# Debt Fragility and Monetary Policy

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## From Corsetti and Dedola

> *[T]he proposition [is] that countries without a printing press are subject to self-fulfilling crises in a way that nations that still have a currency of their own are not.*

P. Krugman, August 17, 2011

### Research question

Can monetary interventions eliminate expectations driven debt fragility?

### Answer

- NO if there is inflation targeting
- NO if there is discretion
- YES if there is one period commitment to 'lean against the winds'

Message: Stability depends on commitment of monetary authority.

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# I - Environment

## OLG economy

- Infinite horizon: $t = 0, 1, 2, ...$
- Two periods lived agents with preferences:

$$E\left[u(c') - g(n')\right] - g(n)$$

assume $u(c) = c$ and $g(n) = \frac{n^2}{2}$

- Linear production
  - Young: $y = z_\theta n$, where $z_\theta \in \{1, z\}$, heterogeneity
  - Old: $y = An$, where $A \sim [A_l, A_h]$, CDF $F(.)$, aggregate shock

## OLG economy

- Saving technologies
  - Money
  - Intermediated claims, participation cost $\Gamma$
    - $\longrightarrow$ Storage with real risk-free return $R$
    - $\longrightarrow$ Government one-period nominal bonds

- Policy Interventions Impact Agents
  - Labor taxes on old agents
  - Inflation tax from money printing
  - Outright default

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## Poor agents - money holder

Mass $\nu^m$ with young-age productivity $z_\theta = 1$

$$\max E\left[u(c') - g(n')\right] - g(n)$$

s.t. real budget constraints:

$$n = m$$
$$c' = An'(1-\tau) + m\tilde{\pi}$$

Notes:

- $m$ is real money holding
- $\tilde{\pi}$ is the inverse gross inflation rate
- $\tau$ is labor income tax

## Rich agents - intermediated

Mass $\nu^I$ with young-age productivity $z_\theta = z > 1$

$$\max E\left[u(c') - g(n')\right] - g(n)$$

s.t. real budget constraints:

$$zn = m + s + \Gamma$$
$$s = b^I + k$$
$$c' = An'(1-\tau) + Rk + (1+i)\tilde{\pi} b^I + m\tilde{\pi}$$

## No-arbitrage condition on nominal bond

$$\underbrace{(1+i)\tilde{\pi}^e}_{real\ return}\overbrace{(1-P(d))}^{repayment} = R \qquad \text{(NAC)}$$

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## Fiscal Policy

- Issue nominal bond $B$ to finance given real expenses $b$
- Under repayment - real budget constraint by generation

$$(1+i)\tilde{\pi}b = \tau\big(\nu^m A n_o^m(\tau) + \nu^l A n_o^l(\tau)\big) + \frac{\Delta M}{P}$$

- Resources from Seignorage $\sigma$ is money growth

$$\frac{\Delta M}{P} = \frac{\sigma}{1+\sigma}\nu^m m = (1-\tilde{\pi})\nu^m m$$

- Using policy functions:

$$\Rightarrow \quad (1+i)\tilde{\pi}b = A^2\tau(1-\tau) + (1-\tilde{\pi})\nu^m m \qquad \textbf{(GBC)}$$

where:

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## Default

$$W^r(A,i,m,\tau,\tilde{\pi}^r) = \frac{[A(1-\tau)]^2}{2} + \nu^m m\tilde{\pi}^r + ((1+i)\tilde{\pi}^r - R)\theta b + \nu^l R(Rz^2 - \Gamma). \quad (1)$$

$$W^d(A,i,m,\tilde{\pi}^d) = \frac{[A(1-\gamma)]^2}{2} + \nu^m m\tilde{\pi}^d - R\theta b + \nu^l R(Rz^2 - \Gamma) + T \quad (2)$$

- Optimal strategic default if:

$$\Delta(.) = W^d(.) - W^r(.) \geqslant 0 \qquad \textbf{(OSD)}$$

where, for $h \in \{d, r\}$

$$W^h(.) = \sum_{j \in \{m,l\}} \nu^j \left[ u(c_o^j) - g(n_o^j) \right]$$

Keys:

- $\gamma$ is default cost impacts old productivity
- A share $\theta$ is held by domestic agents
- no intergenerational effects of default

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## Some Structure

### Assumption (Costly Default)

$$\frac{A_l^2\gamma(2-\gamma)}{2} > \nu^m. \qquad \textbf{(A.2)}$$

if no tax, then no default

### Assumption (No Default Equilibrium)

$b < \bar{b}$ *where*

$$\bar{b} = \frac{A_l^2(1-\gamma)\gamma}{R}. \qquad \textbf{(A.3)}$$

# Real Debt Fragility

## Modified Environment

- non-monetary economy
- all savings through intermediary, $\Gamma = 0$
- all debt held abroad
- $n_y = R; n_0 = A(1 - \tau)$
- builds on Calvo(1988), Cooper (2012) not Cole-Kehoe

## REAL Multiplicity

- government defaults for $A < \bar{A}(i)$, with :

$$\bar{A}(i)^2 = \frac{(1+i)b}{\gamma(1-\gamma)}. \tag{3}$$

- No arbitrage

$$(1+i)\Big(1 - F\big(\bar{A}(i)\big)\Big) = R. \tag{4}$$

- Underlying Complementarity as $F(\bar{A}(i))$ increases in $i$

**Proposition**

*If government debt has value, then there are multiple interest rates that solve the no-arbitrage condition (4).*

- independent of $b > 0$

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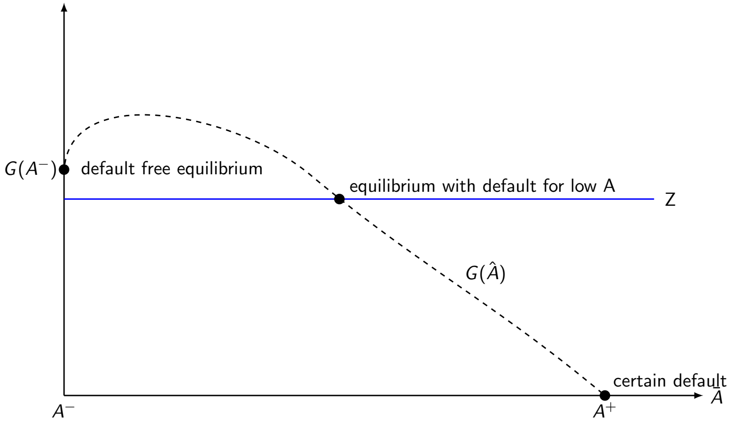
Figure: Real Multiplicity




Key: continuity; equililbrium with certain default

# Monetary policy rules and debt fragility

Question: Does monetary intervention eliminate fragility?

1) Monetary delegation
- Monetary policy independence
- Credible commitment technology
- Forms of intervention
  - Strict inflation targeting
  - state dependent interventions

2) Monetary discretion
- Fiscal and monetary choices jointly determined and discretionary

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## State variables: $\mathcal{S} = (A, i, m, s, s_{-1})$

Exogenous state variables

- $A$ - aggregate technological shock
- Sunspot variable to capture *debt fragility*

$s \in \{s^o, s^p\}$ - shocks to investors confidence

→ if $s = s^o$, debt is "risk-free"

→ if $s = s^p$, pessimistic investors coordinate on lowest price, highest risk

Predetermined state variables

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### Definition

A Stationary Rational Expectations Equilibrium (SREE) is given by:

- *The labor supply and savings decisions of private agents given state contingent monetary and fiscal policies* $(\{\tau(\mathcal{S}), \tilde{\pi}(\mathcal{S}), D(\mathcal{S})\})$, *for all* $\mathcal{S}$.
- *The government maximizes its welfare criterion by choosing a policy* $(\{\tau(\mathcal{S}), \tilde{\pi}(\mathcal{S}), D(\mathcal{S})\})$ subject to the government budget constraint, for all $\mathcal{S}$.
- *All markets clear (goods, money, bonds) for all* $\mathcal{S}$.

# II.1 - Monetary delegation

## Overview

- Inflation target:

$$\tilde{\pi}(\mathcal{S}) = \tilde{\pi}^* \qquad \forall \mathcal{S}. \tag{5}$$

- With strict inflation targeting, nominal debt is "real"

## Steps in Analysis

- critical $A$ partitions repayment and default regions
- multiple solutions to debt valuation equation
- sunspot equilibrium constructed.

### Lemma (Monotone default decision)

*Under Assumption 1, given a level of real expenses* $(1+i)\tilde{\pi}^* b$*, there is a unique* $\bar{A}(i) \in [A_l, A_h]$ *such that if* $A \leq \bar{A}(i)$*, then the treasury defaults on its debt. Otherwise it repays its debt.*

### Lemma (Multiple Solutions)

*Under Assumptions 1 and 2, for any inflation target* $0 < \tilde{\pi}^* \leq 1$*, there are multiple interest rates that solve the no-arbitrage condition,* (NAC).

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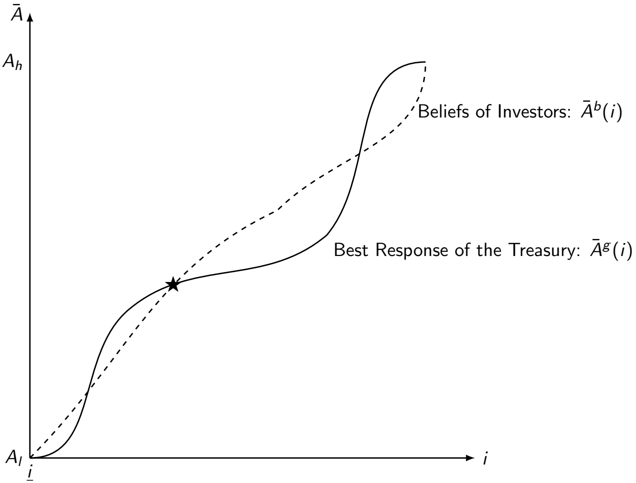
# Fragility under Inflation Target

## Proposition

*Under Assumption 1 and 2, for any* $0 < \tilde{\pi}^* \leq 1$, *there is a sunspot equilibrium with the following characteristics:*

- *If* $s_{-1} = s^o$, *the government security is risk free and the treasury reimburses with probability 1.*
- *If* $s_{-1} = s^p$, *the interest rate incorporates a risk-premium and the treasury defaults on its debt with positive probability.*

Figure: Multiplicity of Interest Rates under Monetary Delegation



## Proposition

*In the equilibrium characterized in Proposition 2, for $\tilde{\pi}^* \geq \tilde{\pi}^L$, an increase in the target inflation rate will increase seignorage and lower the probability of default if and only if the equilibrium is locally stable.*

($\tilde{\pi}^L$ is the top of the Laffer Curve)

## Discretion

1) *ex post* choice of $(\tau, \tilde{\pi}, D)$ given $\mathcal{S}$
2) maximizes welfare of home agents
3) money holdings of old are given
4) inflation ceiling: $\tilde{\pi} \geq \underline{\tilde{\pi}}$

## Choice Problem of Government

$$D \in \{r, d\} = \text{argmax}\left[\max_{\tau,\tilde{\pi}^r} W^r(A, i, m, \tau, \tilde{\pi}^r), W^d(A, i, m, \tilde{\pi}^d)\right] \tag{6}$$

subject to:

$$(1+i)\tilde{\pi}^r b = A^2(1-\tau)\tau + \nu^m m(1-\tilde{\pi}^r) \qquad \text{(if repayment)} \tag{7}$$

$$\tau \geq 0, \qquad \tilde{\pi}^d \in [\underline{\tilde{\pi}}, 1]. \tag{8}$$

## Choice

### Lemma

*Under Assumption 1, given $(A, m, i)$, the policy choices of the discretionary government are:*

- *if the government chooses to repay its debt, then*
  - $\tilde{\pi}^r = \max\{\underline{\tilde{\pi}}, \Pi(i,m)\}$, *where* $\Pi(i,m) = \frac{\nu^m m}{(1+i)b+\nu^m m}$,
  - $\tau > 0$ *and solves the government budget constraint if and only if* $\tilde{\pi}^r = \underline{\tilde{\pi}}$.
- *if the government chooses to default, then* $\tau = 0$ *and* $\tilde{\pi}^d = \underline{\tilde{\pi}}$.
- *the government chooses to default if and only if*

$$\frac{[A(1-\gamma)]^2}{2} - \frac{[A(1-\tau)]^2}{2} + \nu^m m(1-\underline{\tilde{\pi}}) - (1+i)\underline{\tilde{\pi}}\theta b > 0 \quad (9)$$

# Fragility under Discretion

## Proposition

*Under Assumptions 1 and 2, there is a SREE under discretion with the following properties:*

1. *If $s_{-1} = s^o$, government debt is risk free as the treasury reimburses with probability 1, with either:*
   a. *if $0 < b < \hat{b}$, then $\tilde{\pi}^e(s^o) > \underline{\tilde{\pi}}$ and for all $A$ all $s$, $\tilde{\pi}(A, s, \cdot) > \underline{\tilde{\pi}}$, $\tau(A, s, \cdot) = 0$, $D(A, s, \cdot) = r$,*
   b. *if $\hat{b} \leq b < \bar{b}$, then $\tilde{\pi}^e(s^o) = \underline{\tilde{\pi}}$ and for all $A$ all $s$, $\tilde{\pi}(A, s, \cdot) = \underline{\tilde{\pi}}$, $\tau(A, s, \cdot) > 0$, $D(A, s, \cdot) = r$.*
2. *If $s_{-1} = s^p$, the interest rate incorporates a risk-premium. For all $A$, $\tilde{\pi}(A, \cdot) = \underline{\tilde{\pi}}$. The treasury defaults on its debt for all $A < \bar{A}$ where $\bar{A} \in (A_l, A_h)$ and $\tilde{\pi}^e(s^p) = \underline{\tilde{\pi}}$.*

## Modeling $\tilde{\underline{\pi}}$

- triggers punishment in reputation equilibrium Chari et al.
- direct costs of inflation Aguiar, et al., Corsetti-Dedola
- sticky prices
- partial commitment
- costly redistribution
- comparative static:

**Proposition**

*In the sunspot equilibrium characterized in Proposition 4, if $\tilde{\underline{\pi}} > \tilde{\pi}^L$, then a reduction in the inflation ceiling (i.e. an increase in $\tilde{\underline{\pi}}$) will (i) increase the magnitude of taxes when $s_{-1} = s^o$ and (ii) increase the probability of default under $s_{-1} = s^p$ , if and only if the equilibrium is locally stable.*

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# Leaning Against the Winds

## The Policy

- $\tilde{\pi}(A, i, s^o) = \tilde{\pi}^*$ for all $(A, i)$
- pessimism
- $\tilde{\pi}(A, i, s^p)$ deters partial default: given $A$ and $i$
- $\int_A \tilde{\pi}(A, i, s^p) dF(A) = \tilde{\pi}^*$

## Why Leaning?

- $A$
  - for low $A$, use inflation tax
  - for high $A$, use fiscal policy
- debt burden
  - for high $i$, allow default for all $A$
  - for low $i$, repay for all $A$

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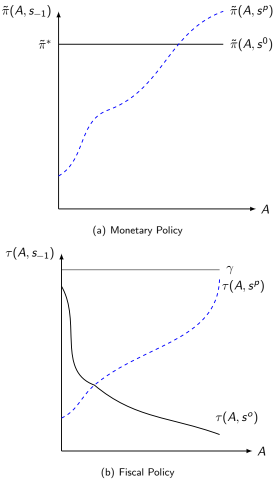
Figure: State Dependent Monetary and Fiscal Policy




The left panel represents the state dependent monetary policy to which the central bank commits. The right panel represents the induced fiscal policy. The dependence of the policies on the sunspot and realized productivity are displayed.

**Proposition**

*Under Assumptions 2 and 3, when the monetary authority commits to $\tilde{\pi}(A, i, s_{-1})$ debt is uniquely valued and risk-free. Debt fragility is eliminated.*

## Conclusions

- interventionist monetary policy can eliminate debt fragility
- inflation target will not
- discretion creates some revenue but does not generally eliminate fragility

## To Ponder

- European Union and bailout
- adding more or modifying asset structure
- reputation effects
- costly inflation
- partial commitment
- sticky prices