Discussion of 'A New Model of Trend Inflation' by Joshua C.C. Chan, Gary Koop and Simon Potter

Timo Teräsvirta

CREATES, Aarhus University

at:

Seventh ECB Workshop on Forecasting Techniques

'New directions for forecasting'

Frankfurt am Main, 4–5 May 2012

1 Main contributions

- A new model for inflation which restricts trend inflation to lie within bounds.
- A computational algorithm which allows for the efficient estimation of state space models involving inequality restrictions.
- An application to the US quarterly inflation 1947(1)–2011(3), defined as

$$\pi_t = 400(\log(z_t) - \log(z_{t-1}))$$

where z_t is the quarterly US Consumer Price Index.

The approach is Bayesian.

2 The argument

’[Letting trend inflation evolve in an unbounded fashion] is inconsistent with the idea that central banks may, implicitly or explicitly, be targeting inflation and acting decisively when inflation moves outside of a desirable range.’ (p. 5)

There exist several models in which trend inflation does evolve in an unbounded fashion.

3 Main idea

Bound the behaviour of the trend inflation τ_t and the time-varying AR coefficient ρ_t in the model

$$\begin{aligned}\pi_t - \tau_t &= \rho_t(\pi_{t-1} - \tau_{t-1}) + \varepsilon_t \exp(h_t/2) \\ \tau_t &= \tau_{t-1} + \varepsilon_t^\tau \\ \rho_t &= \rho_{t-1} + \varepsilon_t^\rho \\ h_t &= h_{t-1} + \varepsilon_t^h\end{aligned}$$

by truncating the distributions of ε_t^τ and ε_t^ρ :

$$\begin{aligned}\varepsilon_t^\tau &\sim TN(a - \tau_{t-1}, b - \tau_{t-1}; 0, \sigma_\tau^2) \\ \varepsilon_t^\rho &\sim TN(a_\rho - \rho_{t-1}, b_\rho - \rho_{t-1}; 0, \sigma_\rho^2).\end{aligned}$$

In the application, a_ρ and b_ρ are chosen such that $0 < \rho_t < 1$.

How to choose the bounds?

The bounds a and b can be chosen by
★ trying various choices and looking at the results
★ estimating them.

Should they be time-varying?

CKP: No. Even in high inflation times it is likely that central bankers desire low trend inflation.

How to choose the bounds?

The bounds a and b can be chosen by
★ trying various choices and looking at the results
★ estimating them.

Should they be time-varying?

CKP: No. Even in high inflation times it is likely that central bankers desire low trend inflation.

Me (speculating): Maybe. In high inflation times (15% or so), central bankers may decide to be somewhat realistic and concede that they cannot bring the annual inflation down to 2% in the near future.

4 Results I

The application of the AR-trend-bound model (shown above) gives the following:

- The posterior mean of τ_t lies between 1.8% (1955) and 3.3% (1988).
- The 16%-tilde quantile of the distribution lies between 1.2% and 2.8%.
- The 84%-tilde quantile of the distribution lies between 2.5% and 4%.

Given the inflation series, the posteriors look quite tight and the posterior mean low overall.

But then, there is no unique definition of trend inflation. Everyone can have his or her own.

5 Results II

Furthermore,

- The posterior mean of the AR coefficient fluctuates between 0.3 (2007) and 0.9 (1973-4).
- This can be understood by writing the AR equation as follows:

$$\pi_t = \rho_t \pi_{t-1} + (1 - \rho_t)\tau_{t-1} + \{\varepsilon_t^\tau + \varepsilon_t \exp(h_t/2)\}.$$

- The 16%-tilde–84%-tilde range is widest at the end (2011), 0.12–0.45.

6 Results III

Forecasting

- The AR-trend-bound model yields the most accurate forecasts (measured in RMSFE) of the six models considered.
- The set of competitors contains a warning example (TVP-AR):

$$\begin{aligned} \pi_t &= \phi_{0t} + \phi_{1t}\pi_{t-1} + \phi_{2t}\pi_{t-2} + \varepsilon_t \\ \boldsymbol{\phi}_t &= \boldsymbol{\phi}_{t-1} + \boldsymbol{\varepsilon}_t^{\phi} \end{aligned}$$

where $\boldsymbol{\phi}_t = (\phi_{0t}, \phi_{1t}, \phi_{2t})'$, $\boldsymbol{\varepsilon}_t^{\phi} \sim$ iid$\mathcal{N}(\mathbf{0}, \mathbf{\Omega})$ with $\mathbf{\Omega} =$ diag$(\omega_0, \omega_1, \omega_2)$.

★ Was the opposition too easy to beat?
★ How would a simple benchmark $\pi_{t+h|t} = \pi_t$ do?
★ Where (which period(s)) does the AR-trend-bound model forecast best?

7 Other models I

Beechey and Österholm (2010): A ’single-equation Villani’ model

$$g(\mathsf{L})(\pi_t - \alpha - D_t) = \varepsilon_t$$

where

$g(\mathsf{L}) = 1 - \sum_{j=1}^{p} g_j \mathsf{L}^j$, the roots of this lag polynomial lie outside the unit circle,
α is the steady state of inflation
D_t represents shifts in the steady state,
$\pi_t - \alpha - D_t$ is the inflation gap $(= c_t)$.

The treatment is Bayesian.

8 Other models II

The shifting mean autoregressive (SM-AR) model (applied to modelling and forecasting inflation by González, Hubrich and Teräsvirta, 2009, 2011):

$$\pi_t = \delta(t) + \sum_{j=1}^{p} \phi_j \pi_{t-j} + \varepsilon_t \tag{1}$$

where the roots of $1 - \sum_{j=1}^{p} \phi_j \mathsf{L}^j$ lie outside the unit circle.

The shifting mean (the ’trend inflation’) equals

$$\mathsf{E}_t \pi_t = (1 - \sum_{j=1}^{p} \phi_j \mathsf{L}^j)^{-1} \delta(t).$$

Discussion of ’A New Model of Trend Inflation’

In (1),

$$\delta(t) = \delta_0 + \sum_{i=1}^{q} \delta_i G(\gamma_i, c_i, t/T)$$

where
δ_0, and $\delta_i, \gamma_i(> 0), c_i$, $i = 1, \ldots, q$, are parameters,
T is the number of observations, and
$G(\gamma_i, c_i, t/T)$, $i = 1, \ldots, q$, are logistic transition functions or sigmoids:

$$G(\gamma_i, c_i, t/T) = \big(1 + \exp\{-\gamma_i(t/T - c_i)\}\big)^{-1}.$$

An aside:

In forecasting with the shifting mean autoregressive model, the inflation target (if any) of the central bank can be taken explicitly into account by penalising the log-likelihood.

Discussion of ’A New Model of Trend Inflation’

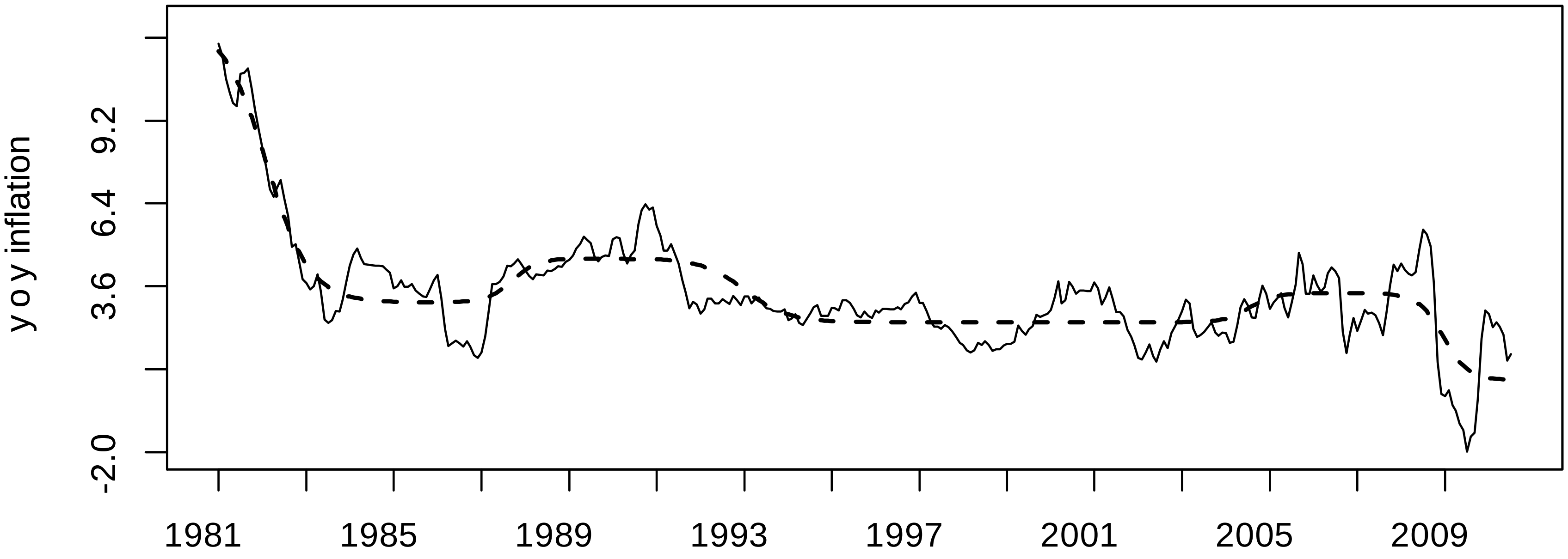

Figure: The monthly US year-on-year inflation rate 1981(1)–2010(6) (solid curve) and the shifting mean from an estimated SM-AR model (dashed curve). Source: González et al. (2011).

Discussion of ’A New Model of Trend Inflation’

Inspired by these two models: Would

$$\pi_t - \tau_t = \sum_{j=1}^{p} \rho_j(\pi_{t-j} - \tau_{t-j}) + \varepsilon_t$$

where the roots of

$$1 - \sum_{j=1}^{p} \rho_j \mathsf{L}^j = 0$$

lie outside the unit circle, be already a sufficiently flexible model?

Discussion of 'A New Model of Trend Inflation'

Inspired by these two models: Would

$$\pi_t - \tau_t = \sum_{j=1}^{p} \rho_j (\pi_{t-j} - \tau_{t-j}) + \varepsilon_t$$

where the roots of

$$1 - \sum_{j=1}^{p} \rho_j \mathsf{L}^j = 0$$

lie outside the unit circle, be already a sufficiently flexible model? Replace τ_t

- by $\mu(s_t)$, $\{s_t\}$ is a Markov chain $\Longrightarrow$ Hamilton (1989),
- by $\alpha + D_t \Longrightarrow$ Beechey and Österholm (2010),
- by $\delta(t) \Longrightarrow$ reparameterised SM-AR model ($\mathsf{E}_t \pi_t = \delta(t)$).

9 Summary of questions

- Could one after all think of time-varying bounds for τ_t? If yes, would doing so change the interpretation of ’trend inflation’ in the paper?
- Forecasting:
  - ★ Was the opposition too easy to beat?
  - ★ How would a simple benchmark $\pi_{t+h|t} = \pi_t$ do (h is the forecasting horizon)?
  - ★ Where (which period(s)) does the AR-trend-bound model forecast best?
- Could one think of adding more lags of $\pi_t - \tau_t$ to the model and thereby be able to replace ρ_t by constant AR coefficients?