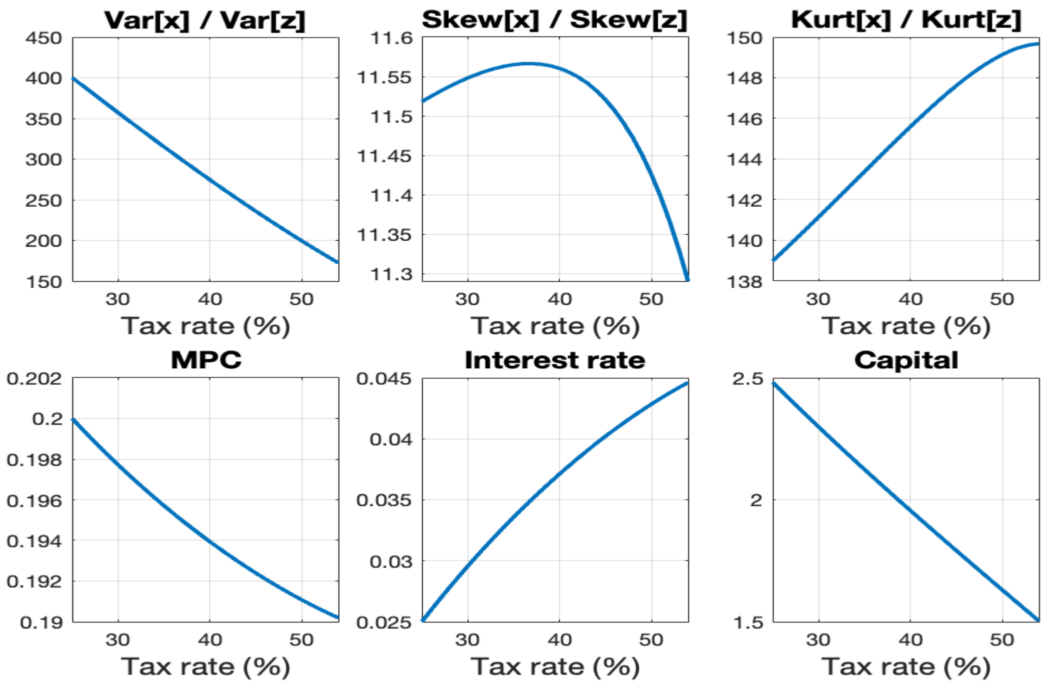
$$\frac{\mathrm{Var}[x]}{\mathrm{Var}[z]} = \frac{\phi^2}{\rho_x\left(\rho_x+\rho_\ell\right)} + \frac{\lambda_k K \zeta_2}{2\rho_x \mathrm{Var}[z]}, \tag{29}$$

$$\frac{\mathrm{Skew}[x]}{\mathrm{Skew}[z]} = \frac{2\sqrt{\rho_x\left(\rho_x+\rho_\ell\right)}}{2\rho_x+\rho_\ell}\left[1+\frac{\left(\lambda_k K \zeta_2\right)\left(\rho_x+\rho_\ell\right)}{2\mathrm{Var}[z]\phi^2}\right]^{-3/2} + \frac{\lambda_k K \zeta_3}{3\rho_x \mathrm{Skew}[z]\left(\mathrm{Var}[x]\right)^{3/2}}, \tag{30}$$

[15]This distribution can be derived numerically using the transform analysis of Duffie, Pan, Singleton (2000).

[16]Notice that when a limiting stationary distribution exists, the limiting exponential moment of $x_t$ does not depend on the initial value $x_0$.

Figure 3: Inequality of Wealth and Income with Different $\tau_k$ in Equilibrium




Note: we use parameters obtained from calibration below and change the capital tax rate $\tau_k$.

$$\begin{aligned}\frac{\text{Kurt}[x]}{\text{Kurt}[z]} &= \frac{\rho_x\left(5\rho_\ell+6\rho_x\right)}{\left(3\rho_x+\rho_\ell\right)\left(2\rho_x+\rho_\ell\right)}\left(1+\frac{\varpi_1\rho_x\left(\rho_x+\rho_\ell\right)}{\phi^2}\right)^{-2} \\ &+\frac{3}{\text{Kurt}[z]}\left[\frac{\phi^2\left(\rho_\ell\left[\rho_x\left(\rho_x+\rho_\ell\right)\varpi_1+\phi^2\right]+3\phi^2\rho_x\right)}{\left(3\rho_x+\rho_\ell\right)\left[\rho_x\left(\rho_x+\rho_\ell\right)\varpi_1+\phi^2\right]^2}-1\right] \\ &+\left[\frac{3\phi^2}{\rho_x\left(3\rho_x+\rho_\ell\right)}\frac{\sigma_z^2 Z\text{Var}[z]}{2\left(\rho_x+\rho_\ell\right)}\varpi_1+\varpi_2\right]\frac{1}{\text{Var}[x]^2\text{Kurt}[z]},\end{aligned}$$

where

$$\zeta_m \equiv \mathbb{E}_\nu\left[q^m\right] = m!\sum_{j=1}^{n} p_j\mu_j^m \quad \text{for } m \geq 1.$$

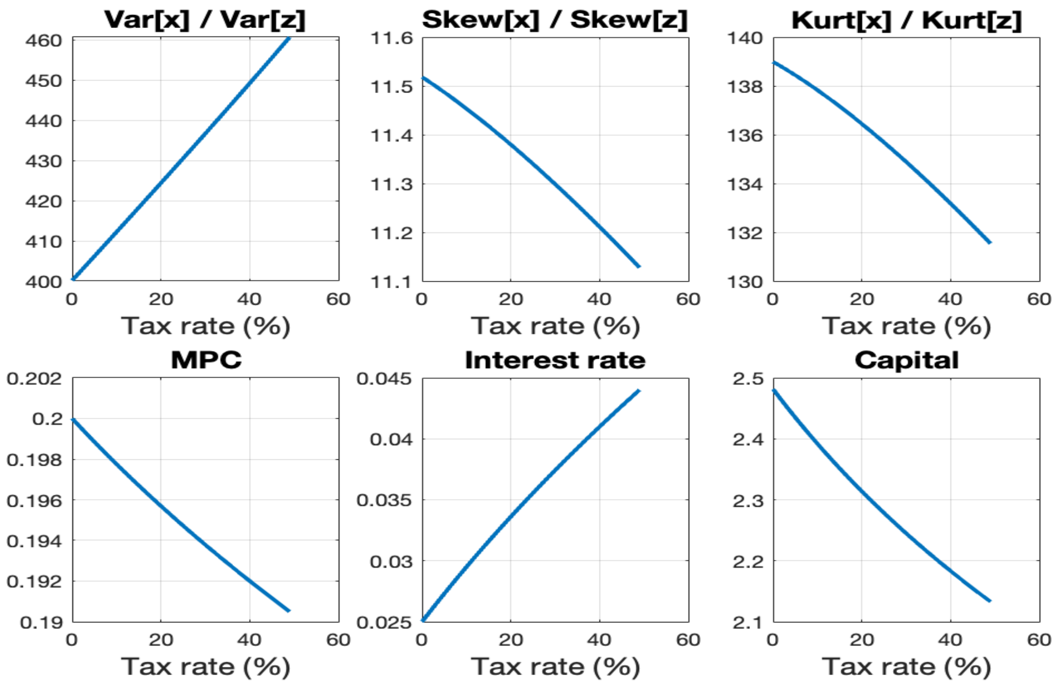
and $\varpi_1 > 0$ and $\varpi_2 > 0$ are defined in the appendix of BCM.

Using the above expressions, we can easily show that without the capital income jump risk (i.e., $\lambda_k = 0$), increasing the tax rate $\tau_k$ reduces the relative skewness $Skew[x]/Skew[z]$ and the relative kurtosis $Kurt[x]/Kurt[z]$ if the equilibrium interest rate $r$ rises or $\rho_x$ declines.

By contrast, in the presence of the capital income jump risk ($\lambda_k > 0$), capital tax may not necessarily reduce the relative skewness because the additional terms in (30) change in a complex way as $K$, $\rho_x$, and $\phi$ all change with the capital tax rate.

Given our calibration, the equilibrium interest rate $r$ increases with $\tau_k$, leading to a fall in $\rho_x = \psi(\beta - r)$ as shown in Figure 3. In the absence of jump risks, this scenario would result

Figure 4: Inequality of Wealth and Income with Different Jump Tax $\tau_J$ in Equilibrium




Note: we use parameters obtained from calibration below and change the capital tax rate $\tau_J$.

in a reduction of $Skew[x]/Skew[z]$, indicating that capital taxation can mitigate the inequality generated by the labor income risk. However, the presence of capital jump income risks can change this outcome. Figure 3 shows that $Skew[x]/Skew[z]$ initially rises with the tax rate $\tau_k$ and even displays a hump-shaped pattern. The same pattern is true for the relative kurtosis (albeit the turning point will be seen at a higher tax rate $\tau_k$).

Notice that MPC goes down when the tax rate $\tau_k$ goes up. This means that everyone in the economy saves more or borrows less out of their wealth. Therefore, wealth skewness or kurtosis relative to income skewness can go up as the rich saves more and this effect dominates under our calibrated parameter values. Recall that the decay rate of the wealth tail is higher with a higher tax rate $\tau_k$, so the relative skewness and kurtosis of wealth eventually decline with the tax rate $\tau_k$.

By contrast, when the jump capital income tax rate $\tau_J$ goes up (Figure 4), the relative skewness and kurtosis of wealth decline, although the effects on aggregate variables are similar to those of raising $\tau_k$. Intuitively, a higher $\tau_J$ reduces the sudden large fortunes of the rich and reduces the wealth inequality.

As both tails and moment statistics cannot fully characterize the whole wealth distribution, we provide additional statistics in the next section.

# 5 Fiscal Policy Experiments

In this section, we consider four fiscal policy experiments with policy instruments $(\tau_k, \tau_\ell, \tau_J, B, G, \Upsilon)$. For all these experiments, we hold $G$ and $\tau_\ell$ fixed and study the effects of changes in some of the other policy instruments on the equilibrium aggregate variables and wealth distribution.

Specifically, we study the following four policy experiments starting from the same calibrated parameter values in Table 1:

1. Fixing $(\tau_J, B, G)$, increase $\tau_k$ and transfer the increased tax revenue to all households equally by raising $\Upsilon$.

2. Fixing $(\tau_k, B, G)$, increase $\tau_J$ and transfer the increased tax revenue to all households equally by raising $\Upsilon$.

3. Fixing $(\tau_J, G, \Upsilon)$, increase $\tau_k$ and use the increased tax revenue to finance additional government debt by raising $B$.

4. Fixing $(\tau_k, G, \Upsilon)$, increase $\tau_J$ and use the increased tax revenue to finance additional government debt by raising $B$.

We focus on a positive analysis only and do not consider household welfare implications. For all 4 policies, we adjust the increase in $\tau_k$ or $\tau_J$ such that the government raises the same amount of tax revenues. We will show by numerical solutions that taxing the capital income jumps to fund more government debt (policy 4) is the best of all four policies as it generates the least wealth inequality.[17] In addition, policy 4 also generates a mild distortion on efficiency.

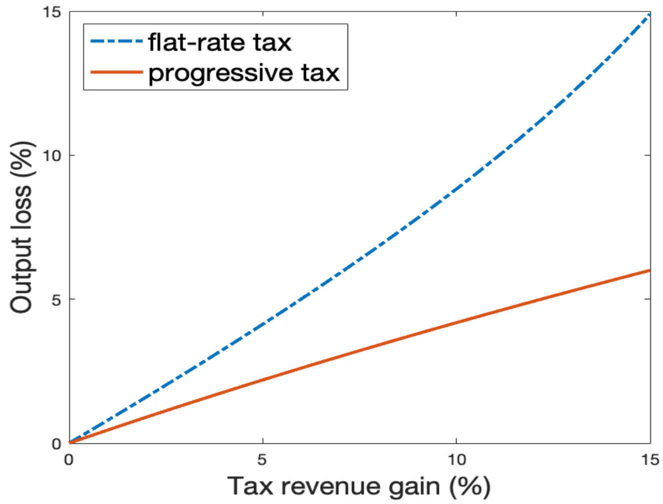
We start our analysis with policies 1 and 2 in the next two subsections, because our definition of equilibrium assumes that $B$ is fixed. We then consider the impact of changes in $B$ in subsection 5.3.

## 5.1 Aggregate Effects

In this subsection, we study the aggregate effects of policies 1 and 2 and leave the distributional effects to the next subsection. We focus on the output losses after raising either $\tau_k$ or $\tau_J$. We normalize the total tax revenues (from the right-hand side of the government budget constraint) in the benchmark calibration to be 100%. Figure 5 shows that the output loss as a function of the extra total tax revenue raised.

[17] The effect of debt policy in incomplete-market models was discovered at least since Aiyagari and McGrattan (1998). A higher level of government debt improves risk-sharing, and our model's result is related to that.

Figure 5: The different effects from flat-rate and progressive taxation




By the analysis in Section 4.1, the aggregate capital stock $K$ declines when either $\tau_k$ or $\tau_J$ is increased, but the change is smaller if the government raises revenues from raising $\tau_J$, compared to raising $\tau_k$. The reason is that the capital demand in (11) is insensitive to the change of $\tau_J$ given our calibrated parameter values. Specifically, notice that $\mu_j = (1 - \tau_J)\tilde{\mu}_j$ and $\tau_J$ only affects the jump risk premium

$$\sum_j \frac{p_j}{\mu_j^{-1} + \gamma\theta},$$

which is not sensitive to $\tau_J$ given the equilibrium value of $\theta$ (which is 25) and other parameters in our calibration. Our calibration specifies two components in the HED: $\tilde{\mu}_2$ is large with a very small $p_2$, while $\tilde{\mu}_1$ is small with a large $p_1$. Intuitively, households would still invest as much whether they earn \$600 million or \$400 million a year, as long as it keeps them at the top. The consequence is that the elasticity of investment decision is less responsive to progressive taxation.

In other words, to raise the same amount of tax revenues, capital will be distorted less under policy 2 and the output loss is thus smaller. Figure 5 shows that to obtain extra 5% tax revenue, policy 1 generates 4.7% fall in output while policy 2 only generates 2.2%. In addition, the output loss is almost a linear function of the tax revenue gain under policy 2, while the output loss under policy 1 is a convex function. That is, to achieve higher tax revenues, the distortion on output becomes more severe if the government raises only the flat-rate tax $\tau_k$.

## 5.2 Distributional Effects

In this subsection, we study both the aggregate and distributional effects of policies 1 and 2. Suppose that the government raises either $\tau_k$ or $\tau_J$ such that the government can receive an extra tax revenue, say 5% out of the original level. Table 2 shows the outcomes of lump-sum transfer redistribution. There are three effects.

Table 2: Taxation with lump-sum transfer policy

| | Capital | Wealth | $r(\%)$ | MPC(%) | Bottom 50% (%) | Top 10% (%) | Top 1% (%) | Top 0.1% (%) | Gini Coeff. (%) |
|---|---|---|---|---|---|---|---|---|---|
| $\tau_k = 0.25$, $\tau_J = 0$ | 2.48 | 4.05 | 2.50 | 20.00 | 1.5 | 62.7 | 32.2 | 15.0 | 77.0 |
| $\tau_k = 0.3115$, $\tau_J = 0$ | 2.26 | 3.94 | 3.05 | 19.72 | 0.50 | 63.4 | 32.0 | 15.3 | 77.5 |
| $\tau_k = 0.25$, $\tau_J = 0.1373$ | 2.36 | 4.08 | 3.11 | 19.70 | 0.87 | 63.1 | 32.0 | 15.0 | 77.3 |

Notes: For each parameter, the corresponding row shows the result in the stationary equilibrium. Both taxation policies raise additional 5% more revenues compared to the benchmark (first row).

First, as previously demonstrated, taxing capital income jumps in our environment better targets individuals with less elastic capital investment incentives, resulting in lower aggregate distortion compared to the flat-rate tax $\tau_k$. Thus, aggregate investment and capital falls less with raising $\tau_J$ than with raising $\tau_k$, and so does the arrival rate $\lambda_k K$ of capital income jump. This effect will reduce inequality, but the progressive taxation reduces less because the jump probability $\lambda_k K$ falls less.

Second, the government transfer increases households' net worth and decreases precautionary savings (influencing $\xi_0$ in (9) and the wealth accumulation process (26)), prompting poorer households to borrow in order to boost their consumption. In our model, the absence of borrowing constraints results in some poor individuals accumulating debt, causing the bottom 50% to hold significantly less wealth. Consequently, the top 10% hold more wealth, and the wealth Gini coefficient rises. Therefore, the lump-sum transfer policy exacerbates inequality under both types of tax policies.[18]

Third, we find that the increase in wealth inequality is less severe due to the general equilibrium price feedback effect. In particular, the wealth shares of top 1% and top 0.1% are similar to those

[18] For similar results, see Kaymak and Poshke (2016). By contrast, in a model without considering the distribution of tax revenues, Benhabib et al. (2011) show that capital tax on $r$ and $R^k$ reduces wealth inequality when wealth follows a Kesten process with a stochastic rate of return. The reason is that the thickness of the right tail depends on the probability of the stochastic wealth return (net of consumption out of wealth) that is above 1 in discrete time and above 0 in continuous time. Inequality is the result of some lucky agents that get long streaks of realizations of high returns. Taxing the stochastic return of wealth/capital reduces the range of its realized values and therefore the inequality.

in the benchmark calibrated model. The wealth Gini coefficients are slightly higher (around 0.774 under the two types of tax policies) than the benchmark Gini coefficient. The main reason is that the equilibrium features a higher interest rate after an increase in the capital tax rate, which puts a downward pressure on the borrowing from the poor households. Though the wage rate declines as $K$ declines, this negative effect on wealth inequality is dominated because of the higher debt incurred by the poor.

Overall, the redistribution effects of the two types of taxation are similar, particularly for the wealthiest households, as shown in Table 2. While the progressive tax policy is designed to reduce inequality more effectively, it also maintains a higher capital level, leading to a higher jump arrival rate and increasing the wealth share of the top households. These two effects balance each other out quantitatively, resulting in the redistribution effect of raising the progressive tax rate being similar to that of increasing only the flat capital tax rate.

Combining the aggregate and distributional results, we find the following. Taxing capital income via the all-component rate $\tau_k$ together with lump-sum transfer redistribution not only reduces efficiency (measured by less investment and less output) but also increases inequality, which is usually considered as a bad policy outcome. Therefore, it is natural to examine the second spending policy that allows the government debt to vary (see Table 3).

## 5.3 Debt Policy

For polices 3 and 4, the government uses the increased tax revenues to finance additional government debt instead of transfers. An increase in public debt injects public liquidity and raises the level of safe assets for precautionary saving purposes similar to Aiyagari and McGrattan (1998). We assume that the increased debt $\Delta B$ in policy 3 and transfers $\Delta \Upsilon$ in policy 1 satisfy $r\Delta B = \Delta \Upsilon$, where $r$ is the equilibrium interest rate in policy 1. Then by the limited Ricardian equivalence discussed before, we deduce that policies 1 and 3 give the same equilibrium $r$, $w, K,$ and $Y$. The same is true for policies 2 and 4.

However, the higher level of debt turns out to generate opposite distributional consequences compared to the case of lump-sum transfers.

The main difference between the transfer policy and the debt policy is that the former changes each household's consumption decision rule directly (see (9) and (10)), while the latter does not, holding prices fixed. As a result, these two types of polices generate different wealth dynamics and hence wealth distributions. In general equilibrium, there are three effects on inequality for the debt policy, similar to the case of the transfer policy. First, the decrease of the capital stock reduces the arrival rate of capital income jumps, thereby reducing inequality. Second, raising the government bond supply provides more assets for the households to save and reduces precautionary

Table 3: Taxation with bond policy

| | Capital | Wealth | $r(\%)$ | MPC(%) | Bottom 50% (%) | Top 10% (%) | Top 1% (%) | Top 0.1% (%) | Gini (%) |
|---|---|---|---|---|---|---|---|---|---|
| $\tau_k = 0.25$, $\tau_J = 0$ | 2.48 | 4.05 | 2.50 | 20.00 | 1.5 | 62.7 | 32.2 | 15.0 | 77.0 |
| $\tau_k = 0.3115$, $\tau_J = 0$ | 2.26 | 4.33 | 3.05 | 19.72 | 4.88 | 58.7 | 29.4 | 14.0 | 73.2 |
| $\tau_k = 0.25$, $\tau_J = 0.1373$ | 2.36 | 4.42 | 3.11 | 19.70 | 4.75 | 58.9 | 29.5 | 13.8 | 73.5 |

Notes: For each parameter, the corresponding row shows the result in the stationary equilibrium. Both taxation policy raises 5% more tax revenues compared to the benchmark (first row).

savings. This effect raises inequality. Third, the rise of the equilibrium interest rate discourages poor households to borrow.

Unlike the transfer policy, the net effect of raising the government debt under both types of capital taxation is to reduce inequality in a similar magnitude quantitatively. For example, the Gini coefficient falls from 0.77 to just slightly above 0.73 in the two cases. The bottom 50% wealth share and the top 1% wealth share are quite close too. However, the capital stock is 4.42% higher when raising $\tau_J$ than that when raising $\tau_k$. These results lead to us to conclude that taxing capital income jumps to fund the government debt can achieve a good balance between efficiency and redistribution.

# 6 Conclusion

In this paper we study the impacts of capital taxation in a tractable heterogeneous-agent model with incomplete markets in continuous time. The environment features that rich people can build wealth from rare capital return/income jumps through technology innovations or R&D and the jump size is stochastic. The jump size distribution is important to explain the wealth distribution in the extreme right tail.

We find that taxing capital income surges can be effective, because the investment incentive may not be sensitive to jump risks. When tax revenues are transferred to all households evenly, reducing the precautionary savings of the poor, such a policy raises wealth inequality. But when tax revenues are used to finance more government debt (public liquidity provision), such a policy reduces wealth inequality. In future research, an optimal policy design in this kind of environment with jump risks could shed light on the optimal tax schedules, especially if the jump risks themselves are time-varying.

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