The impact of central bank digital currency on bank deposits and the interbank market[*]

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Abstract

This paper proposes a theoretical model in which a central bank digital currency (CBDC) and bank deposits are imperfect substitutes. Deposits are subject to liquidity shocks. In the absence of a CBDC, the interbank market can redistribute liquidity between banks. The introduction of CBDC leads to a greater reliance of the banking sector on central bank standing facilities. Calibrating the model to the Eurozone, the model shows that adjusting the remuneration rate of CBDC has little pass-through to the deposit rate set by banks and also has implications for the transmission of monetary policy.

Keywords: central bank digital currency, banking, money, interbank market

JEL codes: E42, E52, E58, G21

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1 Introduction

The introduction of a retail central bank digital currency (CBDC) is currently under active consideration by central banks around the world. A key motivation for the introduction of a central bank digital currency (CBDC) is the decrease in cash use. According to the ECB Study on the payment attitudes of consumers in the euro area (SPACE) from 79% of all point-of-sale transactions in the Eurozone in 2016 to 72% in 2019 and 59% in 2022, (ECB, 2022). The 2022 wave of the SPACE survey suggests that cash may no longer be the preferred means of payment within the Eurozone; 55% of consumers within the Eurozone stated a preference for using cards and other cashless payments in stores, while only 22% preferred to use cash.

In the context of the decline in physical cash use, the introduction of a CBDC can be considered a way to modernize fiat currency for the digital age. In addition, a CBDC is likely to have technical features that make it a closer substitute for bank deposits than physical cash. Thus, CBDC is likely to be a greater source of competition for banks in the deposit market. This gives rise to potential risks associated with the introduction of a CBDC, for example, the financial stability impact of an increase in the cost of bank funding highlighted by Broadbent (2016).

Through the introduction of a retail CBDC, households will essentially be able to hold a bank account directly with the central bank. To maintain a well-functioning payment system, the introduction of a CBDC is likely to require additional settlement transactions between the banking sector and the central bank. This paper focuses on the consequences of this channel for the structure of the deposit market and the implementation of monetary policy.

I propose a theoretical model where CBDC and bank deposits are imperfect substitutes and where deposits are subject to liquidity shocks. Banks are able to transfer liquidity between themselves through an interbank market. I assume that it is more costly for banks to trade with the central bank than in the interbank market. This cost could materialize, for example, due to central banks requiring better quality collateral than would be required in the interbank market. In this paper, I assume the increased costs occur as, following the liquidity shocks, banks are only able to trade with the central bank via standing facilities. As a consequence, the introduction of a CBDC increases the banking sector's use of the central bank standing facilities, and thus increases the costs associated with deposits. In this setting, CBDC raises the cost of bank funding in two ways; directly by competing for depositors and indirectly by increasing the number of transactions with the central bank.

The deposit market is modeled as in the spatial competition model of Salop (1979) with the addition of a central bank. A continuum of atomistic depositors choose to deposit their funds at one of a finite number of banks or, through a CBDC, at the central bank. This deviates from existing models of CBDC, where households hold portfolios of liquid assets consisting of both bank deposits and CBDC. There is some evidence that many households do not hold multiple deposit accounts simultaneously. As part of the UK Competition and Markets Authority's investigation into the retail banking market, they commissioned a survey by GfK NOK which found that only 22% of UK households actively used a personal current account at more than one bank, (Moon et al., 2015). Whether households are more willing to hold both bank deposits and CBDC simultaneously is likely to depend on the specific design choices of CBDC.

In order to study the impact of a CBDC on the structure of the deposit market, I consider two equilibria; a short-run equilibrium where the number of banks is fixed and a long-run equilibrium where the number of banks adjusts according to a free entry condition. With the number of banks fixed, banks respond to the introduction of CBDC by increasing the deposit rate as banks attempt to maintain market share. If the number of banks is able to adjust, the deposit market becomes more concentrated following the introduction of CBDC. This provides an additional channel for adjustment, which dampens the effect on deposit rates.

Using this model, I study the effect of introducing a CBDC on the structure of the deposit market and its monetary policy implications. In particular, I focus on two parts of the policy debate around CBDC. First, the effectiveness of the CBDC remuneration rate as an additional tool in the monetary policy toolkit and second, the implications CBDC has on the transmission of the policy rate through the deposit market. To assess the empirical relevance of the theoretical results, I calibrate the model for the Eurozone.

The model makes several predictions that have important policy implications. First, if the banks do not face liquidity risk from deposit financing, then in the short-run the introduction of a CBDC results in a fall in the market shares of banks in the deposit market and upward pressure on banks to raise deposit rates in the face of greater competition. In the absence of liquidity risk, the bank deposit rate is strictly increasing in the CBDC remuneration rate. This leads to a decrease in bank profitability; therefore, the model predicts that in the long-run the number of banks active in the deposit market will fall following the introduction of CBDC. In the long run, as the banking sector becomes more concentrated, the remaining banks have more market power, which puts downward pressure on the bank deposit rate. As a consequence, the interest rate on bank deposits may not be increasing in the remuneration rate of CBDC in the long-run.

The model proposes a novel liquidity risk channel through which the introduction of a CBDC can further increase the costs of banks operating in the deposit market. As deposits are subject to liquidity risk, the model predicts that as the market share of CBDC increases, so does the size of transactions between the banking sector and the central bank. As a consequence, an increase in the CBDC remuneration rate will increase the cost banks face due to this liquidity risk. Thus, in the presence of this liquidity risk, there is additional downward pressure on the bank deposit rate, which may lead to the bank deposit rate not strictly increasing with the CBDC remuneration rate even in the short-run. These results cast doubt on the use of the CBDC remuneration rate as an additional tool for monetary policy.

This paper also highlights the importance of the liquidity risk channel for the transmission of monetary policy more generally. In the absence of liquidity risk, the model predicts that the bank deposit rate increases one-for-one following an increase in the policy rate, even after the introduction of a CBDC. However, if banks face liquidity risk in the deposit market, introducing a CBDC impacts the transmission of the policy rate and there will be imperfect pass-through to the bank deposit rate. This occurs because raising the policy rate also increases the cost banks face when obtaining additional liquidity from the central bank. Furthermore, the impact of monetary policy will now impact the structure of the deposit market and thus monetary policy will impact the deposit rate to differing degrees in the short-run and long-run.

This paper is complementary to the growing literature on the policy implications of CBDC. A large literature focuses on financial stability issues; in particular, both Böser and Gersbach (2020) and Fernández-Villaverde et al. (2021) consider the increased risk of bank runs that can occur if bank depositors had access to a CBDC so that they could transfer their deposits in times of financial stress. Both Brunnermeier and Niepelt (2019) and Niepelt (2020) discuss an equivalence result where appropriate transfers from the central bank to the financial system are capable of neutralizing the impact of introducing a CBDC and mitigate the risk of a CBDC-induced bank run. This paper also introduces liquidity risk of deposits; the focus is not on bank runs, but on the costs imposed on banks when they obtain liquidity from a central bank lending facility.

In casting doubt on the use of the CBDC remuneration rate in the monetary toolkit, this paper contributes to the literature on how CBDC should be remunerated. Agur et al. (2022) consider the welfare trade-off for the central bank when choosing a non-interest-bearing versus an interest-bearing CBDC. Barrdear and Kumhof (2022) find that a countercyclical remuneration rate rule for CBDC can contribute to stabilizing the business cycle. Similarly, Bordo (2021) finds that an interest-bearing CBDC may improve the transmission mechanism of monetary policy. On the other hand, Chiu and Davoodalhosseini (2021) find that a non-interest-bearing CBDC increases bank intermediation and thus welfare, while an interest-bearing CBDC results in bank disintermediation and lower welfare. Williamson (2022) studies various implementations of CBDC and shows how an interest-bearing CBDC can increase welfare by competing with private means of payment.

The results of this paper on the possible impact of CBDC on the transmission of monetary policy can be considered alongside a growing literature on the implications of CBDC for monetary policy. A summary of the possible monetary policy implications of CBDC can be found in Bindseil (2019). For example, Keister and Sanches (2019) suggests that, while CBDC can promote efficient exchange, it can also increase funding costs. Meaning et al. (2021) provide a detailed discussion on the monetary transmission mechanism in general, as well as other possible policy implications. Burlon et al. (2022) study the welfare implications of a CBDC and provide a characterization of the welfare-maximizing CBDC policy rules. Kumhof and Noone (2021) discuss the remuneration of CBDC in detail and its possible use for monetary policy. Kumhof and Noone (2021) propose a two-tier remuneration system, while Barrdear and Kumhof (2022) propose a quantity rule and a price rule for CBDC.

This paper is also closely related to the literature on the impact of CBDC on the banking sector. In a macroeconomic framework, Bacchetta and Perazzi (2021) assume a constant elasticity of substitution between a CBDC and a continuum of monopolistically competitive banks. Andolfatto (2021) analyzes the case of a single monopoly bank where CBDC and bank deposits are perfect substitutes, but there is a fixed cost for depositors to switch between the two. Chiu et al. (2019) study a model of Cournot oligopoly with a finite number of banks where banks compete in quantity rather than the remuneration of deposits. CBDC is assumed to be a perfect substitute for bank deposits, and so imposes a minimum remuneration rate on bank deposits.

This paper is also related to the literature on spatial models of imperfect competition, as the deposit market is based on the classic paper by Salop (1979). Spatial competition models have been widely used to study deposit markets. For example, Chiappori et al. (1995) study the regulation of deposit rates using a Salop circle model of both loans and deposit markets, while Matutes and Vives (1996) study the impact of deposit insurance in a model of spatial competition in the deposit market. Along similar lines Repullo (2004) investigates the effect of capital requirements on bank behavior when imperfect competition in the deposit market is modeled using a Salop circle. Empirical support for spatial models of the deposit market is provided by Park and Pennacchi (2008) and Ho and Ishii (2011), among others. The structure of competition after the introduction of a CBDC is closely related to Salop Circle models with a center such as Bouckaert (2000) and Madden and Pezzino (2011).

Finally, this paper is also related to the literature on interbank markets. In particular, the theoretical treatment of the interbank market in this paper is closest to that of Hauck and Neyer (2014) and Bucher et al. (2020), who both study the operation of an interbank market within the framework of the Eurozone.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 goes into further detail on the bank problem. In Section 4 the equilibrium is presented. Section 5 provides comparative statics on both the impact of the CBDC remuneration rate on the bank deposit rate and the implications for the transmission of monetary policy. Section 6 calibrates and provides a quantitative assessment of the model. Section 7 concludes.

2 Model

I consider a model of the retail deposit market with three discrete periods, $t = 1, 2, 3$. The economy consists of three types of agents; risk-neutral banks, a central bank, and a continuum of depositors.

In the first period $t = 1$, banks enter the deposit market, paying a fixed cost $F > 0$. I consider two different equilibria: In a short-run equilibrium a fixed number $N \geq 2$ of banks enter in $t = 1$. In a long-run equilibrium, the number of banks adjusts endogenously subject to a free-entry condition. This distinction allows for the study of different channels of adjustment. In the short-run equilibrium banks adjust their intensity of competition solely through the deposit rate. In the long-run equilibrium, the banking sector can adjust through changes in the deposit rate and a change in the concentration of banks in the deposit market.

Banks have access to a technology that yields an exogenously given return R_L on liquidity. Banks must obtain an exogenously given quantity of liquidity $L > 1$ to operate this technology. Banks obtain liquidity in period $t = 2$, either from the central bank or from depositors. The liquidity bank i obtains from the central bank in $t = 2$ is denoted by B_i and is remunerated at the main policy rate R_f. Bank i sets a deposit rate R_i and obtains a share q_i of the retail deposit market. Deposits are subject to aggregate liquidity risk that is realized in $t = 3$.

At the end of the final period, $t = 3$, banks must return to a liquidity neutral position. To do so, banks may borrow or lend liquidity through the interbank market or the central bank standing facilities. The central bank offers a deposit facility with an interest rate R_{DF} and a lending facility with an interest rate R_{LF}. The central bank charges penalty rates on these standing facilities such that $R_{DF} < R_f < R_{LF}$. Banks can trade liquidity between themselves in an interbank market. Trade in the interbank market takes place at a state-dependent interbank rate R_{IB}^s, where the superscript s denotes the state associated with the realized liquidity risk. The interest rates on the central bank standing facilities define a corridor that sets an upper and lower bound on the interbank rate.

The retail deposit market is modeled as a Salop circle as in Salop (1979). There is a continuum of depositors located around a circle with unit mass. The banks are located equidistant from each other around the circle. A depositor located at a distance $x \geq 0$ from the bank must pay a linear transportation cost $t_B x \geq 0$ to deposit their funds. Banks compete in prices *à la* Bertrand. The interest rate paid on deposits by bank i, denoted as R_i.

The central bank may also enter the deposit market in $t = 1$ by issuing a CBDC and setting a remuneration rate R_{CB}. The entry of CBDC into the deposit market and its remuneration rate are known to all participants in advance. All bank decisions are made in full knowledge of whether they will compete against a CBDC and are conditional on the CBDC remuneration rate R_{CB}.

Each depositor pays a fixed transport cost to obtain CBDC, with this transport cost drawn uniformly from the interval $t_{CB} \in [0, t_B]$. The structure of CBDC transport costs serves two purposes. First, it captures the idea that CBDC may have specific characteristics that differentiate it from retail deposits. Examples given in the literature include privacy concerns or preferences over additional security of deposited funds. Second, it allows competition between neighboring banks and CBDC to occur simultaneously. This would not be the case if CBDC transport costs were identical among depositors.

Deposits are subject to liquidity shocks that occur in the final period ($t = 3$). With probability λ, the banking sector sees a net outflow of deposits. A fraction $\xi \in [0, 1]$ of bank deposits relocate to other locations, evenly distributed around the circle. As CBDC depositors do not relocate, the net outflow of deposits will depend on the market share of CBDC. With probability $1 - \lambda$, the banking sector does not see a net outflow of deposits and the aggregate liquidity of the banking sector remains unchanged.

Given the structure of the liquidity shocks, the presence of CBDC introduces additional liquidity risk in the banking sector. Although these liquidity shocks are similar in spirit to those in papers such as Fernández-Villaverde et al. (2021) that focus on the possibility of CBDC generated bank runs, here there is no risk of bank runs. Instead, liquidity risk generates additional costs of deposits for banks. This cost occurs regardless of whether the banking sector has excess liquidity in aggregate or not. The presence of CBDC in conjunction with the liquidity shocks generates volatility in the aggregate liquidity of the banking sector. This in turn means that banks need to increase their use of the central bank standing facilities in at least one of the states.

To summarize the timing of the model, in the first period ($t = 1$), the central bank decides whether to introduce a CBDC and sets its remuneration rate R_{CB}. In a short-run equilibrium, a fixed number, N, banks enter in $t = 1$ while in a long-run equilibrium, N banks enter subject to a free-entry condition. In the second period ($t = 2$), commercial banks compete in the deposit market by setting a deposit rate R_i and obtaining liquidity B_i from the central bank. In the third period ($t = 3$) the liquidity shock is realized and commercial banks use the central bank standing facilities and the interbank market to obtain a liquidity neutral position. In what follows, I focus on the symmetric equilibrium and solve for the Subgame Perfect Nash Equilibrium in pure strategies using backward induction.

3 Banking Sector

3.1 Bank Liquidity

I begin the analysis of the banking sector with the final period, $t = 3$. The $N \geq 2$ banks, indexed by i, have made their decisions about their funding structure. The bank's funding structure consists of a quantity of deposits q_i and central bank liquidity B_i.

The bank's choice of liquidity B_i and deposits q_i implies that before the realization of the liquidity shocks the banks have the following *ex ante* liquidity deficit

$$\epsilon_i \equiv L - B_i - q_i. \tag{1}$$

With probability $1-\lambda$, a fraction ξ of all depositors relocate to locations evenly distributed around the Salop circle. Here banks face the same liquidity inflows as liquidity outflows and their ex post liquidity deficit is simply equal to their *ex ante* liquidity deficit

$$\epsilon_i^0 = \epsilon_i. \tag{2}$$

With probability λ, a fraction ξ of bank depositors relocate while CBDC depositors do not. Bank i receives a liquidity outflow equal to $q_i\xi$, while each bank receives an inflow of liquidity equal to $(1 - q_{CB})\, q_i\xi$. Each bank faces a net outflow of liquidity and has an *ex post* liquidity deficit equal to

$$\epsilon_i^+ = \epsilon_i + q_{CB} q_i \xi. \tag{3}$$

It is assumed that banks must return to a liquidity neutral position at the end of $t = 3$. The amount of liquidity they must trade to achieve this depends on the bank's *ex post* liquidity deficit ϵ_i^s, where $s \in \{0, +\}$ denotes the realization of the liquidity shock. If $\epsilon_i^s > 0$, bank i needs to obtain additional liquidity through the interbank market or through the central bank liquidity facility. If, on the other hand, $\epsilon_i^s < 0$ bank i must reduce its liquidity by lending in the interbank market or depositing liquidity in the central bank deposit facility.

The interest rates on the central bank standing facilities act as upper and lower bounds on the interbank rate. Whether these bounds are reached depends on the aggregate liquidity deficit of the banking system $\sum_i \epsilon_i^s$. The relationship between the interbank market and the realization of the aggregate liquidity deficit is set out in the following equation.

$$R_{IB}^s \begin{cases} = R_{LF} & \text{if } \sum_i \epsilon_i^s > 0 \\ = R_{DF} & \text{if } \sum_i \epsilon_i^s < 0 \\ \in [R_{DF}, R_{LF}] & \text{otherwise.} \end{cases} \tag{4}$$

The expected liquidity cost of deposits is simply the expected cost of returning to a liquidity neutral position

$$E[C_i] = (1 - \lambda) R_{IB}^0 \epsilon_i^0 + \lambda R_{IB}^+ \epsilon_i^+. \tag{5}$$

By combining equations (2) and (3) with equation (5), the bank's expected cost of deposits can be written as

$$E[C_i] = \epsilon_i R_f + \lambda \xi R_{IB}^+ q_{CB} q_i. \tag{6}$$

An important property of equation (6) is that if the expected liquidity shock is positive, $\lambda\xi > 0$, the expected cost of deposits is strictly increasing in the market share of the central bank. This is an important feedback mechanism of the model. An increase in the market share of CBDC increases the liquidity risk of deposits, and thus increases the expected cost of deposits for banks.

3.2 Bank's Problem

Consider the bank's problem in the intermediate period, $t = 2$, taking the number of banks N as given. In this period, the bank decides on its funding structure by obtaining liquidity from the central bank and sets the interest rate it offers to depositors R_i. Banks are risk neutral and maximize expected profits. The profit function of bank i is

$$\pi_i = \max_{B_i, R_i} \left\{ R_L L - R_f B_i - R_i q_i - E\left[C_i\right] - F \right\}, \tag{7}$$

where $E\left[C_i\right]$ is defined in equation (5) and $F > 0$ is the fixed cost that banks are assume to pay in order to enter the deposit market. Banks compete for depositors in prices à la Bertrand, taking as given both the deposit rates set by other banks and the funding structure of other banks.

If the central bank does not introduce a CBDC, competition between banks in the deposit market is identical to the Salop circle model. If bank i offers a deposit rate equal to R_i and other banks offer a deposit rate equal to R_{-i}, then a depositor located at a distance x from bank i, where $x \in \left[0, \frac{1}{N}\right]$, will choose to deposit their funds at bank i rather than the neighboring bank so long as

$$R_i - t_B x \geq R_{-i} - t_B \left(\frac{1}{N} - x \right), \tag{8}$$

where t_B is the linear transport cost that is incurred by depositors. Bank i thus faces the following demand function

$$q_i = \frac{1}{N} + \frac{1}{t_B} \left(R_i - R_{-1} \right). \tag{9}$$

If the central bank introduces a CBDC, bank i faces competition not only from the two banks that neighbor it, but also from the CBDC. I assume that the central bank sets a fixed remuneration rate R_{CB} and that depositors incur a transport cost t_{CB} if they deposit funds in the CBDC. The transport costs associated with CBDC are assumed to be drawn randomly from a uniform distribution over the interval $[0, t_B]$. Thus a depositor located at distance x from bank i would prefer to deposit funds in bank i rather than in the CBDC so long as

$$x \leq \frac{1}{t_B} \left(R_i - R_{CB} + t_{CB} \right). \tag{10}$$

Following the introduction of CBDC, for a depositor to deposit funds in bank i, they must prefer bank i to the CBDC, as well as all other banks and both equations (8) and (10) must be satisfied.

The market share of deposits obtained by bank i depends on the deposit rate that it sets, R_i, relative to the deposit rate set by the neighboring banks, R_{-i} and the CBDC remuneration rate R_{CB}. To characterize the demand function of bank i, it is helpful to define some additional variables.

First, define the distance x_i^* as the point where a depositor is indifferent between bank i and the neighboring bank $-i$. The equation for x_i^* follows from equation (8) and is given by

$$x_i^* = \frac{1}{2}\left(\frac{1}{N} + \frac{1}{t_B}\left(R_i - R_{-i}\right)\right). \tag{11}$$

Now consider a depositor with the lowest possible CBDC related cost, $t_{CB} = 0$. From equation (10) this depositor's preference for bank i's deposits depends on their location relative to bank i and the spread between bank deposits and the CBDC remuneration rate. I now introduce a variable, z_i, that will be important in describing the equilibrium with CBDC that is defined as

$$z_i \equiv x_i^* - \frac{1}{t_B}\left(R_i - R_{CB}\right). \tag{12}$$

If $z_i < 0$, the CBDC remuneration rate is sufficiently low that any depositor that prefers bank i deposits to those of another bank would also prefer bank i deposits to CBDC. Banks compete for deposits with their neighboring banks. In a symmetric equilibrium where $R_i = R_{-i}$ each bank obtains a market share of $1/N$ as in the standard Salop setup and the demand for deposits follows from (11) as

$$q_i = 2x_i^* \quad \text{ if } z_i < 0. \tag{13}$$

If $0 \leq z_i \leq x_i^*$, all depositors located at a distance $x_i^* - z_i > 0$ or closer to bank i prefer bank i deposits to CBDC. However, for depositors located at some distance $x \in [x_i^* - z_i, x_i^*]$, some depositors with a low realization of the cost of CBDC t_{CB} will prefer CBDC to bank i deposits. For these depositors, there exists a function $t_i^*(x)$ that defines the smallest value of t_{CB} that depositors located a distance x from bank i must have in order to prefer depositing in bank i rather than depositing in the CBDC. Bank i faces competition from neighboring banks and partial competition from the CBDC. The demand function facing bank i is

$$q_i = 2\left(\int_{x_i^*-z_i}^{x_i^*}\left(\frac{t_B - t^*(x)}{t_B}\right)dx + x_i^* - z_i\right) \quad \text{ if } 0 \leq z_i \leq x_i^* \tag{14}$$

To have meaningful competition between banks, some depositors located equidistant between bank i and its neighboring bank must prefer to deposit in bank i over CBDC. From equation (10) this occurs whenever

$$x_i^* \leq \frac{1}{t_B}\left(R_i - R_{CB} + t_B\right). \tag{15}$$

Combining this with the definition of z_i, equation (12), yields the condition that $z_i \leq 1$. Therefore, if $x_i^* < z_i \leq 1$, some depositors with a low realization of the CBDC cost t_{CB}

will prefer CBDC to bank i deposits regardless of the distance they are located from bank i. In this case, some fraction of all depositors located at a distance of x_i^* or closer to bank i will choose CBDC over depositing at bank i. Bank i faces competition from neighboring banks and full competition from CBDC. The demand function facing bank i is then

$$q_i = 2\left(\int_0^{x_i^*}\left(\frac{t_B - t^*(x)}{t_B}\right)dx\right) \qquad \text{if } x_i^* < z_i \leq 1 \tag{16}$$

Banks are able to attract some depositors as long as $z_i \leq 1 + x_i^*$. Therefore, if $1 < z_i \leq 1 + x_i^*$, the bank i does not compete directly with its neighboring banks and instead operates a local monopoly where it attracts a fraction of depositors located a distance z_i from it. Bank i competes only with CBDC for deposits, and only depositors who have a high realization of the CBDC cost t_{CB} will prefer deposits to CBDC. The demand function facing bank i is then

$$q_i = 2\left(\int_0^{z_i}\left(\frac{t_B - t^*(x)}{t_B}\right)dx\right) \qquad \text{if } 1 < z_i \leq 1 + x_i^* \tag{17}$$

Finally, if $z_i > 1 + x_i^*$, the CBDC remuneration rate is sufficiently high that $R_{CB} \geq R_i + t_B$ and the CBDC dominates the deposit market. Here, the CBDC remuneration rate is sufficiently high that no depositor would prefer to deposit in bank i over CBDC, and all depositors hold CBDC. Banks do not obtain a market share, and thus $q_i = 0$.

Explicitly evaluating the integrals, the demand function bank i faces can be specified in a piece-wise fashion as

$$q_i = \begin{cases} 2x_i^* & \text{if } z_i < 0 \\ 2x_i^* - z_i^2 & \text{if } 0 \leq z_i \leq x_i^* \\ 2x_i^* - x_i^*\left(2z_i - x_i^*\right) & \text{if } x_i^* < z_i \leq 1 \\ \left(1 + x_i^* - z_i\right)^2 & \text{if } 1 < z_i \leq 1 + x_i^* \\ 0 & \text{if } z_i > 1 + x_i^*. \end{cases} \tag{18}$$

As depositors are assumed not to have an outside option, there will be full coverage in the deposit market and all deposits will be deposited either at a retail bank or at the central bank. Thus the market share of CBDC can be written as

$$q_{CB} = 1 - \sum_i q_i. \tag{19}$$

3.3 Bank Entry

As discussed in Section 2, I consider both short-run and long-run equilibria. In a short-run equilibrium, the number of banks competing in the deposit market is fixed exogenously at some $N \geq 2$. In a long-run equilibrium, the number of banks is set according to a free-entry condition where banks pay a fixed entry cost $F > 0$ in $t = 1$ and enter as long as their expected profits defined by equation (7) are weakly positive. The number of banks that enter, N, then adjusts until expected profits are driven to zero.

4 Equilibrium

I focus on solving for a symmetric equilibrium in which all banks make identical decisions about their funding structure: B_i and set the same deposit rate R_i. As banks set the same deposit rate, they obtain an equal share of deposits q_i. I distinguish between a short-run equilibrium, where the number of banks N is fixed, and a long-run equilibrium where $N \geq 2$ adjusts according to a free-entry condition such that all banks make zero expected profits.

4.1 Interbank Market and Bank Funding Structure

I begin by characterizing the equilibrium funding structure of the bank chosen in $t = 2$ and the equilibrium interest rate of the interbank market in $t = 3$. In choosing their funding structure, banks take the interest rates in the interbank market, the policy rate and the interest rates on standing facilities as given. Conditional on the deposit rate they set, banks perfectly anticipate the market share of deposits they obtain. Obtaining one additional unit of liquidity from the central bank in $t = 2$ has a marginal cost of R_f, while also reducing by one unit the bank's *ex ante* liquidity deficit ϵ_i. Therefore, in equilibrium, banks adjust B_i so that the marginal cost of increasing B_i, R_i, is equal to the expected marginal cost of increasing its *ex ante* liquidity deficit ϵ_i and the following condition holds

$$\frac{\partial E\left[C_i\right]}{\partial \epsilon_i} = R_f. \tag{20}$$

In equilibrium, the interbank rates that hold in $t = 3$ must be consistent with the bank's funding decisions made in $t = 2$. Given a bank's choice of B_i and its market share q_i, a bank's *ex post* liquidity deficit ϵ_i^s is conditional on the realization of the liquidity shock $s \in \{0, +\}$. In equilibrium, the interbank rate conditional on liquidity shock s can be found from equation (4). The equilibrium interbank rate and the bank's equilibrium funding structure are summarized in Proposition 1 below.

Proposition 1. *In both a long-term and short-term equilibrium, banks obtain liquidity B_i from the central bank in $t = 2$ such that*

I. *When $\lambda \leq \left(\frac{R_f - R_{DF}}{R_{LF} - R_{DF}}\right)$, $B_i = L - q_i$. The equilibrium interbank market rates are $R^0_{IB} = R_{LF} - \left(\frac{1}{1-\lambda}\right)(R_{LF} - R_f)$ and $R^+_{IB} = R_{LF}$.*

II. *When $\lambda > \left(\frac{R_f - R_{DF}}{R_{LF} - R_{DF}}\right)$, $B_i = L - (1 - q_{CB}\xi)\, q_i$. The equilibrium interbank market rates are $R^0_{IB} = R_{DF}$ and $R^+_{IB} = R_{DF} + \frac{1}{\lambda}(R_f - R_{DF})$.*

Proof. See the Appendix. □

An implication of Proposition 1 is that interest rates in the interbank market depend on λ, the probability that the bank is hit by a net liquidity outflow. If λ is low, then banks have a neutral liquidity position if they are not hit by an outflow of liquidity, $\epsilon^0_i = 0$. However, if λ is sufficiently high, then banks have a neutral liquidity position if they are hit by an outflow of liquidity and therefore hold surplus liquidity if they are not hit by a liquidity outflow $\epsilon^+_i = 0$. As the probability of being hit by a liquidity shock increases, the incentive banks have a greater incentive to accumulate liquidity in $t = 2$, and thus the supply of liquidity in the banking sector in $t = 3$ increases. As a result, conditional interbank rates are weakly decreasing in λ.

4.2 Deposit Market Equilibrium without CBDC

I now turn to the equilibrium in the deposit market. In $t = 2$ bank i sets a deposit rate R_i that in combination with its funding decision set out in Proposition 1 maximizes its expected profit.

The equilibrium deposit rate can be found by differentiating the bank's profit function given by equation (7) with respect to the deposit rate chosen by the bank, R_i yielding

$$-q_i - \frac{\partial q_i}{\partial R_i}\left(R_i + \frac{\partial E\left[C_i\right]}{\partial q_i}\right) - \frac{\partial q_{CB}}{\partial R_i}\frac{\partial E\left[C_i\right]}{\partial q_{CB}} = 0. \tag{21}$$

First, I focus on the case where CBDC does not have a share of the deposit market, $q_{CB} = 0$. By combining equations (11) and (12), it follows that there exists a sufficiently low CBDC remuneration rate, $\underline{R}_{CB}$ such that no depositor prefers CBDC to bank deposits. As depositors are assumed to have no outside option, with $R_{CB} \leq \underline{R}_{CB}$ depositors will always choose to deposit their funds in a bank. In the symmetric equilibrium studied here, banks obtain equal market shares, and thus bank i's market share is $q_i = \frac{1}{N}$.

In the case where $R_{CB} \leq \underline{R}_{CB}$, the first-order condition for the deposit rate described in equation (21) can be simplified, leading to a closed-form solution for the deposit rate $R_i = R_f - \frac{1}{N} t_B$, as in a textbook Salop circle model.

The profit bank i makes in the case where $R_{CB} \leq \underline{R}_{CB}$ can be found by substituting the equilibrium deposit rate and market share into equation (7), yielding

$$\pi_i = \bar{\pi} - \left(R_i - R_f + \lambda \xi R_{IB}^{+} q_{CB}\right) q_i, \tag{22}$$

where

$$\bar{\pi} \equiv (R_L - R_f) L - F. \tag{23}$$

Absent CBDC, a necessary requirement for a finite number $N \geq 2$ of banks to be profitable is $-\frac{1}{4} t_B \leq \bar{\pi} < 0$. Then, in a long-run equilibrium, the number of banks adjusts subject to a free entry condition such that banks make zero profit in expectation. I assume that this parameter restriction is satisfied and thus focus on the case where a long-run equilibrium exists in the absence of CBDC.

The equilibrium without CBDC is fully characterized by Proposition 2.

Proposition 2. *If* $R_{CB} \leq \underline{R}_{CB} \equiv R_f - \frac{3}{2} t_B \frac{1}{N}$ *then:*

I. *When the number of banks is fixed at* $N \geq 2$ *there exists a unique symmetric short-run equilibrium where banks compete such that every bank* i *sets the same deposit rate* $R_i = R_f - \frac{1}{N} t_B$ *and obtains the same share of deposits* $q_i = \frac{1}{N}$*. CBDC has zero market share* $q_{CB} = 0$*.*

II. *When banks enter subject to* $\pi_i \geq 0$ *and if* $-\frac{1}{4} t_B \leq \bar{\pi} < 0$ *then there exists a unique symmetric long-run equilibrium where banks compete such that every bank* i *sets the same deposit rate* $R_i = R_f - \frac{1}{N} t_B$ *and obtains the same share of deposits* $q_i = \frac{1}{N}$*. The number of banks is* $N = t_B^{\frac{1}{2}} \left(F - (R_L - R_f) L\right)^{-\frac{1}{2}}$ *and CBDC has zero market share* $q_{CB} = 0$*.*

Proof. See the Appendix. □

With $q_{CB} = 0$, the bank's equilibrium funding decision combined with equation (6) implies that $E[C_i] = 0$ and thus the expected liquidity cost of deposits is zero. This is a direct consequence of the structure of the liquidity shock. In an economy without CBDC, banks face net inflows of liquidity that exactly offset the net outflows of liquidity regardless of the realization of the aggregate liquidity shock, s. It is optimal for banks to accumulate sufficient liquidity so that they do not need to make use of the central bank's standing facilities.

It follows from Proposition 2 that the cutoff CBDC remuneration rate $\underline{R}_{CB}$ is increasing in N. Thus, as the banking sector becomes more concentrated, the threshold remuneration rate required for CBDC to obtain a positive market share also falls. With fewer banks active in the deposit market, banks offer lower deposit rates, and therefore CBDC poses greater competitive pressure on banks at a given remuneration rate, R_{CB}.

4.3 Deposit Market Equilibrium with CBDC

Now, consider the bank's choice of deposit rate when $R_{CB} > \underline{R}_{CB}$, and thus the CBDC remuneration rate is sufficiently high that it poses meaningful competition to banks. With $q_{CB} > 0$, the market share of each bank in a symmetric equilibrium is no longer equal to $\frac{1}{N}$ and instead depends on the deposit rate offered by the banks. As a consequence, the short-run deposit rate is now determined by a system of two equations; the first-order condition for the deposit rate, equation (21), and the definition of z_i set out by equation (12). The long-run equilibrium will also require that the free-entry condition of banks holds.

To simplify the analysis, I focus on the case where the liquidity cost facing banks is not so large that banks are forced out of the deposit market almost immediately. Specifically, I assume that the following parameter restriction holds

$$\frac{1}{t_B}\lambda\xi R_{IB}^{+} \leq 1. \tag{24}$$

In a symmetric short-run equilibrium, the number of banks is fixed at N and all banks set identical deposit rates. Equation (11) simplifies to $x_i^* = \frac{1}{2N}$. If $R_{CB} > \underline{R}_{CB}$, the CBDC remuneration rate is sufficiently high that $q_{CB} > 0$. In this case, by combining equations (12) and (21) the short-run equilibrium can be found as the z_i that solves the following equation

$$\Gamma \equiv -q_i - \frac{\partial q_i}{\partial R_i}\left(R_{CB} + t_B\left(x_i^* - z_i\right) - R_f + \lambda\xi R_{IB}^{+} q_{CB}\right) - \frac{\partial q_{CB}}{\partial R_i}\lambda\xi R_{IB}^{+} q_i = 0, \tag{25}$$

where q_i and q_{CB} are functions of x_i^* and z_i given by equations (18) and (19), respectively.

The bank takes the deposit rates set by the other banks, as well as the CBDC remuneration rate, as given. It chooses its deposit rate R_i taking into account the effect that a change in the deposit rate has on both its own market share, q_i, and on the market share of CBDC, q_{CB}.

Equation (25) also depends on the impact of an increase in R_i on the market share of CBDC, holding the deposit rates of other banks fixed. This can be obtained through the definition of q_{CB} set out by equation (19). As depositors do not have an outside option, they must deposit their funds at a bank or in the CBDC. Therefore, the market share of CBDC is the mass of depositors who choose not to deposit funds at any bank. An increase in the deposit rate set by bank i, R_i, affects the market share not only of bank i, but also of neighboring banks; the impact of R_i on the market share of CBDC can be calculated from

$$\frac{\partial q_{CB}}{\partial R_i} = -\frac{\partial q_i}{\partial R_i} - \frac{\partial q_{i+1}}{\partial R_i} - \frac{\partial q_{i-1}}{\partial R_i}, \tag{26}$$

where q_{i+1} and q_{i-1} denote the market shares of neighboring banks.

The short-run equilibrium is summarized by the z_i that solves equation (25). In cases where the CBDC remuneration rate is sufficiently high that $R_{CB} > \underline{R}_{CB}$, it follows from equation (25) that $z_i > 0$ and CBDC obtains a positive share of the deposit market, $q_{CB} > 0$. There exists a threshold CBDC remuneration rate R^*_{CB}, found from equations (11) and (12), above which banks do not directly compete with each other. Instead, banks operate a local monopoly in which they compete only with the CBDC for depositors. Should the CBDC remuneration rate increase above some upper limit $\bar{R}_{CB}$, then it follows from equation (12) that banks will not operate in the deposit market and all depositors hold CBDC. These results are summarized in the following proposition.

Proposition 3. *Given that $\frac{1}{t_B}\lambda\xi R^+_{IB} \leq 1$ and the number of banks is fixed at $N \geq 2$ then if $R_{CB} > R_f - \frac{3}{2}t_B\frac{1}{N}$*

- **I.** *When $R_{CB} \leq R^*_{CB} \equiv R_f + t_B\left(1 - \frac{3}{4N}\right) - \lambda\xi R^+_{IB}\left(1 - \frac{1+N}{4N^2}\right)$ there is short-run competition between banks and some depositors hold CBDC ($q_{CB} > 0$). The market share of banks, q_i is strictly decreasing in R_{CB} while the market share of CBDC is strictly increasing in R_{CB}.*
- **II.** *When $R_{CB} > \bar{R}_{CB} \equiv R_f + t_B - \lambda\xi R^+_{IB}$ banks do not operate in the deposit market in the short-run and CBDC dominates ($q_{CB} = 1$)*
- **III.** *When $R^*_{CB} < R_{CB} \leq \bar{R}_{CB}$ banks operate local monopolies in the short-run and directly compete only with the CBDC. Some depositors hold CBDC ($q_{CB} > 0$) and the market share of banks, q_i, is strictly decreasing in R_{CB}, while the market share of CBDC is strictly increasing in R_{CB}.*

Proof. See the Appendix. □

In a short-run equilibrium, the market share of banks is strictly decreasing in R_{CB} over the interval $\left(\underline{R}_{CB}, \bar{R}_{CB}\right]$. As the number of banks in the short-run equilibrium is fixed, this also results in an increase in the market share of CBDC. A higher remuneration rate of CBDC leads to more depositors choosing CBDC over bank deposits. As banks face stiffer competition from CBDC and declining market shares, their profit also decreases.

In the case where $q_{CB} > 0$, the profit a bank makes by setting the deposit rate at the profit maximizing level can be written as a function of z_i and x_i^*. Substituting equation (12) into (22) and noting that both q_{CB} and q_i will be functions of z_i and x_i^* in equilibrium yields

$$\pi_i = \bar{\pi} - \left(R_{CB} - R_f + t_B\left(x_i^* - z_i\right) + \lambda\xi R_{IB}^{+} q_{CB}\right) q_i. \tag{27}$$

The long-run equilibrium in the deposit market can be summarized as the pair $\{x_i^*, z_i\}$ that satisfies equations (25) and (27).

I focus on the case where $-\frac{1}{4}t_B \leq \bar{\pi} < 0$ and therefore at least two banks compete in a long-run equilibrium where the CBDC remuneration rate is sufficiently low that CBDC has zero market share. By Proposition 3 if the number of banks is kept fixed, the bank profits fall. In the long-run equilibrium, an increase in the CBDC remuneration rate, R_{CB}, results in an increase in market concentration, the number of banks in the deposit market decreases, so that banks return to profitability. As the CBDC remuneration rate increases, the number of banks decreases further. At some point, it becomes unprofitable for a single banks to enter in the deposit market in the long-run equilibrium. I denote this threshold value by R_{CB}^{**}, the formal definition of which is set out in the following proposition which describes the long-run equilibrium.

Proposition 4. *Given that $\frac{1}{t_B}\lambda\xi R_{IB}^{+} \leq 1$, $-\frac{1}{4}t_B \leq \bar{\pi} < 0$ and banks enter subject to $\pi_i \geq 0$, there exists some $R_{CB}^{**} > R_f - t_B\frac{3}{2N}$ such that if $R_{CB} = R_{CB}^{**}$, a single bank ($N = 1$) is indifferent between entering in $t = 1$ or not.*

- **I.** *When $R_{CB} \leq R_{CB}^{**}$ there is long-run competition between banks and some depositors hold CBDC ($q_{CB} \geq 0$).*
- **II.** *When $R_{CB} > R_{CB}^{**}$ in the long-run equilibrium banks do not operate in the deposit market and CBDC dominates ($q_{CB} = 1$).*

Proof. See the Appendix. □

In contrast to the short-run equilibrium, a long-run equilibrium in which multiple banks operate local monopolies does not occur. From equation (6), as long as $\lambda\xi > 0$, the liquidity cost increases in the market share of CBDC, lowering bank profits. When operating a local monopoly, there exist gaps in bank coverage of the deposit market.

More precisely, there exists some location in which all depositors prefer CBDC to bank deposits. In this case, banks would strictly prefer a larger number of banks to enter in order to remove these gaps and to lower the market share of the CBDC.

Although the central bank balance sheet is not explicitly modeled, Proposition 1 highlights that in equilibrium, each bank increases its holdings of central bank liquidity (B_i) as its market share decreases. Summing over all N banks and using the definition of q_{CB} given in equation (19) yields the following equation for the aggregate liquidity borrowed from the central bank by the banking sector

$$\sum_i B_i = NL - (1 - q_{CB})\,. \tag{28}$$

Thus, as the market share of CBDC increases, the aggregate banking sector holds more central bank liquidity, and therefore the introduction of a CBDC increases both the liabilities (q_{CB}) and the assets ($\sum_i B_i$) on the central bank balance sheet.

5 Comparative Statics

5.1 Impact of CBDC on deposit rates

I now present the impact of a change in the CBDC remuneration rate, R_{CB}, on the equilibrium deposit rate R_i offered by the banks. The impact of the CBDC remuneration rate depends on whether the bank faces liquidity costs in its use of deposits. I consider the impact of a change in R_{CB} in two liquidity scenarios. First, when the expected size of the liquidity shock is zero, $\lambda\xi = 0$, and banks do not face a liquidity cost. Second, consider the case where the size of the liquidity shock is positive, $\lambda\xi > 0$, and banks face a liquidity cost to hold deposits that increases in the market share of CBDC. The modeling framework I use also allows me to distinguish between the change in R_{CB} in a short-run equilibrium, where the number of banks is held fixed, versus a long-run equilibrium, where the number of banks adjusts according to the free-entry condition. The results presented in this section are especially relevant to the policy question of whether the remuneration rate of a CBDC can be used as an additional tool in the central bank's toolbox, as has been discussed among others in Meaning et al. (2021).

In the case where $R_{CB} \leq \underline{R}_{CB}$ and $q_{CB} = 0$, CBDC has no market share and the deposit rate is given by Proposition 2. An increase in the CBDC remuneration rate will not have an impact on the bank deposit rate. This holds regardless of the value $\lambda\xi R_{IB}^{+}$ takes.

If $R_{CB} > \underline{R}_{CB}$ so that in an equilibrium CBDC has a positive market share, $q_{CB} > 0$, the impact of an increase in R_{CB} on the deposit rate in the short-run equilibrium can be found by using the implicit function Theorem and rearranging equation (12) and differentiating with respect to R_{CB} which yields

$$\frac{\partial R_i}{\partial R_{CB}} = 1 + t_B \left. \frac{dz_i}{dR_{CB}} \right|_{\Gamma=0} . \tag{29}$$

The impact of an increase in R_{CB} in a long-run equilibrium with $q_{CB} > 0$ can be calculated in a similar way by applying the Implicit Function Theorem to equations (25) and (27).

In general, the pass-through of an increase in R_{CB} to the deposit rate will be imperfect since an increase in the CBDC remuneration rate would not lead to an equal increase in the bank deposit rate, $\frac{\partial R_i}{\partial R_{CB}} < 1$. In a short-run equilibrium with $\lambda\xi = 0$, the equilibrium deposit rate will be strictly increasing in the CBDC remuneration rate, R_{CB}, for all $R_{CB} \in \left[\underline{R}_{CB}, \bar{R}_{CB}\right]$. An increase in the CBDC remuneration rate will result in banks losing market share to CBDC and banks will raise their deposit rates in response to this additional competition.

In a long-run equilibrium, as R_{CB} increases, the number of banks in the deposit market decreases. Thus, while banks face additional competition from higher CBDC remuneration rates, this is counteracted in the long-run equilibrium by a more concentrated deposit market resulting in lower competition from other banks. In a long-run equilibrium, the pass-through of the CBDC remuneration rate to the deposit rate is always lower than in the short-run equilibrium. Furthermore, in a long-run equilibrium it is possible for the market concentration effect to dominate the increased competition from CBDC, as a consequence even without liquidity risk ($\lambda\xi = 0$), the deposit rate is not guaranteed to be strictly increasing in the CBDC remuneration rate.

If the expected value of the liquidity shock is positive $\lambda\xi > 0$, then an increase in the market share of CBDC results in banks facing a higher expected liquidity cost from holding deposits. Therefore, an increase in the CBDC remuneration rate, R_{CB}, not only increases the competition faced by banks in the deposit market, but also increases the liquidity cost of deposits, making deposits a less desirable form of liquidity for banks to hold. As a consequence of this, the pass-through of the CBDC remuneration rate to the deposit rate is lower than it would be in the case without liquidity shocks $\lambda\xi = 0$. Furthermore, if deposits become less desirable for banks to hold, they may prefer to switch to other sources of liquidity rather than raise deposit rates to compete for additional market share. As a result in the presence of liquidity risk, the deposit rate is not guaranteed to increase in the CBDC remuneration rate, even in the short-run equilibrium.

These results are summarized in the following proposition.

Proposition 5. *Given that* $\frac{1}{t_B}\lambda\xi R_{IB}^{+} \leq 1$ *then*

I. *For any equilibrium that features* $q_i > 0$*, the pass-through of the CBDC rate to the deposit rate is imperfect* $\left(\frac{\partial R_i}{\partial R_{CB}} < 1\right)$.

II. *For any equilibrium, the pass-through is positive for any* R_{CB} *larger but sufficiently close to* $\underline{R}_{CB}$. $\left(\lim_{R_{CB}\downarrow \underline{R}_{CB}} \left\{\frac{\partial R_i}{\partial R_{CB}}\right\} > 0\right)$

III. *For a short-run equilibrium if* $\lambda\xi R_{IB}^{+} = 0$ *the pass-through is positive* $\left(\frac{\partial R_i}{\partial R_{CB}} > 0\right)$ *for all* $R_{CB} \in \left[\underline{R}_{CB}, \bar{R}_{CB}\right]$.

IV. *The pass-through* $\left(\frac{\partial R_i}{\partial R_{CB}}\right)$ *will be strictly lower in a long-run equilibrium than in a short-run equilibrium.*

Proof. See the Appendix. □

The results set out in Proposition 5 have important policy implications. In particular, regarding the use of the CBDC remuneration rate as an additional tool in the central bank's toolkit. Even in the most benign scenario where the number of banks is fixed and there is no liquidity, the pass-through of the CBDC remuneration rate to the bank deposit rate is imperfect. In this scenario, while banks raise their deposit rates in response to increased competition from CBDC, as a consequence of the imperfect competition in the deposit market, do so less than one-for-one. This section also highlights that if the central bank chose to use the CBDC remuneration rate as a policy tool, there may be long-run consequences on the structure of the banking sector, which would serve to dampen the pass-through to the bank deposit rate. Finally, in the case where there is risk of liquidity flowing from bank deposits to a CBDC, the additional cost this imposes on banks further weakens the pass-through of the CBDC remuneration rate to the bank deposit rate, and in some cases an increase in the CBDC remuneration rate may result in a fall in the bank deposit rate.

5.2 Implications for Monetary Policy Transmission

In this section, I consider the implications of CBDC for the transmission of monetary policy within the context of the model. To this end, I add some additional structure to the model in the following way. First, I assume that the spreads on the central bank standing facilities are held fixed and that the interest rate on the liquidity facility and on the deposit facility are of the form

$$R_{LF} = R_f + \Delta_{LF}, \tag{30}$$

and

$$R_{DF} = R_f - \Delta_{DF}, \tag{31}$$

with $\Delta_{LF} > 0$ and $\Delta_{DF} > 0$. Second, I assume that the interest rate on bank loans is equal to the policy rate plus a fixed mark-up such that

$$R_L = R_f + \Delta_L, \tag{32}$$

with $\Delta_L > 0$. Finally, I assume that the central bank sets the remuneration rate of CBDC such that there is a fixed spread between the remuneration rate and the policy rate such that

$$R_{CB} = R_f + \Delta_{CB}. \tag{33}$$

Here, Δ_{CB} could be positive or negative, depending on the remuneration rate of CBDC. It should also be noted that this is just one possible remuneration policy that central banks could choose for CBDC. However, as will be shown later, the remuneration policy considered here is the most neutral implementation of CBDC remuneration in the model. Other remuneration policies can be obtained by combining a change in the policy rate with a change in R_{CB}.

In the case where CBDC has no market share ($q_{CB} = 0$) it follows from Proposition 2 that the deposit rate increases one-for-one with the policy rate. The most interesting case occurs when CBDC has a share of the deposit market ($q_{CB} > 0$). Given the above assumptions on interest rates, the two key equations that determine the short- and long-run equilibrium, equations (25) and (27), can be rewritten as follows

$$\tilde{\Gamma} \equiv -q_i - \frac{\partial q_i}{\partial R_i}\left(\Delta_{CB} + t_B\left(x_i^* - z_i\right) + \lambda\xi\left(R_f + \Delta_{IB}^+\right) q_{CB}\right) - \frac{\partial q_{CB}}{\partial R_i}\lambda\xi\left(R_f + \Delta_{IB}^+\right) q_i = 0, \tag{34}$$

and

$$\tilde{\pi} \equiv \Delta_L L - F - \left(\Delta_{CB} + t_B\left(x_i^* - z_i\right) + \lambda\xi\left(R_f + \Delta_{IB}^+\right) q_{CB}\right) q_i = 0, \tag{35}$$

where

$$\Delta_{IB}^+ = \begin{cases} \Delta_{LF} & \text{if } R_{IB}^+ = R_{LF} \\ \left(\frac{1-\lambda}{\lambda}\right)\Delta_{DF} & \text{if } R_{IB}^+ = R_{DF} + \frac{1}{\lambda}\left(R_f - R_{DF}\right). \end{cases}$$

In the case where $q_{CB} > 0$, the CBDC remuneration rate is sufficiently high that $R_{CB} > \underline{R}_{CB}$ and the deposit rate can be written in terms of R_f as

$$R_i = t_B\left(x_i^* - z_i\right) + R_f + \Delta_{CB}. \tag{36}$$

Equation (36) shows that if the spread between the deposit rate and the CBDC rate, z_i, is held fixed then the deposit rate moves one-for-one with the policy rate.

If banks do not face liquidity risk and $\lambda\xi = 0$, then banks can pass on the increase in the policy rate to depositors without affecting their market shares. As a consequence, the two equations that describe the equilibria, equations (34) and (35), do not respond to a change in the policy rate R_f. It follows that in both a short-run and long-run equilibrium, the value of z_i is also unchanged, and so the deposit rate will increase one-for-one with the policy rate.

If, on the other hand, banks face liquidity risk and $\lambda\xi > 0$, an increase in the policy rate increases the liquidity cost of deposits that banks face. Passing on the increase in the policy rate to depositors will no longer leave bank profits unchanged. Both equation (34) and equation (35) are now affected by changes in policy rate R_f as it is no longer optimal for banks to pass on the full policy rate change to depositors. As a consequence of this, an increase in the policy rate affects the spread between the deposit rate and the CBDC remuneration rate, z_i, and thus the market share of CBDC is also impacted by the policy rate. Furthermore, since bank profit is also affected by a change in the policy rate, the pass-through of R_f to the deposit rate will be lower in the long-run equilibrium than in the short-run equilibrium.

These results are summarized in the following proposition.

Proposition 6. *Given that* $0 < \frac{1}{t_B}\lambda\xi R_{IB}^{+} \leq 1$ *then for any equilibrium that features* $q_i > 0$*, the pass-through of the policy rate to the deposit rate is imperfect* $\left(\frac{\partial R_i}{\partial R_f} \neq 1\right)$ *and the pass-through will be lower in a long-run equilibrium than in a short-run equilibrium.*

Proof. See the Appendix. □

The key mechanism driving the imperfect pass-through of the policy rate is the liquidity cost of deposits described in equation (6). If banks face liquidity risk $\lambda\xi > 0$, an increase in the policy rate also increases the cost of obtaining additional liquidity should the bank require it. This results in lower profits, and thus a more concentrated banking sector in the long-run equilibrium. As deposits become less desirable for the bank to hold, there is downward pressure on bank deposit rates, and banks require larger spreads to compensate for the additional liquidity risk.

Proposition 6 highlights a possible risk that the introduction of CBDC poses to the transmission of monetary policy to the economy. In the model, monetary policy transmission occurs solely through pass-through of the policy rate to the deposit rate set by banks. In the case without deposit liquidity risk, a CBDC can be introduced without impacting this transmission channel. However, if banks face a liquidity risk in obtaining liquidity from retail deposits, this cost will increase in the deposit rate and, in turn, will affect the transmission of monetary policy through the deposit rate. This occurs because the cost of this liquidity risk that banks face depends on the cost of obtaining additional liquidity through the central bank lending facility. The cost of obtaining this liquidity increases with the policy rate.

6 Quantitative Analysis

The previous section provides some theoretical results on the impact of a CBDC on both the deposit rate and the transmission of monetary policy. To address whether these theoretical results are quantitatively important, I present a simple calibration of the model to the Eurozone economy without CBDC.

6.1 Calibration

Data are obtained from the ECB Statistical Data Warehouse. The data obtained are averaged over the year 2021, which is the last year data are available for all of the series. The policy rates in the model are calibrated to the corresponding ECB rates. The main policy rate R_f is calibrated to the ECB's Main Refinancing Rate which was 0 throughout 2021. The interest rates on the standing facilities, R_{LF} and R_{DF} are calibrated to the ECB's Lending Facility Rate and Deposit Facility Rate which were 25 basis points and -50 basis points, respectively.

The parameters affecting the banking sector are calibrated so that in equilibrium there is no CBDC ($q_{CB} = 0$) and the free-entry condition holds. The number of banks in this equilibrium is set to $N = 7$. This is chosen to match the Herfindahl Hirschman Index (HHI) of Eurozone credit institutions which averaged 0.145 in 2021. Given that the model assumes banks of equal size, the HHI corresponds to $1/N$.

Given $q_{CB} = 0$, the equilibrium deposit rate is given by Proposition 2 as

$$R_i = R_f - \frac{1}{N} t_B. \tag{37}$$

Therefore, the transport cost t_B can be set such that the equilibrium deposit rate R_i matches the average deposit rate in the Eurozone, which was -1.44 basis points. This deposit rate is calculated as the weighted average deposit rate on overnight household deposits and overnight corporate deposits.

In an equilibrium without CBDC, the model predicts that banks hold sufficient liquidity that they do not require additional liquidity from central bank standing facilities or the interbank market. As a consequence, the following equation holds $L = B_i + q_i$. As the size of the banks is normalized by $q_i = 1/N$, L and B_i are calibrated such that the ratio of deposits to total liabilities (q_i/L) in the model matches the ratio of deposit to liabilities of Eurozone credit institutions which in the data is 0.42.

The bank lending rate R_L is chosen to match the interest rate on short-term loans to non-financial corporations, which stood at 150 basis points. It follows from the assumption that the free-entry condition binds in this calibration, which implies that a value of F can be found from the following equation

$$F = (R_L - R_f)\, L + t_B \frac{1}{N^2}. \tag{38}$$

Finally, the size of the liquidity shock ξ is set to match the percentage of total deposit liabilities that are traded daily in the Target 2 market which in 2021 was 3.13%. The calibration is summarized in Table 1.

Table 1: Calibrated Parameters

Parameter	Notation	Value	Calibration Target
Main Policy Rate	R_f	1.0	ECB Main Refinancing Rate
Lending Facility Rate	R_{LF}	1.0025	ECB Lending Facility Rate
Deposit Facility Rate	R_{DF}	0.995	ECB Deposit Facility Rate
Number of Banks	N	7	Herfindahl Hirschman Index (HHI) of Eurozone credit institutions
Bank Lending Rate	R_L	1.015	Interest rate on short-term loans to non-financial corporations
Bank Deposit Rate	R_i	0.99986	Overnight deposit rate of household and corporate deposits
Deposit to Liability Ratio	q_i/L	0.42	Ratio of Deposit to Liabilities of Eurozone Credit institutions
Size of Liquidity Shock	ξ	0.00313	Ratio of Euro short-term rate volume to Eurozone Credit institutions Deposits

6.2 Impact of the CBDC remuneration rate

Using the benchmark calibration, I now plot how the deposit rate R_i changes as R_{CB} varies in the range $\underline{R}_{CB}$ to $\bar{R}_{CB}$. First I consider the case where $\lambda = 0$ and thus the liquidity shock channel is shut down. This case is shown in Figure 1.

In Figure 1, when the CBDC remuneration rate is sufficiently low such that $q_i = 0$, $R_{CB} \leq \underline{R}_{CB}$, the deposit rate, R_i, does not respond to the CBDC remuneration rate, R_{CB}. As the market share of CBDC is zero at these low remuneration rates, CBDC does not pose meaningful competition to bank deposits. As the CBDC remuneration rate increases above $\underline{R}_{CB}$, the deposit rate, R_i, is strictly increasing in the short-run equilibrium. In the long-run equilibrium, the deposit rate, R_i, increases in the CBDC remuneration rate only for R_{CB} above but sufficiently close to $\underline{R}_{CB}$. As the CBDC remuneration rate increases further there becomes a point above which, in the long run, the deposit rate decreases in R_{CB}. This matches the results stated in Proposition 5 that the long-run response of R_i to an increase in R_{CB} will always be lower than in the short-run.

In the short-run, with the number of banks fixed, an increase in R_{CB} decreases the market share of banks, and banks raise deposit rates to mitigate the loss of market share. The fall in market share and the increase in the cost of deposits results in a fall in bank profits in the short-run. Thus, in the long run, there is a consolidation of the banking sector and a fall in the number of banks. As banks face less competition in the long-run following an increase in R_{CB}, banks can reduce deposit rates even in the face of greater competition from CBDC.

There are two kinks in the response of R_i to R_{CB} that exist in Figure 1. The first of these occurs at the point where $z_i = 0$ This is the point at which CBDC starts to put competitive pressure on bank deposits. The second kink occurs at the point where $z_i = x_i^*$. This is the point at which some proportion of depositors choose CBDC over bank deposits, no matter how far away they are located from a bank. These kinks correspond to the intervals over which the piece-wise continuous function for q_i is defined by equation (18).

Setting $\lambda = 0.05$ such that banks now face deposit liquidity risk, the response of the deposit rate, R_i, to a change in R_{CB} is plotted in Figure 2. Apart from λ, all other parameters remain the same as in Figure 1. Figure 2 illustrates the case where the expected size of the liquidity shock, $\lambda\xi$, is large enough that even in the short-run the deposit rate decreases in R_{CB} at sufficiently high levels of R_{CB}. In the previous case where $\lambda = 0$ and thus banks did not face deposit liquidity risk, an increase in R_{CB} placed

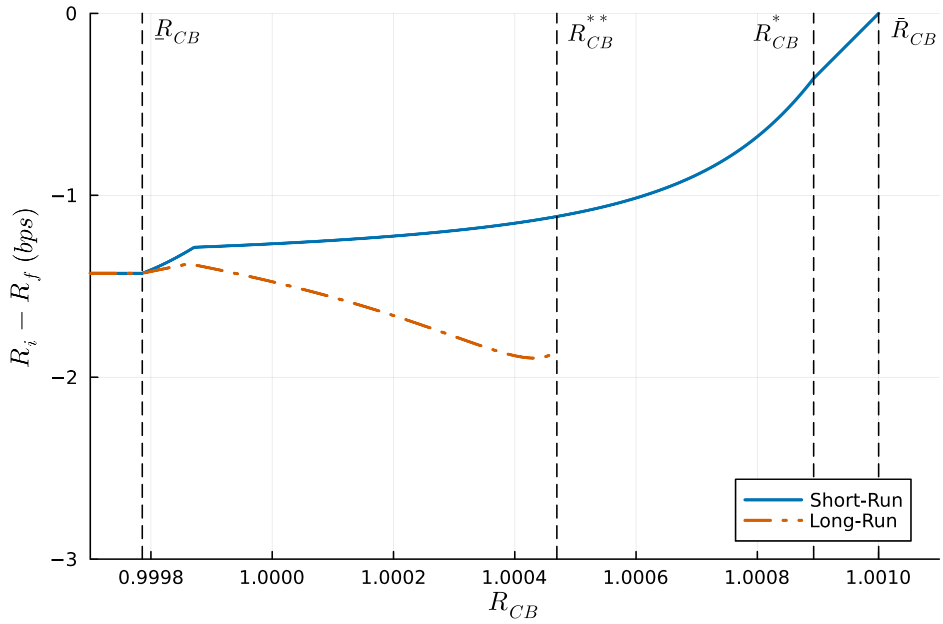

Figure 1: Impact of R_{CB} on the deposit rate ($\lambda = 0$)

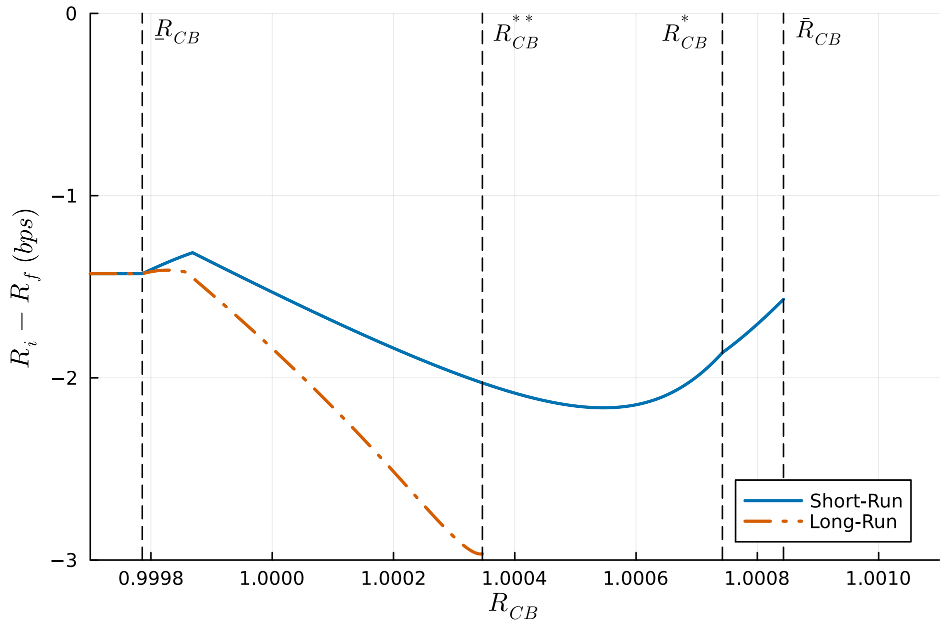

Figure 2: Impact of R_{CB} on the deposit rate ($\lambda = 0.05$)

additional competitive pressure on the banking sector in the short-run and thus forced banks to set higher deposit rates to compete for market share. However, in the case where $\lambda > 0$ and banks face liquidity risk, increasing R_{CB} has an additional effect, which is to increase the cost of deposits for banks. The expected liquidity cost of deposits increases in the market share of CBDC and decreases in the bank's own market share q_i, as evidenced by equation (5). Therefore, if the CBDC remuneration rate (R_{CB}) and the market share of CBDC (q_i) are sufficiently large banks can choose to lower their deposit rate to reduce their own market share, thus lowering the liquidity cost of deposits.

6.3 Implications for Monetary Policy Pass-through

I now plot how the pass-through of the deposit rate interacts with the market share of CBDC. As discussed in the previous section, in the case without liquidity risk, $\lambda = 0$, the policy rate has perfect pass-through to the deposit rate with $\frac{dR_i}{dR_f} = 1$. Thus, I focus only on the parameterization that features $\lambda > 0$ and where banks face liquidity risk. In the quantitative exercise, I make the same simplifying assumptions regarding the policy rate as in the previous numerical exercise.

Figure 3 illustrates the pass-through of the policy rate to the deposit rate, $\frac{dR_i}{dR_f}$, in the case where $\lambda = 0.05$. There is a non-linear response of the pass-through, $\frac{dR_i}{dR_f}$, to the increase in the market share of CBDC, q_{CB}. The figure uses the same calibration as that of Figure 2, with an increase in q_{CB} generated by a corresponding increase in R_{CB}. It should be noted that while the numerical analysis confirms the theoretical results of Proposition 6, it also suggests that, at least with the current parameterization, the magnitude of this effect may not be very large.

It is instructive to consider what generates the non-linear response of $\frac{dR_i}{dR_f}$ to the increase in q_{CB}, especially in the short-run. From equation (34) it follows that an increase in R_f would lead to a change in z_i and hence each bank's share of the deposit market. Therefore, in the presence of liquidity shocks, the increase in the policy rate will have an impact on the deposit market similar to the increase in R_{CB}. The main mechanism occurs as an increase in R_f will, if q_{CB} is sufficiently large, lead to an increase in z_i and therefore, through equation (36), to a lower increase in the bank deposit rate with $\frac{dR_i}{dR_f} < 1$. Analogously to the result stated in Proposition 5, the increase in z_i for a given increase in R_f will vary with q_{CB}, hence the non-linear response of $\frac{dR_i}{dR_f}$.

Figure 3 also shows a discontinuity in the pass-through response in the short-run equilibrium. This occurs at the point where $R_{CB} = R_{CB}^*$ and the banking sector transitions to an equilibrium without direct competition between banks. As stated in Proposition

4, this equilibrium does not occur in the long-run, hence why this discontinuity is only visible in the short-run response.

Next, I consider the reason for the low magnitude of the pass-through distortion highlighted in Figure 3. Equation (34) and equation (35) show that an important determinant of the pass-through distortion is the expected size of the liquidity shock, $\lambda\xi$. The values of λ and ξ in the benchmark calibration are quite modest and are the key to driving the low magnitude exhibited in Figure 3. To illustrate this, Figure 4 shows the response of the policy rate pass-through when the probability of liquidity shocks takes a higher value, $\lambda = 0.15$. Here, the magnitude of the effect is much stronger and the differences between the long-run and short-run responses are more stark. A similar increase in magnitude would occur if the size of the liquidity shock, ξ, were to take a higher value.

The quantitative analysis highlighted the importance of both allowing for long-run structural changes in the banking sector and the size of the liquidity risk channel. This is especially true when attempting to quantify the impact of CBDC on the transmission of monetary policy.

7 Conclusion

As the policy debate surrounding the potential introduction of a retail CBDC grows, so does the need for further analysis of its potential implications. This paper focuses on the impact of CBDC on the structure of the market for retail bank deposits and on bank liquidity. In this paper, CBDC is modeled as a source of direct competition for bank deposits. Competition in the deposit market is modeled using a Salop circle model, and thus there is imperfect substitutability between deposits of different banks and the CBDC. This framework allows us to distinguish between the short-run impact of CBDC, where the number of banks is fixed, and the long-run impact where the number of banks may adjust. Additionally, the model suggests a liquidity risk channel through which CBDC can further increase the costs of banks operating in the deposit market.

In the absence of liquidity risk, the model suggests that in a short-run equilibrium, the introduction of CBDC will result in an increase in interest rates on bank deposits. This leads to a reduction in the market shares of banks in the deposit market. Banks substitute these deposits by obtaining additional liquidity from the central bank through open market operations, and bank profitability falls. In the long run, the model suggests that the introduction of CBDC will reduce the number of banks active in the deposit market and lead to greater concentration in the banking sector. The pass-through of the CBDC remuneration rate to the bank deposit rate is lower in the long-run than in

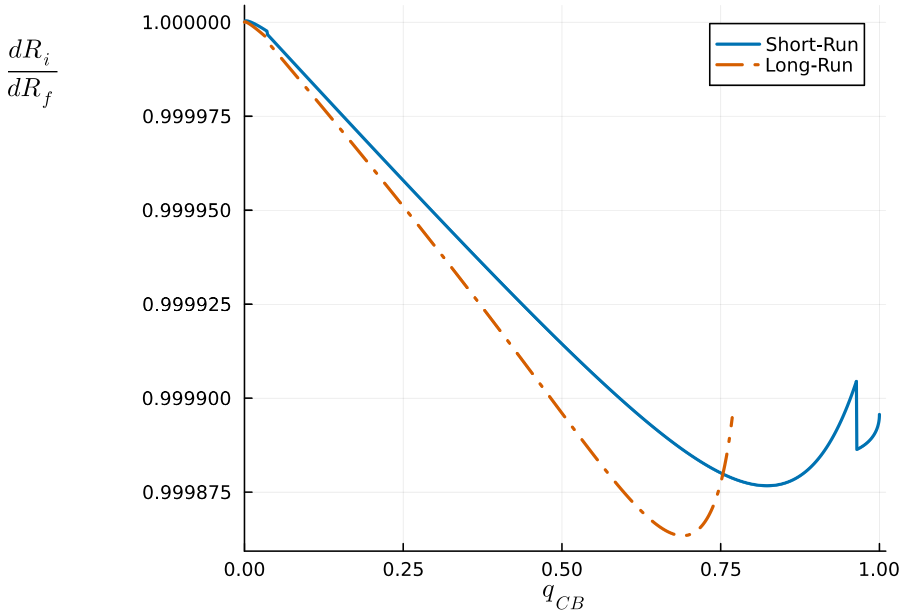

Figure 3: Impact of CBDC on Monetary Policy Transmission ($\lambda = 0.05$)

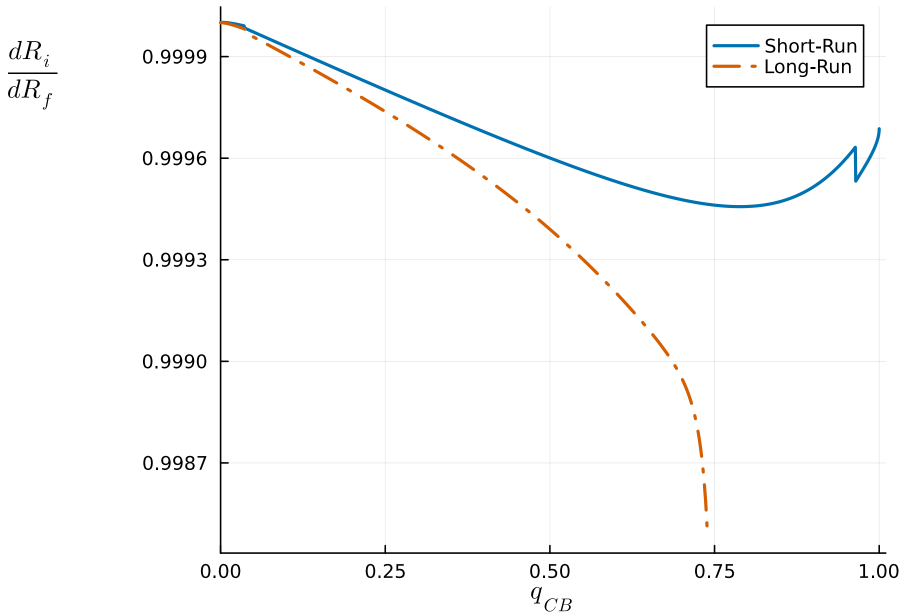

Figure 4: Impact of CBDC on Monetary Policy Transmission ($\lambda = 0.15$)

the short-run, and the deposit rate may even be decreasing in the CBDC remuneration rate. This effect is amplified if banks face liquidity risk in holding deposits, and in this case the bank deposit rate may be decreasing in the CBDC remuneration even in the short-run. Thus, the paper casts doubt on the use of the remuneration rate of CBDC as an additional tool in the monetary policy toolkit of central banks.

The paper also highlights the importance of the liquidity risk channel for monetary policy transmission in general. Absent liquidity risk, the model predicts that banks increase the deposit rate one-for-one following an increase in the policy rate, even after the introduction of a CBDC. However, if banks face liquidity risk in the deposit market, the introduction of a CBDC also affects the transmission of monetary policy through the bank deposit rate. Furthermore, the impact of monetary policy impacts the structure of the deposit market, and thus monetary policy will have a different impact in the short- and long-run.

Although this paper makes no claims regarding the welfare implications of the introduction of CBDC, it would be prudent for policymakers to take into account the welfare implications of a more concentrated banking sector that may follow the introduction of a CBDC, as well as possible implications for the transmission of monetary policy.

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Appendix

Proof of Proposition 1

By combining (5) and (7) the single bank's problem can be written as follows

$$\pi_i = \max_{B_i, R_i} \left\{ R_L L - R_f B_i - R_i q_i - (1-\lambda) R_{IB}^0 \epsilon_i^0 - \lambda R_{IB}^+ \epsilon_i^+ - F \right\} \tag{A.39}$$

where ϵ_i^0 and ϵ_i^+ are defined by equations (2) and (3) respectively.

Differentiating with respect to B_i yields the following first-order condition

$$-R_f + (1-\lambda) R_{IB}^0 + \lambda R_{IB}^+ = 0. \tag{A.40}$$

Given that $\epsilon_i^0 < \epsilon_i^+$ it follows from equation (4) and $R_f \in (R_{DF}, R_{LF})$ that $R_{IB}^0 < R_{IB}^+$. Furthermore, since obtaining too much or too little liquidity from the central bank in $t = 2$ is costly, the following inequality constraints must hold, $\epsilon_i^0 \leq 0$ and $\epsilon_i^+ \geq 0$, with one of these inequality constraints holding with equality.

Thus, there are two cases to consider. First, if $\epsilon_i^0 = 0$, then banks will have exactly enough liquidity to ensure that if they are in a neutral liquidity position if they do not receive a net liquidity outflow. In this case, $R_{IB}^+ = R_{LF}$ and $R_{IB}^0 \in [R_{DF}, R_{LF})$. From equation (A.40) it follows that the value of R_{IB}^0 that ensures the first condition holds is

$$R_{IB}^0 = R_{LF} - \left(\frac{1}{1-\lambda} \right) (R_{LF} - R_f), \tag{A.41}$$

and that for $R_{IB}^0 \geq R_{DF}$ it must be the case that

$$\lambda \leq \left(\frac{R_f - R_{DF}}{R_{LF} - R_{DF}} \right). \tag{A.42}$$

Finally, for $\epsilon_i^0 = 0$ it follows from equation (2) that

$$B_i = L - q_i. \tag{A.43}$$

The second case to consider occurs if $\epsilon_i^+ = 0$ where banks have exactly enough liquidity so that they do not require additional liquidity should they suffer a net outflow of liquidity. In this case, $R_{IB}^0 = R_{DF}$ and $R_{IB}^+ \in (R_{DF}, R_{LF})$. From equation (A.40) it follows that the value of R_{IB}^{+} that ensures the first condition holds is

$$R_{IB}^{+} = R_{DF} + \frac{1}{\lambda}\left(R_f - R_{DF}\right), \tag{A.44}$$

and for $R_{IB}^{+} < R_{LF}$ it follows that

$$\lambda > \left(\frac{R_f - R_{DF}}{R_{LF} - R_{DF}}\right). \tag{A.45}$$

Finally, for $\epsilon_i^{+} = 0$ it follows from equation (3) that

$$B_i = L - \left(1 - q_{CB}\xi\right) q_i. \tag{A.46}$$

The proposition follows.

Proof of Proposition 2

First, differentiating the bank's profit function given in equation (A.39) with respect to R_i and combining with equation (A.40) gives the following first-order condition for the bank given by equation (21).

In the case where $q_{CB} = 0$, the demand function that the bank faces is given by equation (9) and thus

$$\frac{\partial q_i}{\partial R_i} = \frac{1}{t_B}, \tag{A.47}$$

while from equation (6) if $q_{CB} = 0$ then

$$\frac{\partial E\left[C_i\right]}{\partial q_i} = -R_f, \tag{A.48}$$

and

$$\frac{\partial E\left[C_i\right]}{\partial q_{CB}} = 0. \tag{A.49}$$

In the short-run equilibrium with $q_{CB} = 0$, all banks have an equal market share and since there is full coverage, $q_i = \frac{1}{N}$. Combining the above with the first-order condition yields the following equation for the deposit rate in the short-run

$$R_i = R_f - t_B\frac{1}{N}. \tag{A.50}$$

Finally, from equation (11), the distance from bank i where a depositor is indifferent between holding a bank i deposit and the CBDC is $x_i^* = \frac{1}{2N}$. The highest possible remuneration rate in CBDC such that all depositors prefer bank deposits to CBDC, $\underline{R}_{CB}$, can be found by combining equations (12) and (A.50) and substituting $x_i^* = \frac{1}{2N}$ to produce

$$\underline{R}_{CB} = R_f - \frac{3}{2} t_B \frac{1}{N}. \tag{A.51}$$

In a long-run equilibrium, N adjusts so that banks enter and make zero expected profits. The bank's profit function is given by equation (7). Substituting the expected liquidity of the deposits given by equation (6), the deposit rate given by equation (A.50) and the optimal funding decision as defined by Proposition 1 produce the following equation for profit in the case where $q_{CB} = 0$

$$\pi_i = \left(R_L - R_f\right) L - F + t_B \frac{1}{N^2}. \tag{A.52}$$

Denote the number of firms that drive bank profit to zero as N^*. Assuming that $F - \left(R_L - R_f\right) L > 0$, a positive N^* exists and can be written as

$$N^* = t_B^{\frac{1}{2}} \left(F - \left(R_L - R_f\right) L\right)^{-\frac{1}{2}}. \tag{A.53}$$

If $F - \left(R_L - R_f\right) L \leq \frac{1}{4} t_B$ then $N^* \geq 2$ and it follows that when $R_{CB} \leq R_f - \frac{3}{2} t_B \frac{1}{N^*}$ there exists a long-run equilibrium with $q_{CB} = 0$ where banks obtain equal market shares with $q_i = \frac{1}{N^*}$ and set deposit rates as in equation (A.50).

Proof of Proposition 3

A short-run equilibrium with $R_{CB} > R_f - \frac{3}{2} t_B \frac{1}{N}$ can be summarized as z_i that solves equation (25).

From equation (18), the derivative of a bank's share of the deposit market with respect to the deposit rate is

$$\frac{\partial q_i}{\partial R_i} = \begin{cases} \frac{1}{t_B} & \text{if } z_i < 0 \\ \frac{1}{t_B}\left(1 + z_i\right) & \text{if } 0 \leq z_i \leq x_i^* \\ \frac{1}{t_B}\left(1 + 2x_i^* - z_i\right) & \text{if } x_i^* < z_i \leq 1 \\ 2\frac{1}{t_B}\left(1 + x_i^* - z_i\right) & \text{if } 1 < z_i \leq 1 + x_i^* \\ 0 & \text{if } z_i > 1 + x_i^*. \end{cases} \tag{A.54}$$

From equation (19), the derivative of the CBDC's share of the deposit market with respect to the deposit market is

$$\frac{\partial q_{CB}}{\partial R_i} = \begin{cases} 0 & \text{if } z_i < 0 \\ -2\frac{1}{t_B}\left(x_i^* - \hat{x}_i\right) & \text{if } 0 \leq z_i \leq x_i^* \\ -2\frac{1}{t_B}x_i^* & \text{if } x_i^* < z_i \leq 1 \\ -2\frac{1}{t_B}\left(1 + \hat{x}_i\right) & \text{if } 1 < z_i \leq 1 + x_i^* \\ 0 & \text{if } z_i > 1 + x_i^*. \end{cases} \tag{A.55}$$

Equation (25) can be written as

$$\Gamma = \begin{cases} R_f - R_{CB} - t_B\left(x_i^* - z_i\right) - \lambda\xi R_{IB}^+ \frac{z_i^2}{2x_i^*} & \\ -t_B\left(1 - 2z_i\frac{1}{t_B}\lambda\xi R_{IB}^+\right)\left(\frac{2x_i^* - z_i^2}{1+z_i}\right), & \text{if } 0 \leq z_i \leq x_i^* \\ R_f - R_{CB} - t_B\left(x_i^* - z_i\right) - \lambda\xi R_{IB}^+\left(z_i - \frac{1}{2}x_i^*\right) & \\ -t_B\left(1 - 2x_i^*\frac{1}{t_B}\lambda\xi R_{IB}^+\right)\left(\frac{2x_i^* - x_i^*\left(2z_i - x_i^*\right)}{1 - z_i + 2x_i^*}\right), & \text{if } x_i^* < z_i \leq 1 \\ R_f - R_{CB} - t_B\left(x_i^* - z_i\right) - \lambda\xi R_{IB}^+\left(1 - \frac{1}{2x_i^*}\left(1 + x_i^* - z_i\right)^2\right) & \\ -t_B\left(1 - 2\left(1 + x_i^* - z_i\right)\frac{1}{t_B}\lambda\xi R_{IB}^+\right)\left(1 + x_i^* - z_i\right), & \text{if } 1 < z_i \leq 1 + x_i^*. \end{cases} \tag{A.56}$$

A solution to $\Gamma = 0$ exists at the limit as $z_i \to 0$, which is equal to the equilibrium without CBDC set out by Proposition 2 where the solution to $\Gamma = 0$ occurs if $R_{CB} = \underline{R}_{CB}$ where $\underline{R}_{CB} \equiv R_f - t_B\frac{3}{4N}$.In the case where $R_{CB} < \underline{R}_{CB}$, the CBDC remuneration rate is sufficiently low that CBDC does not compete with bank deposits and the equilibrium without CBDC set out by Proposition 2 holds.

Next, consider how the function Γ changes with z_i

$$\frac{\partial \Gamma}{\partial z_i} = \begin{cases} t_B - \lambda\xi R_{IB}^+\frac{z_i}{x_i^*} + \left(\frac{2\lambda\xi R_{IB}^+\left(2x_i^* - z_i^2(2z_i+3)\right) + t_B\left(2x_i^* + 2z_i + z_i^2\right)}{(1+z_i)^2}\right) & \text{if } 0 \leq z_i \leq x_i^* \\ t_B - \lambda\xi R_{IB}^+ + 3x_i^{*2}\left(\frac{t_B - 2x_i^*\lambda\xi R_{IB}^+}{\left(1 - z_i + 2x_i^*\right)^2}\right) & \text{if } x_i^* < z_i \leq 1 \\ t_B - \lambda\xi R_{IB}^+\frac{1}{x_i^*}\left(1 + x_i^* - z_i\right) + t_B - 4\left(1 + x_i^* - z_i\right)\lambda\xi R_{IB}^+ & \text{if } 1 < z_i \leq 1 + x_i^*. \end{cases} \tag{A.57}$$

At the limit as $z_i \to 0$, $\frac{\partial \Gamma}{\partial z_i} > 0$ and by the implicit function theorem there exists a solution to $\Gamma = 0$ in the neighborhood of $z_i = 0^+$. In general, as long as $\lambda\xi R_{IB}^+ \leq t_B$, $\frac{\partial \Gamma}{\partial z_i} > 0$ for all $z_i \in [0, 1 + x_i^*]$.

Thus, if $\lambda\xi R_{IB}^+ \leq t_B$, a solution to $\Gamma = 0$ exits at the upper limit, as $z_i \to 1 + x_i^*$ and occurs if $R_{CB} = \bar{R}_{CB}$ were $\bar{R}_{CB} = R_f + t_B - \lambda\xi R_{IB}^+$. It follows that there is a solution to $\Gamma = 0$ for any $R_{CB} \in \left[\underline{R}_{CB}, \bar{R}_{CB}\right]$ and that the equilibrium z_i increases in R_{CB}. From equations (18) and (19) it follows that q_i increases in R_{CB} and q_{CB} decreases in R_{CB} over this range.

In the case where $R_{CB} > \bar{R}_{CB}$, the CBDC remuneration rate is sufficiently high that CBDC dominates bank deposits and banks obtain zero market share.

Finally, for any $z_i > 1$, banks do not compete directly in the sense that all depositors who are indifferent between holding deposits at neighboring banks strictly prefer to hold CBDC to bank deposits. In this case, banks face direct competition only from CBDC. This occurs whenever $R_{CB} > R^*_{CB}$ where R^*_{CB} can be found as the limit of equation (A.56) as $z_i \to 1$

$$R^*_{CB} = R_f + t_B\left(1 - \frac{3}{4N}\right) - \lambda\xi R^+_{IB}\left(1 - \frac{1+N}{4N^2}\right). \tag{A.58}$$

Proof of Proposition 4

A long-run equilibrium with $R_{CB} > R_f - \frac{3}{2}t_B\frac{1}{N}$ can be summarized as the tuple (x^*_i, z_i) that solves equations (25) and (27).

Equation (27) can be written explicitly in terms of x^*_i and z_i as

$$\pi = \begin{cases} \bar{\pi} - \left(2x^*_i - z_i^2\right)\left(R_{CB} - R_f\right) \\ -\left(2x^*_i - z_i^2\right)\left(t_B\left(x^*_i - z_i\right) + \lambda\xi R^+_{IB}\frac{1}{2x^*_i}z_i^2\right) & \text{if } 0 \le z_i \le x^*_i \\ \bar{\pi} - \left(2x^*_i - x^*_i\left(2z_i - x^*_i\right)\right)\left(R_{CB} - R_f\right) \\ -\left(2x^*_i - x^*_i\left(2z_i - x^*_i\right)\right)\left(t_B\left(x^*_i - z_i\right) + \lambda\xi R^+_{IB}\left(z_i - \frac{1}{2}x^*_i\right)\right) & \text{if } x^*_i < z_i \le 1 \\ \bar{\pi} - \left(1 + x^*_i - z_i\right)^2\left(R_{CB} - R_f\right) \\ -\left(1 + x^*_i - z_i\right)^2\left(t_B\left(x^*_i - z_i\right) + \lambda\xi R^+_{IB}\left(1 - \frac{1}{2x^*_i}\left(1 + x^*_i - z_i\right)^2\right)\right) & \text{if } 1 < z_i \le 1 + x^*_i. \end{cases} \tag{A.59}$$

As long as $-\frac{1}{4}t_B \le \bar{\pi} < 0$ a solution to this system of equations exists at the limit of $z_i \to 0$ where $q_{CB} = 0$. This is simply the long-run no-CBDC equilibrium set out by Proposition 2 and occurs if $R_{CB} = \underline{R}_{CB}$ where $\underline{R}_{CB}$ is defined in Proposition 2.

In the case where $R_CB < \underline{R}_{CB}$, the CBDC remuneration rate is sufficiently low that CBDC does not compete with bank deposits and the no-CBDC equilibrium set out by Proposition 2 holds.

At the limit as $z_i \to 0$, $\det\begin{pmatrix} \frac{\partial\Gamma}{\partial z_i} & \frac{\partial\Gamma}{\partial x_i^*} \\ \frac{\partial\pi}{\partial z_i} & \frac{\partial\pi}{\partial x_i^*}\end{pmatrix} > 0$ for any $N \geq 2$. From the implicit function theorem, there exists a solution to $[\Gamma, \pi] = [0, 0]$ in the neighborhood of $z_i = 0^+$. In general, for there to exist a solution to this system of equations, we require

$$\det\begin{pmatrix} \frac{\partial\Gamma}{\partial z_i} & \frac{\partial\Gamma}{\partial x_i^*} \\ \frac{\partial\pi}{\partial z_i} & \frac{\partial\pi}{\partial x_i^*}\end{pmatrix} = \frac{\partial\Gamma}{\partial z_i}\frac{\partial\pi}{\partial x_i^*} - \frac{\partial\Gamma}{\partial x_i^*}\frac{\partial\pi}{\partial z_i} > 0. \tag{A.60}$$

Consider first the following derivative.

$$\frac{\partial\Gamma}{\partial z_i}+\frac{\partial\Gamma}{\partial x_i^*} = \begin{cases} -t_B\left(\frac{2-2x_i^*-z_i^2}{(1+z_i)^2}\right) - \lambda\xi R_{IB}^+\left(2\left(\frac{2-2x_i^*-z_i^2}{(1+z_i)^2}\right) + \left(1-\frac{1}{2}\frac{z_i}{x_i^*}\right)\frac{z_i}{x_i^*}\right) & \text{if } 0 \leq z_i \leq x_i^* \\ t_B\left(1 - \frac{3x_i^{*2}+6x_i^*(1-z_i)+3(1-z_i)^2}{\left(1-z_i+2x_i^*\right)^2}\right) & \\ -\lambda\xi R_{IB}^+\left(\frac{1}{2} - \frac{2x_i^*\left(x_i^{*2}+7x_i^*(1-z_i)-4(1-z_i)^2\right)}{\left(1-z_i+2x_i^*\right)^2}\right) & \text{if } x_i^* < z_i \leq 1 \\ -\lambda\xi R_{IB}^+\left(\left(2+\frac{1}{x_i^*}\right)(1+x_i^*-z_i) + \frac{1}{2}\left(\frac{1+x_i^*-z_i}{x_i^*}\right)^2\right) & \text{if } 1 < z_i \leq 1+x_i^*. \end{cases} \tag{A.61}$$

From equation (A.61), if $\lambda R_{LF}\xi < t_B$ then for any $x_i^* < \frac{1}{2N}$ and $N \geq 2$, it follows that

$$\frac{\partial\Gamma}{\partial z_i} + \frac{\partial\Gamma}{\partial x_i^*} < 0. \tag{A.62}$$

Given this,the following two inequalities must hold

$$\det\begin{pmatrix} \frac{\partial\Gamma}{\partial z_i} & \frac{\partial\Gamma}{\partial x_i^*} \\ \frac{\partial\pi}{\partial z_i} & \frac{\partial\pi}{\partial x_i^*}\end{pmatrix} < \left(-\frac{\partial\Gamma}{\partial x_i^*}\right)\left(\frac{\partial\pi}{\partial x_i^*} + \frac{\partial\pi}{\partial z_i}\right) \tag{A.63}$$

and

$$\det\begin{pmatrix} \frac{\partial\Gamma}{\partial z_i} & \frac{\partial\Gamma}{\partial x_i^*} \\ \frac{\partial\pi}{\partial z_i} & \frac{\partial\pi}{\partial x_i^*}\end{pmatrix} > \frac{\partial\Gamma}{\partial z_i}\left(\frac{\partial\pi}{\partial x_i^*} + \frac{\partial\pi}{\partial zi}\right) \tag{A.64}$$

It follows from this that a necessary and sufficient condition for the existence of a long-run equilibrium is the following

$$\frac{\partial\pi}{\partial x_i^*} + \frac{\partial\pi}{\partial z_i} > 0 \tag{A.65}$$

where

$$\frac{\partial \pi}{\partial z_i}+\frac{\partial \pi}{\partial x_i^*}=\begin{cases} 2t_B\left(1-z_i\right)\left(\frac{2x_i^*-z_i^2}{1+z_i}\right) \\ -\lambda\xi R_{IB}^{+}\left(\frac{2x_i^*-z_i^2}{1+z_i}\right)\left(\left(1+z_i\right)\left(1-\frac{1}{2}\left(\frac{z_i}{x_i^*}\right)\right)\left(\frac{z_i}{x_i^*}\right)+4z_i\left(1-z_i\right)\right) & \text{if } 0\leq z_i\leq x_i^* \\ 2t_B\left(1-z_i\right)\left(\frac{2x_i^*-x_i^*\left(2z_i-x_i^*\right)}{1-z_i+2x_i^*}\right) \\ -\lambda\xi R_{IB}^{+}\left(\frac{2x_i^*-x_i^*\left(2z_i-x_i^*\right)}{1-z_i+2x_i^*}\right)\left(\left(\frac{1}{2}+4x_i^*\right)\left(1-z_i\right)+x_i^*\right) & \text{if } x_i^*<z_i\leq 1 \\ -\frac{1}{x_i^*}\lambda\xi R_{IB}^{+}\left(1+x_i^*-z_i\right)^3\left(1+\frac{1}{x_i^*}\left(x_i^*+z_i-1\right)\right) & \text{if } 1<z_i\leq 1+x_i^*. \end{cases} \tag{A.66}$$

From equation (A.66), the condition set out by equation (A.65) clearly holds for $0 \leq z_i \leq x_i^*$, while it fails to hold for $1 < z_i \leq 1 + x_i^*$. There exists a point $z_i^{**} \in (x_i^*, 1)$ such that at $z_i = z_i^{**}$ the determinant is zero and negative for $z_i > z_i^{**}$. From equation (A.66) z_i^{**} can be written as

$$z_i^{**} \leq \left(\frac{2-\frac{1}{t_B}\lambda R_{LF}\xi\left(\frac{1}{2}+5x_i^*\right)}{2-\frac{1}{t_B}\lambda R_{LF}\xi\left(\frac{1}{2}+4x_i^*\right)}\right). \tag{A.67}$$

By combining equation (A.67) with equation (A.56), an upper-bound in terms of the CBDC remuneration rate, R_{CB}^{**} can be found, where

$$R_{CB}^{**}=R_f-t_B\left(x_i^*-z_i^{**}\right)-\lambda\xi R_{IB}^{+}\left(z_i^{**}-\frac{1}{2}x_i^*\right)-t_B\left(1-2x_i^*\frac{1}{t_B}\lambda\xi R_{IB}^{+}\right)\left(\frac{2x_i^*-x_i^*\left(2z_i^{**}-x_i^*\right)}{1-z_i^{**}+2x_i^*}\right). \tag{A.68}$$

From this it follows that given $\frac{1}{t_B}\lambda\xi R_{IB}^{+} \leq 1$ and $-\frac{1}{4}t_B \leq \bar{\pi} < 0$ there exists a long-run equilibrium with $q_i > 0$ and $q_{CB} > 0$ for any $R_{CB} \in [\underline{R}_{CB}, R_{CB}^{**}]$. In the case where $R_{CB} > R_{CB}^{**}$, banks do not enter the deposit market and CBDC dominates the market with $q_{CB} = 1$.

Proof of Proposition 5

From equation (12), in a symmetric equilibrium,

$$R_i = R_{CB} + t_B\left(x_i^* - z_i\right). \tag{A.69}$$

Given that $\frac{1}{t_B}\lambda\xi R_{IB}^{+} \leq 1$, any equilibrium with $q_i > 0$ features $R_{CB} > \underline{R}_{CB}$. Applying the Implicit Function Theorem to equation (29) yields the following equation for the pass-through of the CBDC rate to the deposit rate in the short-run where x_i^* is fixed

$$\frac{\partial R_i}{\partial R_{CB}} = 1 + t_B\frac{\partial\Gamma/\partial R_{CB}}{\partial\Gamma/\partial z_i}. \tag{A.70}$$

From Proposition 4, $\frac{\partial\Gamma}{\partial z_i} > 0$. Differentiating equation (25) with respect to R_{CB} yields $\frac{\partial\Gamma}{\partial R_{CB}} = -1$. Therefore, it follows that given $\frac{1}{t_B}\lambda\xi R_{IB}^+ \leq 1$, whenever $q_i > 0$, $\frac{\partial R_i}{\partial R_{CB}} < 1$ and the pass-through of the CBDC rate to the deposit rate is imperfect.

Next, consider the limit of the short-run pass-through as R_{CB} approaches $\underline{R}_{CB}$ from above.

$$\lim_{R_{CB}\downarrow\underline{R}_{CB}} \left\{\frac{\partial R_i}{\partial R_{CB}}\right\} = \frac{\left(2\frac{1}{t_B}\lambda\xi R_{IB}^+ + 1\right)2x_i^*}{1+\left(2\frac{1}{t_B}\lambda\xi R_{IB}^+ + 1\right)2x_i^*} > 0 \tag{A.71}$$

Thus, the short-run pass-through is positive (but less than 1) for any x_i^* for R_{CB} above but sufficiently close to $\underline{R}_{CB}$.

In the special case where $\lambda\xi R_{IB}^+ = 0$, from equation (A.57) note that

$$\frac{\partial\Gamma}{\partial z_i} = \begin{cases} t_B\left(1+\frac{2x_i^*+2z_i+z_i^2}{(1+z_i)^2}\right) & \text{if } 0 \leq z_i \leq x_i^* \\ t_B\left(1+\frac{3x_i^{*2}}{\left(1-z_i+2x_i^*\right)^2}\right) & \text{if } x_i^* < z_i \leq 1 \\ 2t_B & \text{if } 1 < z_i \leq 1+x_i^*. \end{cases} \tag{A.72}$$

and thus $\frac{\partial\Gamma}{\partial z_i} > t_B$. From equation (A.70) it follows that in the short-run $\frac{\partial R_i}{\partial R_{CB}} > 0$.

In the long-run, x_i^* is not fixed, and applying the Implicit Function Theorem to equation (29) yields the following equation for the pass-through of the CBDC rate to the deposit rate in the long-run

$$\frac{\partial R_i}{\partial R_{CB}} = 1 + t_B\left(\frac{\partial x_i^*}{\partial R_{CB}} - \frac{\partial z_i}{\partial R_{CB}}\right) \tag{A.73}$$

where

$$\frac{\partial x_i^*}{\partial R_{CB}} - \frac{\partial z_i}{\partial R_{CB}} = \frac{\det\begin{pmatrix} \frac{\partial\Gamma}{\partial R_{CB}} & \frac{\partial\Gamma}{\partial x_i^*} \\ \frac{\partial\pi}{\partial R_{CB}} & \frac{\partial\pi}{\partial x_i^*}\end{pmatrix} - \det\begin{pmatrix} \frac{\partial\Gamma}{\partial z_i} & \frac{\partial\Gamma}{\partial R_{CB}} \\ \frac{\partial\pi}{\partial z_i} & \frac{\partial\pi}{\partial R_{CB}}\end{pmatrix}}{\det\begin{pmatrix} \frac{\partial\Gamma}{\partial z_i} & \frac{\partial\Gamma}{\partial x_i^*} \\ \frac{\partial\pi}{\partial z_i} & \frac{\partial\pi}{\partial x_i^*}\end{pmatrix}}. \tag{A.74}$$

From Proposition 4 we showed that for all $R_{CB} < R_{CB}^{**}$ we have

$$\det\begin{pmatrix} \frac{\partial\Gamma}{\partial z_i} & \frac{\partial\Gamma}{\partial x_i^*} \\ \frac{\partial\pi}{\partial z_i} & \frac{\partial\pi}{\partial x_i^*}\end{pmatrix} > 0. \tag{A.75}$$

Thus the properties of the long-run pass-through depend on

$$\det\begin{pmatrix} \frac{\partial\Gamma}{\partial R_{CB}} & \frac{\partial\Gamma}{\partial x_i^*} \\ \frac{\partial\pi}{\partial R_{CB}} & \frac{\partial\pi}{\partial x_i^*} \end{pmatrix} - \det\begin{pmatrix} \frac{\partial\Gamma}{\partial z_i} & \frac{\partial\Gamma}{\partial R_{CB}} \\ \frac{\partial\pi}{\partial z_i} & \frac{\partial\pi}{\partial R_{CB}} \end{pmatrix} = \left(\frac{\partial\pi}{\partial z_i} + \frac{\partial\pi}{\partial x_i^*}\right)\frac{\partial\Gamma}{\partial R_{CB}} - \left(\frac{\partial\Gamma}{\partial z_i} + \frac{\partial\Gamma}{\partial x_i^*}\right)\frac{\partial\pi}{\partial R_{CB}} \tag{A.76}$$

and thus

$$\frac{\partial R_i}{\partial R_{CB}} = 1 - \left(\frac{1 + q_i\left(-\frac{\frac{\partial\Gamma}{\partial z_i}+\frac{\partial\Gamma}{\partial x_i^*}}{\frac{\partial\pi}{\partial z_i}+\frac{\partial\pi}{\partial x_i^*}}\right)}{\frac{1}{t_B}\frac{\partial\Gamma}{\partial z_i} + \frac{1}{t_B}\frac{\partial\pi}{\partial z_i}\left(-\frac{\frac{\partial\Gamma}{\partial z_i}+\frac{\partial\Gamma}{\partial x_i^*}}{\frac{\partial\pi}{\partial z_i}+\frac{\partial\pi}{\partial x_i^*}}\right)}\right) < 1 \tag{A.77}$$

where we note that from Proposition 4 we found that have shown that for all $R_{CB} < R_{CB}^{**}$ we have

$$\left(\frac{\partial\pi}{\partial z_i} + \frac{\partial\pi}{\partial x_i^*}\right) > 0 \left(\frac{\partial\Gamma}{\partial z_i} + \frac{\partial\Gamma}{\partial x_i^*}\right) < 0 \tag{A.78}$$

and thus

$$\left(-\frac{\frac{\partial\Gamma}{\partial z_i} + \frac{\partial\Gamma}{\partial x_i^*}}{\frac{\partial\pi}{\partial z_i} + \frac{\partial\pi}{\partial x_i^*}}\right) > 0. \tag{A.79}$$

Comparing equation (A.70) to equation (A.77), the long-run pass-through will be strictly less than the short-run pass-through if

$$q_i\frac{\partial\Gamma}{\partial z_i} > \frac{\partial\pi}{\partial z_i} \tag{A.80}$$

This holds for all $z_i \in [0, z_i^{**}]$.

Proof of Proposition 6

Applying the Implicit Function Theorem to equation (29) yields the following equation for the pass-through of the policy rate to the deposit rate in the short-run where x_i^* is fixed is

$$\frac{\partial R_i}{\partial R_f} = 1 + t_B\frac{\partial\Gamma/\partial R_f}{\partial\Gamma/\partial z_i}. \tag{A.81}$$

From Proposition 3 as long as $\frac{1}{t_B}\lambda\xi R_{IB}^+ \leq 1$

$$\frac{\partial\Gamma}{\partial z_i} > 0 \tag{A.82}$$

and that from differentiating equation (34)

$$\frac{\partial\Gamma}{\partial R_f} = -\lambda\xi\left(\left[\frac{\partial q_i}{\partial R_i}\right]^{-1}\frac{\partial q_{CB}}{\partial R_i}q_i + q_{CB}\right) \tag{A.83}$$

which in the case where $0 < \frac{1}{t_B}\lambda\xi R^{+}_{IB}$ is non-zero and in general as $\frac{\partial R_i}{\partial R_f} \neq 1$.

In the long-run, x_i^* is not fixed, and applying the Implicit Function Theorem to equation (29) yields the following equation for the pass-through of the CBDC rate to the deposit rate in the long-run

$$\frac{\partial R_i}{\partial R_f} = 1 + t_B \left(\frac{\partial x_i^*}{\partial R_f} - \frac{\partial z_i}{\partial R_f} \right) \tag{A.84}$$

where

$$\frac{\partial x_i^*}{\partial R_f} - \frac{\partial z_i}{\partial R_f} = \frac{\det \begin{pmatrix} \frac{\partial \Gamma}{\partial R_f} & \frac{\partial \Gamma}{\partial x_i^*} \\ \frac{\partial \pi}{\partial R_f} & \frac{\partial \pi}{\partial x_i^*} \end{pmatrix} - \det \begin{pmatrix} \frac{\partial \Gamma}{\partial z_i} & \frac{\partial \Gamma}{\partial R_f} \\ \frac{\partial \pi}{\partial z_i} & \frac{\partial \pi}{\partial R_f} \end{pmatrix}}{\det \begin{pmatrix} \frac{\partial \Gamma}{\partial z_i} & \frac{\partial \Gamma}{\partial x_i^*} \\ \frac{\partial \pi}{\partial z_i} & \frac{\partial \pi}{\partial x_i^*} \end{pmatrix}} \tag{A.85}$$

From Proposition 4, given $\frac{1}{t_B}\lambda\xi R^{+}_{IB} \leq 1$, we showed that for all $R_{CB} < R^{**}_{CB}$ we have

$$\det \begin{pmatrix} \frac{\partial \Gamma}{\partial z_i} & \frac{\partial \Gamma}{\partial x_i^*} \\ \frac{\partial \pi}{\partial z_i} & \frac{\partial \pi}{\partial x_i^*} \end{pmatrix} > 0. \tag{A.86}$$

Thus the properties of the long-run pass-through depend on

$$\det \begin{pmatrix} \frac{\partial \Gamma}{\partial R_f} & \frac{\partial \Gamma}{\partial x_i^*} \\ \frac{\partial \pi}{\partial R_f} & \frac{\partial \pi}{\partial x_i^*} \end{pmatrix} - \det \begin{pmatrix} \frac{\partial \Gamma}{\partial z_i} & \frac{\partial \Gamma}{\partial R_f} \\ \frac{\partial \pi}{\partial z_i} & \frac{\partial \pi}{\partial R_f} \end{pmatrix} = \left(\frac{\partial \pi}{\partial z_i} + \frac{\partial \pi}{\partial x_i^*} \right) \frac{\partial \Gamma}{\partial R_f} - \left(\frac{\partial \Gamma}{\partial z_i} + \frac{\partial \Gamma}{\partial x_i^*} \right) \frac{\partial \pi}{\partial R_f} \tag{A.87}$$

with

$$\frac{\partial \Gamma}{\partial R_f} = \lambda\xi \left(\left[\frac{\partial q_i}{\partial R_i} \right]^{-1} \frac{\partial q_{CB}}{\partial R_i} q_B + q_{CB} \right) \tag{A.88}$$

and

$$\frac{\partial \pi}{\partial R_f} = -\lambda\xi q_{CB} q_i \tag{A.89}$$

where the derivative will again be nonzero if $\frac{1}{t_B}\lambda\xi R^{+}_{IB} > 0$.

Now note that equation (A.85) can be rewritten as

$$\frac{\partial x_i^*}{\partial R_f} - \frac{\partial z_i}{\partial R_f} = \frac{\frac{\partial \Gamma}{\partial R_f} - \left(\frac{\frac{\partial \Gamma}{\partial z_i} + \frac{\partial \Gamma}{\partial x_i^*}}{\frac{\partial \pi}{\partial z_i} + \frac{\partial \pi}{\partial x_i^*}} \right) \frac{\partial \pi}{\partial R_f}}{\frac{\partial \Gamma}{\partial z_i} - \left(\frac{\frac{\partial \Gamma}{\partial z_i} + \frac{\partial \Gamma}{\partial x_i^*}}{\frac{\partial \pi}{\partial z_i} + \frac{\partial \pi}{\partial x_i^*}} \right) \frac{\partial \pi}{\partial z_i}} \tag{A.90}$$

A sufficient condition for the long-run pass-through to be less than the long-run pass-through is that

$$\frac{\partial \pi}{\partial R_f} < q_i \frac{\partial \Gamma}{\partial R_f} \tag{A.91}$$

which holds for all values of $z_i \in [0, z_i^{**}]$.